TJP TOP TIPS

FOR

IGCSE NUMBER

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First, some important words; know what they mean (get someone to test you):

Integer – a whole number.
2, 0 and -17 are **integers**, but ½, 0.247, √3 and π are not

Factor – a whole number that 'goes into' another number exactly.
the **factors** of 6 are 1, 2, 3 and 6

Prime Number – a whole number with exactly two factors, namely 1 and itself.
2, 3, 5, 7 and 11 are **prime numbers**, but 1, 4, 6, 8 and 9 are not

Highest Common Factor (HCF) – the biggest number that goes into two (or more) others.
the **HCF** of 20 and 28 is 4

Lowest Common Multiple (LCM) – the smallest number that two (or more) others go into.
the **LCM** of 20 and 28 is 140

Product – the result of multiplying several numbers together.
the **product** of 2 and 3 is 6

Square Number – a whole number squared (or multiplied by itself).
1, 4, 9 and 16 are **square numbers** because they are given by 1×1, 2×2, 3×3, 4×4

Surd – a 'nasty' root, one whose decimal goes on for ever without repeating.
√2 (= 1.41421356...) and √3 (= 1.7320508...) are **surds**, but √4 (= 2 exactly) is not

Recurring Decimal – one which contains a digit (or block of digits) repeating for ever.
$^1/_3$ = 0.33333... and $^1/_7$ = 0.142857142857... are **recurring decimals**

Significant Figures (sig figs or sf) – start counting from the first non-zero digit.
The number 0.000016573 has 5 **significant figures**

Decimal Places (dp) – start counting from the decimal point.
The number 0.000016573 has 9 **decimal places**

Indices (plural of **index**) – little raised numbers representing powers (squared, cubed, etc.).
in the example 3^2, 3 is the **base** and 2 is the **index**

Standard Form – a number written as $A \times 10^N$, where $1 \leq A < 10$ and N is a whole number.
$32000 = 3.2 \times 10^4$ and $0.00198 = 1.98 \times 10^{-3}$ when converted to **standard form**

Reciprocal – 'one over' a number, 1 divided by a number; you 'flip' a fraction to do this.
the **reciprocal** of 4 is ¼; the **reciprocal** of $^7/_{11}$ is $^{11}/_7$

Numerator – the top number in a fraction.
the **numerator** of ¾ is 3

Denominator – the bottom number in a fraction.
the **denominator** of ¾ is 4

Common denominator – the LCM of several denominators, used if we add/subtract fractions.
to add $^2/_3$ and $^1/_5$, we use a **common denominator** of 15

Mixed Number or **Mixed Fraction** – a number consisting of an integer and a fraction.
1½ is a **mixed number**, while $^3/_2$ is an **improper** (or **top-heavy**) **fraction**

Evaluate – work out the value of.
to **evaluate** 3.4×1.4^2, work it out according to BIDMAS = $3.4 \times 1.96 = 6.664$

Estimate – work out the rough value of (round numbers in the question to 1 sig fig, usually).
to **estimate** 3.9×9.1, first round the values and then work out $4 \times 9 = 36$

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ARITHMETIC

▷ **Evaluate** means '**work out the value of**'.
You can usually type the numbers straight into your calculator, but beware!

- **Faulty Squares**: if you type -2^2 the calculator gives –4 (it should be 4).
  Get round this by typing $(-2)^2$
- **Big Divide**: $\frac{1+2}{4+5}$ is **not** the same as 1 + 2 ÷ 4 + 5.
  Use the $\frac{\blacksquare}{\square}$ key to enter big fractions.
  Otherwise, insert brackets like this: (1 + 2) ÷ (4 + 5)

IGCSE INSIDER INFO: 27% of marks rely on calculator usage. That's a C grade!

▷ **BIDMAS** stands for **Brackets, Indices, Divide Multiply, Add Subtract**.
This is the order in which you should carry out calculations.

Do anything in brackets first, followed by indices (powers and roots), then ÷ and × together, then + and – together.

TJP TOP TIP: Not a lot of people know this, but Divide Multiply are on the same level, and Add Subtract are on the same level.

This means that if you have 1 – 2 + 3, you **don't** do + before – to get 1 – 5 = –4. You just work from left to right to get 1 – 2 + 3 = 2.

It's the same with × and ÷; just work from left to right.

Your calculator does BIDMAS automatically, but note '**Faulty Squares**' and '**Big Divide**' mentioned above.

▷ **Negative Numbers** – remember these rules!

▶ **SKILL: Add or subtract negative numbers.**
Go up/down in a lift, but '**minus a minus**' **goes up**.
(Imagine making a + sign out of two adjacent – signs.)

TJP TOP TIP: Sending someone naughty out of the class ('subtracting a negative') has a positive effect on the class.

Q: Work out 2 + (-3) – 4 – (-5)
A: Total so far: 2 (↓ 3) -1 (↓ 4) -5 (↑ 5) **0.**

▶ **SKILL: Multiply or divide negative numbers.**
Two minuses make a plus. One minus makes a minus.

Q: Work out (-3) × (-4) ÷ (-2):
A: First, do the numbers: 3 × 4 ÷ 2 = 6.
Then decide if it's + or –: two of the minuses cancel out, leaving **–6**.

