**1** The discrete random variable *X* has a probability distribution function



where *k* is a constant.

**a** Show that *k* = 10. **(2 marks)**

**b** Show that E(*X*) = 3 and find the value of E(*X* 2). **(4 marks)**

**c** Calculate the variance of *X*. **(1 mark)**

The random variable *Y* = 3*X* – 2

**d** Find E(*Y*) and Var(*Y*). **(2 marks)**

**2** Caitlin is designing a game of chance for her school fete.

She has a fair, five-sided spinner marked with the numbers 1, 2, 3, 4 and 5.

Players get 20 virtual points to have a go at spinning an even number.

If they are successful, they win their points back plus k times the number spun.

Points won can then be exchanged for small prizes.

Given that Caitlin’s expected winnings per game is 3 points, show that *k* = 7.5. **(4 marks)**

**3** The discrete random variable *X* has a probability distribution given in the table below.

**Table 1**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| ***x*** | −3 | −2 | −1 | 0 | 1 |
| **P(*X* = *x*)** | 0.1 | 0.2 | *k* | 0.4 | 0.15 |

**a** Write down the value of *k*. **(1 mark)**

**b** Find the expectation of *X* and show that Var(*X*)  1.51. **(4 marks)**

The random variable *Y* = 2*X* + 1.

**c** Find P(*X* > 2*Y*). **(2 marks)**

**4** The discrete random variable *X* has a probability distribution function



**a** Show that  **(2 marks)**

**b** Find the exact values of E(*X*) and Var(*X*). **(4 marks)**

**c** Show that Var **(2 marks)**

**5** The discrete random variable *X* has a probability distribution given in the table below.

**Table 2**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| ***x*** | −1 | 0 | 2 | 4 | 5 |
| **P(*X* = *x*)** | 0.2 | 0.1 | *a* | *b* | 0.3 |

The random variable *Y* is defined as *Y* = 2*X* – 3.

Given that E(*Y*) = 2.4,

**a** Find the values of *a* and *b*. **(5 marks)**

**b** Calculate E(*X* 2) and show that Var(*X*) = 5.61. **(3 marks)**

**c** Write down the value of Var(*Y*). **(1 mark)**

**d** Find P(*X* – 3 > 2*Y*). **(2 marks)**

**6** The discrete random variable ***X*** has a probability distribution given by

**Table 3**

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| ***x*** | −1 | 0 | 1 | 2 | 3 | 4 |
| **P(*X* = *x*)** | *a* | *b* | *a* | *b* | *a* | *c* |

The random variable *Y* is defined as *Y* = 4 – 3*X*.

Given that E(*Y*) = −1.7 and the P(*Y* < 0) = 0.6,

**a** Calculate the values of *a*, *b* and *c*. **(7 marks)**

Given that Var(*Y*) = 27.81,

**b** Find the exact value of Var(*X*). **(2 marks)**

**c** Find P(*X* < *Y*). **(2 marks)**