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| Q | Scheme | Marks | AOs | Pearson Progression Step and Progress Descriptor |
| **1a** | The colonies occur at random | **B1** | 3.5b | 3rdUnderstand the conditions for a Poisson distribution |
| The colonies occur at a constant rate | **B1** | 3.5b |
|  | **(2)** |  |  |
| **1b** | *X* ~ Po(4) | **B1** | 3.3 | 3rdUse the Poisson distribution to model real-world situations |
| P(*X* = 0) | **M1** | 1.1b |
| = 0.0183 | **A1** | 1.1b |
|  | **(3)** |  |  |
| **1c** | *X* ~ Po(6) | **B1** | 3.3 | 3rdUse the Poisson distribution to model real-world situations |
| 1 − P(*X* ⩽ 3) = 1 – 0.1512 | **M1** | 1.1b |
| = 0.8488 | **A1** | 1.1b |
|  | **(3)** |  |  |
| **(8 marks)** |
| **Notes****1a** Accept ‘colonies appear independently of one another’ |

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| **Q** | **Scheme** | **Marks** | **AOs** | **Pearson Progression Step and Progress Descriptor** |
| **2a** | 1.04 | **B1** | 1.1b | 2ndUnderstand the basics of the Poisson distribution |
|  | **(1)** |  |  |
| **2b** | *X* ~ Po(1.04) | **B1** | 3.3 | 3rdUse the Poisson distribution to model real-world situations |
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| ***x*** | **P(*X* = *x*)** | **Expected frequency** |
| 0 | 0.3535 | 53.03 |
| 1 | 0.3676 | 55.14 |
| 2 | 0.1911 | 28.67 |
| 3 | 0.0663 | 9.95 |
| 4 | 0.0172 | 2.58 |

 | **M1****M1****A1** | 1.1b |
|  | **(4)** |  |  |
| **2c** | The Poisson model seems a (fairly) good fit | **B1** | 2.2b | 4thComment on the appropriateness of the Poisson distribution |
| because the expected frequencies are similar to the observed frequencies, oe | **B1** | 2.4 |
|  | **(2)** |  |  |
| **(7 marks)**  |
| **Notes****2b** **M1:** (first) at least three correct probabilities **M1:** (second) at least three correct expected frequencies **A1:** for all correct frequencies |

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| **Q** | Scheme | Marks | AOs | Pearson Progression Step and Progress Descriptor |
| **3a** | *X* ~ Po(3) | **B1** | 3.3 | 3rdUse the Poisson distribution to model real-world situations |
| P(*X* > 4) = 1 − P(*X* ⩽ 4) = 1 − 0.8153 | **M1** | 1.1b |
| = 0.1847 | **A1** | 1.1b |
|  | **(3)** |  |  |
| **3b** | Expectation = 3, Variance = 3 (need both) | **B1** | 1.1a | 4thCalculate the mean of a Poisson distribution |
|  | **(1)** |  |  |
| **3c** | P(*X* ⩽ *x*) ⩾ 0.95P(*X* ⩽ 5) = 0.9161P(*X* ⩽ 6) = 0.9665 | **M1****A1** | 1.1b1.1b | 3rdUse the Poisson distribution to model real-world situations |
| The newsagent should order six copies in order to meet the demand with a probability of at least 0.95 | **A1** | 2.4 |
|  | **(3)** |  |  |
| **(7 marks)** |
| **Notes** |

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| Q | Scheme | Marks | AOs | Pearson Progression Step and Progress Descriptor |
| **4a** | Expectation = 150 × 0.015 = 2.25 | **B1** | 1.1b | 4thCalculate the variance of a binomial distribution |
| Variance = 150 × 0.015 × 0.985 = 2.21625 | **B1** | 1.1b |
|  | **(2)** |  |  |
| **4b** | Expectation ≈ Variance | **B1** | 2.4 | 6thUnderstand when to use a Poisson distribution as an approximation to the binomial distribution |
|  | **(1)** |  |  |
| **4c** | *X* ~ Po(2.25) | **B1** | 3.3 | 6thUse the Poisson distribution as an approximation to the binomial distribution |
| P(*X* > 5) = 1 – P(*X* ⩽ 5) | **M1** | 1.1b |
| = 0.0274 | **A1** | 1.1b |
|  | **(3)** |  |  |
| (6 marks) |
| Notes |