ROUNDING AND ESTIMATION

▷ Rounding numbers

We are often asked to round our answers in IGCSE exams. Remember:
A number rounds **up** if the next digit is **5 or above**.
A number rounds **down** if the next digit is **4 or below**.

IGCSE INSIDER INFO: Really important, this one...

You can't (usually) **lose marks for being too accurate!**

[The mark scheme will say 'award marks for 3 sig figs **or greater**']
So: **if in doubt**, just write down **all the digits of your answer**.

WARNING: The **exception** to this is when you have just given an accurate answer and **then** you are asked to round it.

▶ SKILL: Round a number to N decimal places.

Start counting digits after the **decimal point**, then round up/down as appropriate.

Q: Round 31.5735189 to 3 decimal places.

A: Chop the number off after 3 decimal places: 31.573|5189
Now round up because a 5 comes next. **31.574**

- **Sneaky super-rounding situation**: 99.9999 rounded to 2 dps becomes **100.00**. (You may have to pad out the number with zeros to give it the right number of dps.)

▶ SKILL: Round a number to N significant figures.

Start counting from the **first non-zero digit**, then round up/down as appropriate.

Q: Round 0.00084631 to 2 significant figures.

A: Chop the number off after 2 significant figures: 0.00084|631
Now round up because a 6 comes next. **0.00085**

- **Ever-so-slightly evil example**: 137423 rounded to 3 sf becomes **137000**, not 137. (You have to pad out the number with enough zeros to make it the **right size**.)

▶ SKILL: Estimate the answer to a calculation.

Round all the numbers in the question to **1 significant figure**, then find the answer.

Q: Estimate (3.187 + 6.893) × 0.2096.

A: Round to 1 sig fig first: ≈ (3 + 7) × 0.2 = 10 × 0.2 = **2**.

- The ≈ symbol means “approximately equal to”.
- Make sure you don't round 0.2096 down to zero by mistake... It's sig figs, not dp!

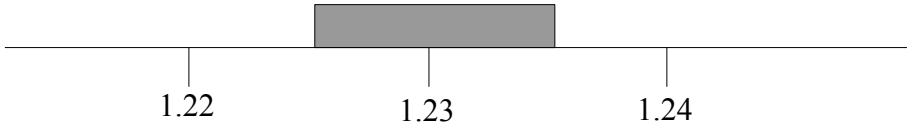
UPPER AND LOWER BOUNDS

▷ **Upper and Lower Bounds**

This is roughly the reverse of the previous section: what is the question to my answer?

Q: If a number has been rounded to 1.23 (correct to 2 dps), what could it have been originally?

A: It can help to sketch out a ruler marked with 1.22, 1.23, 1.24, etc.



Any measurements in the grey region would round to 1.23 rather than to 1.22 or 1.24.

This region clearly extends from 1.225 up to 1.235 (not inclusive).

So the **lower bound is 1.225** and the **upper bound is 1.235**.

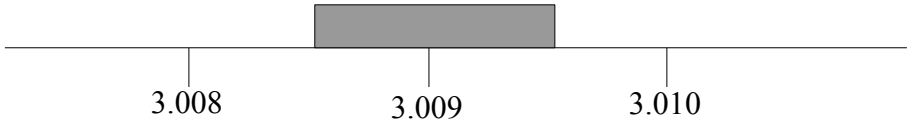
Although that upper bound is not inclusive, it's the answer they want you to give.
Don't say 1.2349, because we could come along with 1.23499999 and beat that...

▶ **SKILL: Find the lower and upper bounds of a rounded number**

Sketch the appropriate ruler, then read off 'half way down' and 'half way up'.

Q: Find the lower and upper bounds of 3.009cm (correct to 4 sig figs).

A: Sketch the ruler:



Lower bound = 3.0085cm **Upper bound = 3.0095cm**

▷ **Calculations with Upper and Lower Bounds**

Q: If a runner covers 200m in 23.3 sec, what is her greatest and least possible speed?

A: Well, if the distance is accurate to the nearest metre and the time is accurate to the nearest 0.1 second, we can find the lower and upper bounds for each of these:

Distance:	lower bound = 199.5m,	upper bound = 200.5m
Time:	lower bound = 23.25 sec,	upper bound = 23.35 sec

Now using the DST triangle, Speed = Distance/Time.

So her greatest possible speed = Max Distance/Min Time = 200.5/23.25 = **8.624 m/s**
and her least possible speed = Min Distance/Max Time = 199.5/23.35 = **8.544 m/s**

PRIMES AND FACTORS

▷ A **factor** is a whole number that 'goes into' another number exactly.
For example, the factors of 10 are 1, 2, 5 and 10.

▶ **SKILL: List all the factors of a number.**

Q: List all the factors of 72.

A: Do this in pairs as follows (each pair multiplies to make 72):

1, 72
2, 36
3, 24
4, 18
6, 12
8, 9

▷ A **prime number** has exactly two factors, namely 1 and itself.
The first prime numbers are 2, 3, 5, 7, 11, 13, 17, 19, 23, etc.

4 isn't prime because 2 goes into it; 6 isn't prime because 2 and 3 go into it.

1 isn't a prime number because it has only one factor, not two.

▷ As you might guess, a **prime factor** is a factor which is a prime number.

