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| Q | Scheme | Marks | AOs | Pearson Progression Step and Progress Descriptor |
| **1a** | The colonies occur at random | **B1** | 3.5b | 3rd  Understand the conditions for a Poisson distribution |
| The colonies occur at a constant rate | **B1** | 3.5b |
|  | **(2)** |  |  |
| **1b** | *X* ~ Po(4) | **B1** | 3.3 | 3rd  Use the Poisson distribution to model real-world situations |
| P(*X* = 0) | **M1** | 1.1b |
| = 0.0183 | **A1** | 1.1b |
|  | **(3)** |  |  |
| **1c** | *X* ~ Po(6) | **B1** | 3.3 | 3rd  Use the Poisson distribution to model real-world situations |
| 1 − P(*X* ⩽ 3) = 1 – 0.1512 | **M1** | 1.1b |
| = 0.8488 | **A1** | 1.1b |
|  | **(3)** |  |  |
| **(8 marks)** | | | | |
| **Notes**  **1a** Accept ‘colonies appear independently of one another’ | | | | |

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| **Q** | **Scheme** | **Marks** | **AOs** | **Pearson Progression Step and Progress Descriptor** |
| **2a** | 1.04 | **B1** | 1.1b | 2nd  Understand the basics of the Poisson distribution |
|  | **(1)** |  |  |
| **2b** | *X* ~ Po(1.04) | **B1** | 3.3 | 3rd  Use the Poisson distribution to model real-world situations |
| |  |  |  | | --- | --- | --- | | ***x*** | **P(*X* = *x*)** | **Expected frequency** | | 0 | 0.3535 | 53.03 | | 1 | 0.3676 | 55.14 | | 2 | 0.1911 | 28.67 | | 3 | 0.0663 | 9.95 | | 4 | 0.0172 | 2.58 | | **M1**  **M1**  **A1** | 1.1b |
|  | **(4)** |  |  |
| **2c** | The Poisson model seems a (fairly) good fit | **B1** | 2.2b | 4th  Comment on the appropriateness of the Poisson distribution |
| because the expected frequencies are similar to the observed frequencies, oe | **B1** | 2.4 |
|  | **(2)** |  |  |
| **(7 marks)** | | | | |
| **Notes**  **2b** **M1:** (first) at least three correct probabilities  **M1:** (second) at least three correct expected frequencies  **A1:** for all correct frequencies | | | | |

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| **Q** | Scheme | Marks | AOs | Pearson Progression Step and Progress Descriptor |
| **3a** | *X* ~ Po(3) | **B1** | 3.3 | 3rd  Use the Poisson distribution to model real-world situations |
| P(*X* > 4) = 1 − P(*X* ⩽ 4) = 1 − 0.8153 | **M1** | 1.1b |
| = 0.1847 | **A1** | 1.1b |
|  | **(3)** |  |  |
| **3b** | Expectation = 3, Variance = 3 (need both) | **B1** | 1.1a | 4th  Calculate the mean of a Poisson distribution |
|  | **(1)** |  |  |
| **3c** | P(*X* ⩽ *x*) ⩾ 0.95  P(*X* ⩽ 5) = 0.9161  P(*X* ⩽ 6) = 0.9665 | **M1**  **A1** | 1.1b  1.1b | 3rd  Use the Poisson distribution to model real-world situations |
| The newsagent should order six copies in order to meet the demand with a probability of at least 0.95 | **A1** | 2.4 |
|  | **(3)** |  |  |
| **(7 marks)** | | | | |
| **Notes** | | | | |

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| Q | Scheme | Marks | AOs | Pearson Progression Step and Progress Descriptor |
| **4a** | Expectation = 150 × 0.015 = 2.25 | **B1** | 1.1b | 4th  Calculate the variance of a binomial distribution |
| Variance = 150 × 0.015 × 0.985 = 2.21625 | **B1** | 1.1b |
|  | **(2)** |  |  |
| **4b** | Expectation ≈ Variance | **B1** | 2.4 | 6th  Understand when to use a Poisson distribution as an approximation to the binomial distribution |
|  | **(1)** |  |  |
| **4c** | *X* ~ Po(2.25) | **B1** | 3.3 | 6th  Use the Poisson distribution as an approximation to the binomial distribution |
| P(*X* > 5) = 1 – P(*X* ⩽ 5) | **M1** | 1.1b |
| = 0.0274 | **A1** | 1.1b |
|  | **(3)** |  |  |
| (6 marks) | | | | |
| Notes | | | | |