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| **Q** | **Scheme** | **Marks** | **AOs** | **Pearson Progression Step and Progress Descriptor** |
| **5a** | *X* ~ Po(3.6) | **B1** | 3.3 | 6thUnderstand when to use a Poisson distribution as an approximation to the binomial distribution |
| *p* is small and *n* is large (*np* < 10) | **B1** | 2.4 |
|  | **(2)** |  |  |
| **5b** | P(*X* ⩽ 3) | **M1** | 1.1b | 6thUse the Poisson distribution as an approximation to the binomial distribution |
| = 0.5152 | **A1** | 1.1b |
|  | **(2)** |  |  |
| **5c** | *Y* ~ B(10, 0.5152) | **B1** | 3.3 | 5thSolve problems involving the mean and variance of a binomial distribution |
| Mean = *np* = 5.152 | **B1** | 1.1b |
| Variance = *npq* = 2.4976896… | **M1** | 1.1b |
| Standard deviation =  = 1.5804… = 1.58 (3 s.f.) | **A1** | 1.1b |
|  | **(4)** |  |  |
| **(8 marks)** |
| **Notes****5a** Accept ‘Poisson’ without stated *λ* if correct value used in **b** |

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| **Q** | **Scheme** | **Marks** | **AOs** | **Pearson Progression Step and Progress Descriptor** |
| **6a** | H0: *λ* = 1.5; H1: *λ* < 1.5 | **B1** | 2.5 | 3rdUnderstand the language of hypothesis testing |
|  | **(1)** |  |  |
| **6b** | *X* ~ Po(9) | **B1** | 3.3 | 4thCarry out one-tailed tests for the mean of a Poisson distribution |
| P(*X* ⩽ 3) = 0.0212P(*X* ⩽ 4) = 0.0550 | **M1** | 1.1b |
| Hence critical region is 3 or fewer | **A1** | 1.1b |
|  | **(3)** |  |  |
| **6c** | 0.0212 | **B1** | 1.1a | 3rdUnderstand the language of hypothesis testing |
|  | **(1)** |  |  |
| **6d** | 4 is not in the critical region … | **B1** | 2.2b | 4thCarry out one-tailed tests for the mean of a Poisson distribution |
| … so accept the null hypothesis: there is no evidence of a reduction in the (average) number of errors. | **B1** | 2.2b |
|  |  | **(2)** |  |  |
| **(7 marks)**  |
| **Notes****6a** Accept hypotheses framed with *λ* = 9 |

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| **Q** | **Scheme** | **Marks** | **AOs** | **Pearson Progression Step and Progress Descriptor** |
| **7a** | *X* ~ Po(4.7) | **B1** | 3.3 | 4thUnderstand the additive property of the Poisson distribution |
| P(*X* ⩾ 9) = 1 − P(*X* ⩽ 8) = 1 – 0.9497 | **M1** | 1.1b |
| = 0.0503 | **A1** | 1.1b |
|  | **(3)** |  |  |
| **7b** | H0: *λ* = 4.7; H1: *λ* ≠ 4.7 | **B1** | 2.5 | 5thCarry out two-tailed tests for the mean of a Poisson distribution |
| 0.0503 > 0.05 | **M1** | 1.1b |
| Therefore, accept null hypothesis | **A1** | 2.2b |
| There is no evidence that the number of cars and vans passing the recording point has changed | **A1** | 2.2b |
|  | **(4)** |  |  |
| **(7 marks)** |
| **Notes****7b** Accept ft their part **a** if conclusions are consistent and in context |