Every number can be written by multiplying its prime factors together.
It's a bit like making compounds out of chemical elements.
Instead of water = H_2O, we can write $12 = 2^2 \times 3$

▶ **SKILL: Write a number as a product of its prime factors.**

TJP TOP TIP: Set out your working like this; it's a method similar to LCM and HCF (see below): learn one, get two free!

Q: Write 420 as a product of its prime factors.

A:

2 |	420	(2 goes into 420, leaving 210 in the next line)
2 |	210	(2 goes into 210, leaving 105)
3 |	105	(3 goes into 105, etc.)
5 |	35	
7 |	7	(7 goes into 7, leaving 1)
|	1	(No primes go into 1, so we stop)

420 = **$2 \times 2 \times 3 \times 5 \times 7$**

- Remember to divide by **prime** numbers only, working your way up from 2.
- If we are asked to write a number using **powers** of prime factors, use indices:

420 = **$2^2 \times 3 \times 5 \times 7$**

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HCF AND LCM

▷ **HCF** stands for **Highest Common Factor**.
It is the **largest** number that **goes into** two (or more) **others** exactly.
The HCF of 12 and 21 is 3.

▷ **LCM** stands for **Lowest Common Multiple**.
It is the **smallest** number that two (or more) **others go into** exactly.
The LCM of 12 and 21 is 84.

▶ **SKILL: Find the HCF and LCM of two numbers.**

TJP TOP TIP: Set out your working like this; it's similar to finding prime factors (above).

Q: Find the LCM and HCF of 45 and 75.

A:

3	30	75
5	10	25
	2	5

(3 goes into 45 and 75, leaving 15 and 25)
(5 goes into 10 and 25)
(nothing goes into 2 and 5, so we stop)

Now draw an L shape... (shaded here)

Multiply all the numbers in the **L** to get the **LCM** (geddit?)
Multiply the numbers in the left-hand column to get the HCF.

LCM = 3×5×2×5 = **150** HCF = 3×5 = **15**.

● Remember: it's **L for LCM**!

Q: Find the LCM and HCF of 240 and 768.

A:

2	240	768
2	120	384
2	60	192
2	30	96
3	15	48
	5	16

(nothing goes into 5 & 16 so we stop)

Now draw an L shape... (shaded here)

LCM = 2×2×2×2×3×5×16 = **3840** HCF = 2×2×2×2×3 = **48**.

Note: if you have **three** numbers, the method will work for **HCF only**.

Don't get carried away; you may be asked for just the LCM or just the HCF...

INDICES

▷ The **little numbers** used for powers (7^2, 7^3, etc.) are called **indices** (singular is **index**). The **big number** underneath (7 in this case) is called the **base**.

There are **three laws for combining indices** – they only work if the **bases match**:

▶

Multiply by **adding the indices**:	$7^4 \times 7^5 = 7^9$
Divide by **subtracting the indices**:	$7^{12} \div 7^2 = 7^{10}$
Do **brackets** by **multiplying the indices**:	$(7^3)^5 = 7^{15}$

LEARN!

There are **three further facts** to know about indices:

▶

Negative indices mean '**1 over**':	$7^{-2} = 1 / 7^2 = {}^1/_{49}$
Fractional indices are **roots**:	$9^{1/2} = \sqrt{9} = 3$
Zero indices always equal **1**:	$7^0 = 1$

LEARN!

TJP TOP TIP: Some hints to help you remember these extra facts:

- Imagine the '–' of the negative index turning into the division line of $^1/_{\ldots}$
- The number underneath the fraction = roots underneath a tree.
- 'The Power of Love' (= 0; think tennis) got to No. 1 in the charts.
  [Huey Lewis and the News, Frankie Goes to Hollywood, Jennifer Rush, Céline Dion, etc.]

▶ **SKILL: Simplify expressions involving indices.**

Use the above laws and facts to do this.

Q: Simplify $\dfrac{6^4 \times 6^9}{6^5}$.

A: $\dfrac{6^4 \times 6^9}{6^5} = 6^{(4+9-5)} = \mathbf{6^8}$.

Q: Simplify $81^{-1/2}$, giving your answer as a fraction.

A: $81^{-1/2} = \dfrac{1}{81^{1/2}} = \dfrac{1}{\sqrt{81}} = \mathbf{\dfrac{1}{9}}$.

Q: Simplify $16^{3/2}$.

TJP TOP TIP: With fractional indices such as 3/2, remember it's like a tree: **"Power from above, roots down below."**

Hint: do the **root first** to keep the number small, **then** do the **power**.

A: $16^{3/2} = (\sqrt{16})^3 = 4^3 = \mathbf{64}$.

STANDARD FORM

▷ **Standard Form** is a convenient way of writing really big and really small numbers. (These often occur in astronomy and atomic physics, for instance.)

For example, $51000 = 5.1 \times 10^4$ and $0.00000492 = 4.92 \times 10^{-6}$.

A number is in **standard form** if it is written as
$A \times 10^N$, where $1 \leq A < 10$ and N is a whole number
(in other words, there is one digit before the decimal point).

▶ **SKILL: Convert a number to standard form.**

TJP TOP TIP:
- Move the decimal point until it's after the first non-zero digit.
- Count how many times you moved it – this gives the index.
- Small numbers (<1) have a negative index.
- Big numbers (>1) have a positive index.