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| **Q** | **Scheme** | **Marks** | **AOs** | **Pearson Progression Step and Progress Descriptor** |
| **5a** | *X* ~ Po(3.6) | **B1** | 3.3 | 6th  Understand when to use a Poisson distribution as an approximation to the binomial distribution |
| *p* is small and *n* is large (*np* < 10) | **B1** | 2.4 |
|  | **(2)** |  |  |
| **5b** | P(*X* ⩽ 3) | **M1** | 1.1b | 6th  Use the Poisson distribution as an approximation to the binomial distribution |
| = 0.5152 | **A1** | 1.1b |
|  | **(2)** |  |  |
| **5c** | *Y* ~ B(10, 0.5152) | **B1** | 3.3 | 5th  Solve problems involving the mean and variance of a binomial distribution |
| Mean = *np* = 5.152 | **B1** | 1.1b |
| Variance = *npq* = 2.4976896… | **M1** | 1.1b |
| Standard deviation =  = 1.5804… = 1.58 (3 s.f.) | **A1** | 1.1b |
|  | **(4)** |  |  |
| **(8 marks)** | | | | |
| **Notes**  **5a** Accept ‘Poisson’ without stated *λ* if correct value used in **b** | | | | |

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| **Q** | **Scheme** | **Marks** | **AOs** | **Pearson Progression Step and Progress Descriptor** |
| **6a** | H0: *λ* = 1.5; H1: *λ* < 1.5 | **B1** | 2.5 | 3rd  Understand the language of hypothesis testing |
|  | **(1)** |  |  |
| **6b** | *X* ~ Po(9) | **B1** | 3.3 | 4th  Carry out one-tailed tests for the mean of a Poisson distribution |
| P(*X* ⩽ 3) = 0.0212  P(*X* ⩽ 4) = 0.0550 | **M1** | 1.1b |
| Hence critical region is 3 or fewer | **A1** | 1.1b |
|  | **(3)** |  |  |
| **6c** | 0.0212 | **B1** | 1.1a | 3rd  Understand the language of hypothesis testing |
|  | **(1)** |  |  |
| **6d** | 4 is not in the critical region … | **B1** | 2.2b | 4th  Carry out one-tailed tests for the mean of a Poisson distribution |
| … so accept the null hypothesis: there is no evidence of a reduction in the (average) number of errors. | **B1** | 2.2b |
|  |  | **(2)** |  |  |
| **(7 marks)** | | | | |
| **Notes**  **6a** Accept hypotheses framed with *λ* = 9 | | | | |

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| **Q** | **Scheme** | **Marks** | **AOs** | **Pearson Progression Step and Progress Descriptor** |
| **7a** | *X* ~ Po(4.7) | **B1** | 3.3 | 4th  Understand the additive property of the Poisson distribution |
| P(*X* ⩾ 9) = 1 − P(*X* ⩽ 8) = 1 – 0.9497 | **M1** | 1.1b |
| = 0.0503 | **A1** | 1.1b |
|  | **(3)** |  |  |
| **7b** | H0: *λ* = 4.7; H1: *λ* ≠ 4.7 | **B1** | 2.5 | 5th  Carry out two-tailed tests for the mean of a Poisson distribution |
| 0.0503 > 0.05 | **M1** | 1.1b |
| Therefore, accept null hypothesis | **A1** | 2.2b |
| There is no evidence that the number of cars and vans passing the recording point has changed | **A1** | 2.2b |
|  | **(4)** |  |  |
| **(7 marks)** | | | | |
| **Notes**  **7b** Accept ft their part **a** if conclusions are consistent and in context | | | | |