| **Question** | **Scheme** | | | **Marks** |
| --- | --- | --- | --- | --- |
| **1(a)** | If *n* = 1,  so **true** for ***n* = 1** | | | B1 |
| Assume result true for *n* = *k* | | |  |
| or | | | M1 |
| or | | | M1 A1 |
|  | | | A1 |
| **True** for ***n* = *k* + 1** if **true** for ***n* = *k***, (and **true** for ***n* = 1**) so **true** by induction for all | | | A1cso |
|  |  | | | **(6)** |
| **1(b)** | If *n* =1, , so **true** for ***n* = 1**. | | | B1 |
| Assume result true for *n* = *k* so | | | M1 |
|  | | | M1 A1 |
| or | | |  |
| = | | | dA1 |
| **True** for ***n* = *k* + 1** if **true** for ***n* = *k***, ( and **true** for ***n* = 1**) so **true** by induction forall | | | A1cso |
|  |  | | | **(6)** |
|  |  | | | **(12 marks)** |
| **2(i)** |  | | |  |
|  | | | B1 |
| So is true when  and | | |  |
| Assume that  and | | |  |
|  | | |  |
|  | | | M1A1 |
|  | | | M1 |
|  | | |  |
|  | | |  |
|  | | | A1 |
| If the result is **true for** ***n* = *k* and *n* = *k*+1** then it is now **true for** ***n* = *k* + 2**. As it is **true for *n* = 1 and *n* = 2** then it is **true** **for all *n***∈ ℤ+ | | | A1 cso |
|  |  | | | **(6)** |
| **2(ii)** | is divisible by 19 | | |  |
| {which is divisible by 19}. | | | B1 |
| { is divisible by 19 when } | | |  |
| {Assume that for | | |  |
| is divisible by 19 for *k* ∈ ℤ+} | | |  |
|  | | | M1 |
|  | | |  |
|  | | |  |
|  | | | A1  A1 |
|  | | |  |
|  | | | dM1 |
| { is divisible by 19 as both  and  are both divisible by 19} | | |  |
| If the result is **true for** ***n* = *k***, then it is now **true for** ***n* = *k* + 1**. As the result has shown to be **true for** ***n* = 1**,then the result is **true** **for all *n*** ∈ ℤ+ | | | A1 cso |
|  |  | | | **(6)** |
|  |  | | | **(12 marks)** |
| **3** | is divisible by 6. | | |  |
|  | | | B1 |
| Assume that for   is divisible by 6. | | |  |
|  | | | M1 |
|  | | |  |
|  | | | M1A1 |
|  | | | A1 |
| If the result is **true for** thenit is now **true for** As the result has been shown to be **true for** then the result is **true for all *n*** ∈ ℤ+ | | | A1cso |
|  |  | | | **(6marks)** |
| **4(a)** |  | | |  |
|  | | |  |
| As the matrix result is true for | | Check to see that the result is true for | B1 |
| Assume that the matrix equation is true for | | |  |
|  | | |  |
| With  the matrix equation becomes | | |  |
|  | |  |  |
| or | | by | M1 |
| or | | Correct unsimplified matrix with no errors seen. | A1 |
|  | | |  |
|  | | Manipulates so that  on at least one term. | dM1 |
| Correct result with no errors seen with some working between this and the previous A1 | A1 |
| If the result is true for  then it is now true for  (2)  As the result has shown to be true for then the result is true for all *n*. (4) | | Correct conclusion with all previous marks earned | A1 cso |
|  |  | |  | **(6)** |
| **4(b)** |  | | Shows that | B1 |
| {which is divisible by 12}.  { is divisible by 12 when } | |  |  |
| Assume that for | |  |  |
| is divisible by 12 for | |  |  |
| So, | | Correct unsimplified expression for | B1 |
| giving, | |  |  |
|  | | Applies  No simplification is necessary and condone missing brackets. | M1 |
|  | |  |  |
|  | | Attempting to isolate | M1 |
|  | |  | A1cso |
| , which is divisible by 12 as both  and  are both divisible by 12.(1)  If the result is true for  (2) then it is now true for  (3)  As the result has shown to be true for then the result is true for all *n*. (5). | | Correct conclusion with no incorrect work. Don’t condone missing brackets. | A1 cso |
|  |  | | | **(6)** |
|  |  | | | **(12 marks)** |
| **5** | and | | |  |
| So is true when | Check that  yields 2 when | | B1 |
| Assume that for that,  is true for |  | |  |
|  |  | |  |
|  | Substituting  into | | M1 |
|  | An attempt to multiply out the brackets by 4 or | | M1 |
|  |  | |  |
|  |  | |  |
|  |  | | A1 |
| Therefore, the general statement,  is true when  (As  is true for ) then  is true for all positive integers by mathematical induction | Require ‘True when n=1’, ‘Assume true when *n*=*k*’ and ‘True when ’ then true for all *n* o.e. | | A1 |
|  |  | | | **(5 marks)** |
| **6** | is divisible by 5. | | |  |
|  | | | B1 |
| Assume that for  is divisible by 5 for | | |  |
|  | | | M1A1 |
|  | | |  |
|  | | |  |
|  | | | M1 |
|  | | |  |
|  | | |  |
|  | | |  |
|  | | |  |
|  | | | A1 |
| If the result is true for  then it is now true for  As the result has shown to be true for then the result is true for all *n*. | | | A1 cso |
|  |  | | | **(6 marks)** |

|  |  |  |  |  |  |
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|  | **Source paper** | **Question number** | **New spec references** | **Question description** | **New AOs** |
| 1 | FP1 2015 | 6 |  | Proof - induction, Matrices | 1.1b, 2.1 |
| 2 | FP1 2017 | 9 |  | Proof | 1.1b, 2.1 |
| 3 | FP1 2014 | 9 |  | Proof | 1.1b, 2.1 |
| 4 | FP1 2011 | 9 |  | Proof | 1.1b, 2.1 |
| 5 | FP1 2011 | 9 |  | Proof | 1.1b, 2.1 |
| 6 | FP1 2012 | 10 |  | Proof | 1.1b, 2.1 |