Q: Convert 299000000 to standard form.

A: 299000000. We need to move the decimal point **8 times**.
The number is **big**, so the index is **8**, not –8.
So the answer is **2.99×10^8**.

▶ **SKILL: Convert a number out of standard form.**

TJP TOP TIP:
- Move the decimal point the number of times given by the index.
- If the index is negative, move the decimal point to make a small number.
- If the index is positive, move the decimal point to make a big number.

Q: Convert 7.4×10^{-3} to a normal number.

A: Move the decimal point 3 times to make a small number. 0.0074
The answer is **0.0074**.

▶ **SKILL: Combine (algebraic) numbers written in standard form.**

Q: Work out $A \times 10^6 + B \times 10^7$ (where $1 \leq A < 10$ and $1 \leq B < 9$).

A: The answer will be 'something $\times 10^7$' so we convert both parts to 'something $\times 10^7$'.
$(A \div 10) \times 10^7 + B \times 10^7$
We multiplied 10^6 by 10, so we have to divide A by 10 to compensate.
So the answer is **$(B + A \div 10) \times 10^7$**.

Q: Work out $(A \times 10^6) \times (B \times 10^7)$ (where $4 \leq A < 10$ and $4 \leq B < 10$).

A: We begin by getting $(A \times B) \times 10^{13}$, but we need to adjust this since $A \times B > 10$.
So we divide $A \times B$ by 10 and multiply 10^{13} by 10 to compensate.
The answer is **$(A \times B \div 10) \times 10^{14}$**.

IGCSE INSIDER INFO: IGCSE examiners like these algebraic questions nowadays...

SURDS

▷ A **surd** is a root of a whole number which is not a whole number; a 'messy' root.
So $\sqrt{2}$ (= 1.41421356...) and $\sqrt{3}$ (= 1.7320508...) are **surds**, but $\sqrt{4}$ (= 2 exactly) is not.
(The word comes from the same root as 'absurd'; in other words, not rational.)

▶ **SKILL: Simplify a surd.**

TJP TOP TIP: Find the biggest square number that goes into the number. You can then square root this part **exactly**.

Q: Simplify $\sqrt{32}$.
A: $\sqrt{32} = \sqrt{(16 \times 2)} = \sqrt{16} \times \sqrt{2} = \mathbf{4\sqrt{2}}$

▶ **SKILL: Rationalise the denominator of a surd.**

TJP TOP TIP: If there is a surd on the bottom of a fraction, multiply top & bottom of the fraction by this same surd. This gets the surd on the top instead.
Why bother? So that we can add it to other surds (see next skill).

Q: Simplify $\frac{2}{\sqrt{6}}$.

A: $\frac{2}{\sqrt{6}} \times \frac{\sqrt{6}}{\sqrt{6}} = \frac{2\sqrt{6}}{6} = \mathbf{\frac{\sqrt{6}}{3}}$.

▶ **SKILL: Add or subtract surds.**

TJP TOP TIP: Simplify the surds **first**!

Q: Simplify $\sqrt{27} + \sqrt{12} - \frac{6}{\sqrt{3}}$.

A: $\sqrt{9\times3} + \sqrt{4\times3} - \frac{6}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = 3\sqrt{3} + 2\sqrt{3} - \frac{6\sqrt{3}}{3} = \mathbf{3\sqrt{3}}$.

▶ **SKILL: Multiply or divide surds.**

TJP TOP TIP: Simplify the surds **last**!

Q: Work out $\sqrt{6} \times \sqrt{42} \div \sqrt{7}$.

A: $\sqrt{6} \times \sqrt{42} \div \sqrt{7} = \sqrt{\frac{6\times42}{7}} = \sqrt{36} = \mathbf{6}$.

Q: Simplify $(3+\sqrt{5})(3-\sqrt{5})$.

A: $(3+\sqrt{5})(3-\sqrt{5}) = 3\times3 + 3\times(-\sqrt{5}) + 3\times\sqrt{5} - \sqrt{5}\times\sqrt{5}$ (using FOIL)
$= 9-5 = \mathbf{4}$.

IGCSE NUMBER

FRACTIONS

▷ The main thing about fraction questions is that **you have to show all your working** to get any marks at all! They know that your calculator can find the answer for you...

First, a quick reminder:

> **TJP TOP TIP**:
> **Numerator** (**N**earer to **N**eptune) – the top number in a fraction.
> **Denominator** (**D**ownstairs) – the bottom number in a fraction.

▶ **SKILL: Cancel a fraction down to its lowest terms.**

Q: Cancel $\frac{16}{24}$ down to a fraction in its lowest terms.

A: Find the biggest number that goes into 16 and 24 (the HCF). This is **8**.
Now divide top and bottom by 8 to get $\frac{2}{3}$.

▶ **SKILL: Convert a fraction to an equivalent fraction with a different denominator.**
(We often need to do this when adding or subtracting fractions.)

Q: Write $\frac{3}{5}$ as a fraction in the form $\frac{x}{35}$.

A: Whatever we do to the denominator, we must also do to the numerator.
Here, we need to multiply the denominator by **7**, turning 5 into 35.
Now do the same to the numerator: 3×7 = 21.
So the answer is $\frac{\mathbf{21}}{\mathbf{35}}$.

▶ **SKILL: Convert a top-heavy to a mixed fraction.**

Q: Convert $\frac{23}{5}$ to a mixed fraction.

A: How many times does 5 go into 23? **4** times.
What's the remainder when we divide 23 by 5? Remainder is **3**.
Answer is $\mathbf{4\frac{3}{5}}$.

▶ **SKILL: Convert a mixed to a top-heavy fraction.**

Q: Convert $5\frac{2}{7}$ to a top-heavy fraction.

A: Multiply the whole number by the denominator. 5×7 = **35**.
Add the numerator. 35 + 2 = **37**.
Answer is $\frac{\mathbf{37}}{\mathbf{7}}$.

▶ **SKILL: Multiply fractions.**

- Convert any **mixed** fractions to **top-heavy** first.
- **Cancel down** (zap) any numbers with **common factors** on the **top** and the **bottom**.
- Then work out **top × top** and **bottom × bottom**.
- If your answer is a top-heavy fraction, convert it to a **mixed fraction** if required.

Q: Evaluate $4\frac{2}{3}\times\frac{5}{7}$, showing all your working.

A: $4\frac{2}{3}\times\frac{5}{7} = \frac{14}{3}\times\frac{5}{7} = \frac{2}{3}\times\frac{5}{1}$ (cancelling down)

$= \frac{10}{3} = \mathbf{3\frac{1}{3}}$.

▶ **SKILL: Divide fractions.**

- Convert any **mixed** fractions to **top-heavy** first.
- Turn the **second fraction upside-down** and then **multiply** (as above).

Remember ***KFC***: **K**eep the first fraction, **F**lip the second fraction, **C**hange the ÷ to ×.

Q: Evaluate $2\frac{3}{4}\div 1\frac{5}{8}$, showing all your working.

A: $2\frac{3}{4}\div 1\frac{5}{8} = \frac{11}{4}\div\frac{13}{8} = \frac{11}{4}\times\frac{8}{13} = \frac{11}{1}\times\frac{2}{13}$ (cancelling down)

$= \frac{22}{13} = \mathbf{1\frac{9}{13}}$

▶ **SKILL: Add and subtract fractions.**

You **don't** have to make top-heavy fractions in this case…

- Add/subtract any **whole numbers** first.
- Put both fractions over a **common denominator**.
- **Add/subtract** the **tops** of the fractions – the denominator stays the same.
- **Combine** with the whole number part to finish.

Q: Evaluate $2\frac{3}{4} + 1\frac{5}{8}$, showing all your working.

A: $2 + 1 = 3$ and $\frac{3}{4} + \frac{5}{8} = \frac{6}{8} + \frac{5}{8} = \frac{11}{8} = 1\frac{3}{8}$

So the answer is $3 + 1\frac{3}{8} = \mathbf{4\frac{3}{8}}$.

Q: Evaluate $4\frac{2}{7} - 1\frac{3}{5}$, showing all your working.

A: $4 - 1 = 3$ and $\frac{2}{7} - \frac{3}{5} = \frac{10}{35} - \frac{21}{35} = -\frac{11}{35}$

So the answer is $3 + -\frac{11}{35} = \mathbf{2\frac{24}{35}}$.

RECURRING DECIMALS

▷ A **recurring decimal** is one which contains a digit (or block of digits) repeating for ever.

Any recurring decimal can be **written as a fraction**; your calculator will do this, but once again **we have to show all our working** to get any marks.

▶ **SKILL: Convert a recurring decimal to a fraction.**

TJP TOP TIP: Set out your working like this.
The idea is to cunningly cancel out all those repeating digits...

Q: Convert 0.433333... to a fraction in its lowest terms.

A: We start by writing F = 0.433333...

If we multiply by 10 we get 10F = 4.333333...

Now subtract (bottom – top): 9F = 3.9 (we cancel all those '3's)

Rearrange to get F = 3.9 / 9

But that isn't a fraction; adjust it: F = 39 / 90

So the fraction in its lowest terms is $\frac{\mathbf{13}}{\mathbf{30}}$.

Q: Convert 0.45454545... to a fraction in its lowest terms.

A: We start by writing F = 0.45454545...

If we multiply by 100 we get 100F = 45.45454545...

Now subtract (bottom – top): 99F = 45 (zap the pesky decimals)

Rearrange to get F = 45 / 99

Now cancel down: F = 5 / 11

So the fraction in its lowest terms is $\frac{\mathbf{5}}{\mathbf{11}}$.

How do we know whether to multiply by 10 or by 100, etc.?

If a single digit repeats, multiply by 10.
If two digits repeat, multiply by 100.
If three digits repeat, multiply by 1000. (Spot the pattern...)

We can also deal with algebraic questions as follows:

Q: Convert 0.abcabcabcabc... to a fraction in terms of a, b and c.

A: We write F = 0.abcabcabcabc...

Multiply through by 1000: 1000F = abc.abcabcabcabc...

Subtract (bottom – top) 999F = abc

Therefore F = $\frac{\mathbf{abc}}{\mathbf{999}}$.

PERCENTAGES

▷ '**Percent**' is simply Latin for '**out of a hundred**'. (It's also written as **%**.)
Remember this and there should be no problem.

▶ **SKILL: Convert between percent, fractions and decimals.**

% to Fraction: Write as a fraction out of 100, then cancel down.
35% = 35/100 = **7/20**

Fraction to %: Multiply the fraction by 100.
7/8 = 7/8 of 100% = (7÷8)×100% = **87.5%**

% to Decimal: Move the decimal point twice to make the number smaller (÷100).
27% = 27÷100 = **0.27**

Decimal to %: Move the decimal point twice to make the number bigger (×100).
0.07 = 0.07×100% = **7%**

Remember: percentages are 100× bigger than decimals.

▶ **SKILL: Find a certain percentage of a number.**

TJP TOP TIP: Write the % as a fraction over 100, and multiply by it.

Q: Find 23% of £120.

A: 120×(23/100) = **£27.60**.

▶ **SKILL: Increase or decrease a number by a certain percentage.**

Work out the percentage amount (see above)
then **add** it to the original number to **increase** or **subtract** it to **decrease**.

Q: Increase £32 by 40%.

A: 40% of £32 = 32×(40/100) = £12.80.
Now add it on! £32 + £12.80 = **£44.80**.

Q: Decrease 70kg by 6%.

A: 6% of 70kg = 70×(6/100) = 4.2kg.
Now subtract it! 70 – 4.2 = **65.8kg**.

TJP TOP TIP: A more powerful way is to think about how many percent we'll have altogether at the end, **including** the original amount (100%).

Q: Increase £32 by 40%.

A: At the end, we'll have 100 + 40 = 140%.
So we find 32×(140/100) = **£44.80**.

Q: Decrease 70kg by 6%.

A: At the end, we'll have 100 – 6 = 94%.
So we find 70×(94/100) = **65.8kg**.

▶ **SKILL: Find a percentage change.**

$$\text{Percentage Change} = \frac{\text{Change}}{\text{Original Amount}} \times 100$$ **Learn!**

(Use the **original** amount, not the final amount!)

Q: A house falls in price from £250,000 to £210,000; find the percentage change.

A: The change is £40,000, so $\frac{40000}{250000} \times 100$ = **16% fall**.

Q: The price of a litre of milk increases from 80p to 86p; find the percentage change.

A: The change is 6p, so $\frac{6}{80} \times 100$ = **7.5% increase**.

▶ **SKILL: Solve a reverse percentage problem.**

For instance, you are told the **sale** price of a coat and you need to find the **original** price.

TJP TOP TIP: This is a **classic danger area**!
If 20% was taken off, **don't** just add 20%; this is **wrong**!
Instead, **learn the method below**.

Q: A coat costs £64 in a sale after being reduced by 20%. Find the original price.

A: If 20% was taken off, £64 must be 100 – 20 = **80%** of the original price.

£64	=	80%	
(÷80)		(÷80)	
£0.80	=	1%	(get 1%)
(×100)		(×100)	
£80	=	100%	(and now get 100%)

So the original price was **£80**.

Q: Becki sells a camera for £170, making a 25% profit.
How much did she buy the camera for originally?

A: If 25% profit was added, £170 must be 100 + 25 = **125%** of the original price.

£170	=	125%	
(÷125)		(÷125)	
£1.36	=	1%	(get 1%)
(×100)		(×100)	
£136	=	100%	(and now get 100%)

So the original price was **£136**.

COMPOUND INTEREST AND DEPRECIATION

▷ If you put money in a savings account, you generally get some **interest** each year; this means you receive **extra money** on top of your original amount. Yippee!

And that's not all; if you leave your money there for several years, you get interest on your interest, too... Yippee squared!

Anyway, this is called **compound interest**.

IGCSE INSIDER INFO: Compound interest is a **brand new topic this year** (2011) so it's quite likely they'll be keen to include a question on it...

▶ **SKILL: Solve a compound interest problem.**

Q: Will puts £500 into a savings account paying 5% compound interest each year. How much money does he have after 10 years?

A: Every year, the money is multiplied by 100 + 5 = 105% = (105/100).
This consists of the original money (100%) plus the interest (5%).
So after 10 years, there will be $500 \times (105/100)^{10}$ = **£814.45**.

Q: Mel saves £200 for 5 years at a compound interest rate of 4%. How much interest does she receive in this time?

A: Every year, the money is multiplied by 100 + 4 = 104% = (104/100).
So after 5 years, there will be $200 \times (104/100)^5$ = £243.33.

But the **interest** earned is 243.33 – 200 = **£43.33**.

▶ **SKILL: Solve a depreciation problem.**

When the value of something decreases over time, this is called **depreciation**.

Q: Ed buys a laptop for £400 but it loses 15% of its value each year. How much is it worth after 3 years?

A: Every year, the value is multiplied by 100 – 15 = 85% = (85/100).
So after 3 years the laptop is worth $400 \times (85/100)^3$ = **£245.65**.

If you prefer, you can work with decimals instead of fractions.
For example, 400×0.85^3 = **£245.65**.

RATIOS

There are two types of ratio questions: sharing out and increase/decrease.

▶ **SKILL: Solve a 'sharing' ratio problem.**

TJP TOP TIP: Add up the number of 'shares', and divide the amount to be shared by this number to get the size of one share. Then answer the question...

Q: £90 is to be shared between Bella, Tori and Jack in the ratio 5:3:2.
How much does Tori receive?

A: There are 5 + 3 + 2 = 10 shares altogether.
Each share is worth 90 ÷ 10 = £9.
Tori gets 3 shares, so she receives 3 × 9 = **£27**.

▶ **SKILL: Solve an 'increase/decrease' ratio problem.**

TJP TOP TIP: Scale all the quantities by the given ratio (turn it into a fraction). Get the fraction the right way up by considering whether the quantities should become larger or smaller.

Q: A recipe to serve 4 people requires 500g of flour, 2 eggs and 200ml of milk.
Adapt it to serve 10 people.

A: We have to increase all quantities in the ratio 4:10, so multiply through by (10/4) to make the quantities bigger.
500 × (10/4) = **1250g of flour**,
2 × (10/4) = **5 eggs**,
200 × (10/4) = **500ml of milk**.

▷ **Converting units**

We may be asked to convert between currencies, between units of length, area and volume, or between units of time. This is another form of increase/decrease ratio.

▶ **SKILL: Convert between units.**

Q: If £1.00 = $1.60 and $1.00 = 83 Japanese Yen, convert £7.99 to Japanese Yen.

A: Convert £7.99 to $ by multiplying by (1.60/1.00): 7.99 × (1.60/1.00) = $12.784
Convert $12.784 to Yen by multiplying by (83/1): 12.784 × (83/1) = **1061 Yen**.

Q: Convert a speed of 10m/s to km/h.

A: There are 1000m in 1km.
To convert m to km, divide by 1000: 10m/s = 0.01km/s

There are 3600 seconds in 1 hour. You travel further in 1 hour than in 1 second.
So we now multiply by 3600: 0.01km/s = **36km/h**.

Q: Convert an area of $3m^2$ to cm^2.

A: There are 100^2 cm^2 in $1m^2$ (area is squarier) so answer is 3×100^2 = **$30000cm^2$**.

IGCSE NUMBER

SETS

▷ A **set** is a collection of 'objects' containing no duplicates.
The objects in the collection can be **listed** between **curly brackets**, for instance:
A = {1, 3, 5, 7, 9}

We can also **define** a set using a **rule**, such as:
B = {even numbers from 0 to 10 inclusive}
C = {n^2 : $1 \le n \le 10$} (this means the square numbers from 1 to 100)

Some important words and symbols:

▶ **Learn!**

Element = an object belonging to a set.
Set {a, b, c} contains the elements a, b and c.
We usually use BIG letters for sets and little letters for elements.

Ɛ = Universal Set.
This is the 'world' for the current question, e.g. Ɛ = {All animals}.

∅ = Empty Set = { }.
This is the set with nothing in it.

A' = Not A = the complement of A.
A' contains every element that isn't in A.

A ∩ B = A intersection B = the 'overlap' of A and B.
A ∩ B contains all elements which are in A and in B.

A ∪ B = A union B = A combined with B.
A ∪ B contains all elements which are in A or in B (or in both).

a ∈ B = a is an element in set B = a is a member of B.
1 ∈ {odd numbers}

A ⊂ B = A is a subset of B = set A is contained inside set B.
If A = {2, 3, 4} and B = {1, 2, 3, 4, 5, 6} then A ⊂ B.

A ⊄ B = A is not a subset of B = set A is not contained inside set B.
If A = {2, 3, 7} and B = {1, 2, 3, 4, 5, 6} then A ⊄ B.

n(A) = number of elements in A.
If A = {a, b, c, d} then n(A) = 4. If B = ∅ then n(B) = 0.

TJP TOP TIP: Here are some ways to remember these symbols:

Ɛ = **Ɛ**verything (**Ɛ** is BIG).

∈ = ∈lement (∈ is small).

∩ = i∩tersection = a∩d = the **overlap** because ∩ goes **over** the top.

∪ = ∪nion = a mixing bowl ∪ that you chuck everything into.

A ⊂ B is like A < B so A is contained in B.

▶ **SKILL: List the elements of a set.**
Memorise all the symbols from the previous page otherwise you'll just be guessing...

Q: If $\mathcal{E}$ = {integers between 1 and 10}, A = {even numbers} and B = {prime numbers}, work out:

(i) $A \cap B$ (ii) A' (iii) $A \cup B'$ (iv) $(A \cup B)'$ (v) $n(B)$

A: (i) $A \cap B$ = **{2}** 2 is the only number which is even **and** prime.

(ii) A' = **{1, 3, 5, 7, 9}** These are the non-even (odd) numbers between 1 and 10.

(iii) $A \cup B'$ = **{1, 2, 4, 6, 8, 9, 10}** Even numbers **or** non-prime numbers.

(iv) $(A \cup B)'$ = **{1, 9}** Only 1 and 9 are not (even or prime).

(v) $n(B)$ = **4** There are 4 prime numbers (2, 3, 5, 7) between 1 and 10.

Q: If $\mathcal{E}$ = {animals}, A = {adult animals}, B = {black animals}, C = {cats}, D = {dogs}, and P = {my pets}, describe the following in English:

(i) Max $\in B \cap C$ (ii) Sam $\in A' \cap D$
(iii) Ellie $\in (C \cup D)'$ (iv) $C \cap D = \varnothing$
(v) $P \subset A \cap B' \cap C$ (vi) $n(P) = 1$

A: (i) Max $\in B \cap C$ Max is a black cat.

(ii) Sam $\in A' \cap D$ Sam is a puppy [**or** a dog which isn't an adult].

(iii) Ellie $\in (C \cup D)'$ Ellie isn't a cat or a dog (but is some other animal).

(iv) $C \cap D = \varnothing$ No animal is a cat and a dog [there are no cats that are dogs].

(v) $P \subset A \cap B' \cap C$ My pets are all adult non-black cats.

(vi) $n(P) = 1$ I have one pet.

Q: If $\mathcal{E}$ = {quadrilaterals}, A = {shapes with one or two pairs of parallel sides}, B = {shapes with all angles equal to 90°}, C = {shapes whose diagonals are equal}, decide whether the following are true or false.

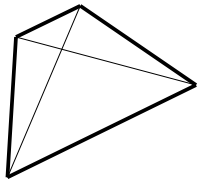
(i) trapezium $\in A$ (ii) $A \subset B$ (iii) $B \subset C$ (iv) $B' \cap C = \varnothing$

A: (i) trapezium $\in A$ True; a trapezium has one pair of parallel sides.

(ii) $A \subset B$ False; for instance, a parallelogram need not have 90° angles.

(iii) $B \subset C$ True; all squares or rectangles have equal diagonals.

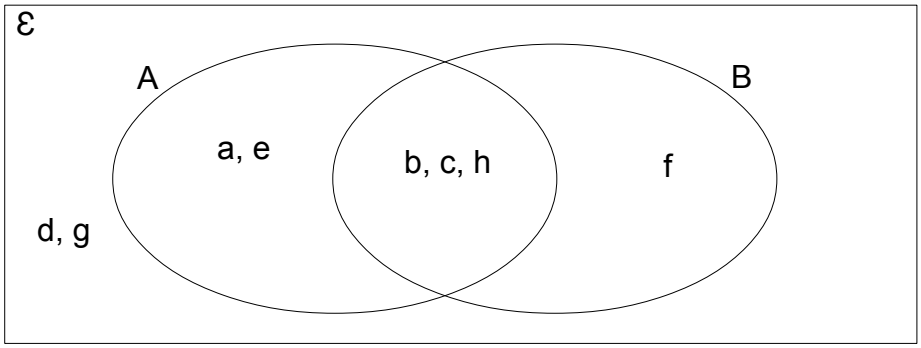
(iv) $B' \cap C = \varnothing$ False; if two equal diagonals don't cross at their midpoints, the angles aren't all 90°. For instance, this (flat) shape...



VENN DIAGRAMS

▷ We can show sets graphically using a **Venn diagram**.

Here is an example:



From this diagram we can see that:

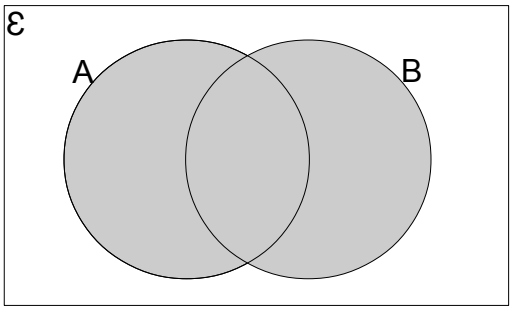
A = {a, b, c, e, h}	(everything inside ellipse A)
A' = {d, f, g}	(everything outside ellipse A)
$A \cap B$ = {b, c, h}	(the **overlap** of A and B; anything inside A **and** B)
$A' \cap B$ = {f}	(anything outside A **and** also inside B)
$A' \cup B'$ = {a, d, e, f, g}	(anything outside A **or** anything outside B)
$(A \cap B)'$ = {a, d, e, f, g}	(anything **not** in the overlap of A and B)
n(A) = 5	(there are five elements inside A)
$f \in B$	(f is an element of B; f is a member of B)

▶ **SKILL: Shade and interpret Venn diagrams.**

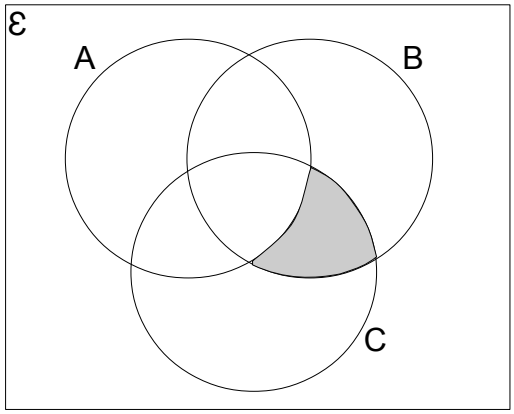
Q: Shade the following regions in the Venn diagrams:

(i) $A \cup B$ (ii) $A' \cap B \cap C$

A:



Anything in A **or** in B.



Anything **not** in A **and** in B **and** in C.

TJP TOP TIP: If you find this difficult, turn the symbols into words, then go through each region of the Venn diagram in turn to check if it 'qualifies'; shade it if it does.