TJP TOP TIPS

FOR

IGCSE GRAPHS

IGCSE GRAPHS

GRAPHS AND CO-ORDINATES

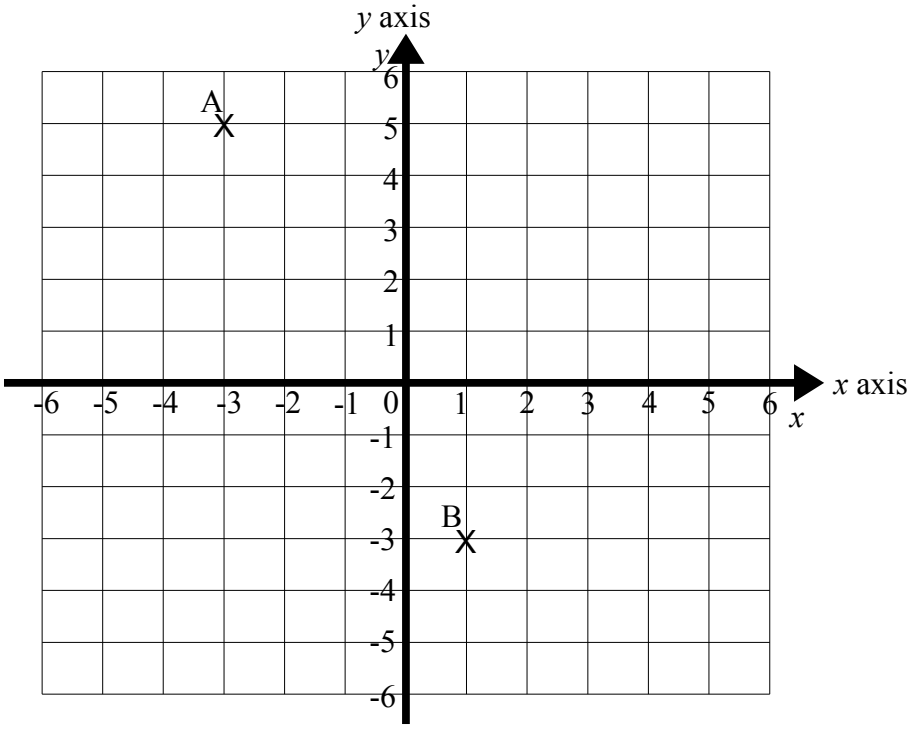
▷ A **graph** is a set of points displayed on a grid.

This grid usually has a **horizontal axis** and a **vertical axis** (plural **axes**), each marked with numbers to help locate points on the graph.

The **position of a point** on a graph is given by its **(x, y) co-ordinates**.
This position is always measured from the **origin**, **(0, 0)** where the axes cross.

TJP TOP TIP: **x is a cross** (=across), and **y to the sky**.

Right and **Up** are the **positive** directions.
Left and **Down** are the **negative** directions.



Point A is at (-3, 5) and point B is at (1, -3).

It is best to plot points by marking an **X** using a pencil (just like voting, really...)

An X marks the exact point more accurately than a blob (if your pencil is sharp).
An X also shows up better on graph paper than a + or a dot.

Once you have plotted your points, you may be asked to join them to make a polygon or a straight line, or else draw a smooth curve passing through them.

Once again, do this using a sharp pencil – just in case you need to change anything.

If you have a **graph using different letters**, for example (t, v), then the **first letter** goes **across** (like x) and the **second letter** goes **up** (like y).

IGCSE GRAPHS

STRAIGHT LINES

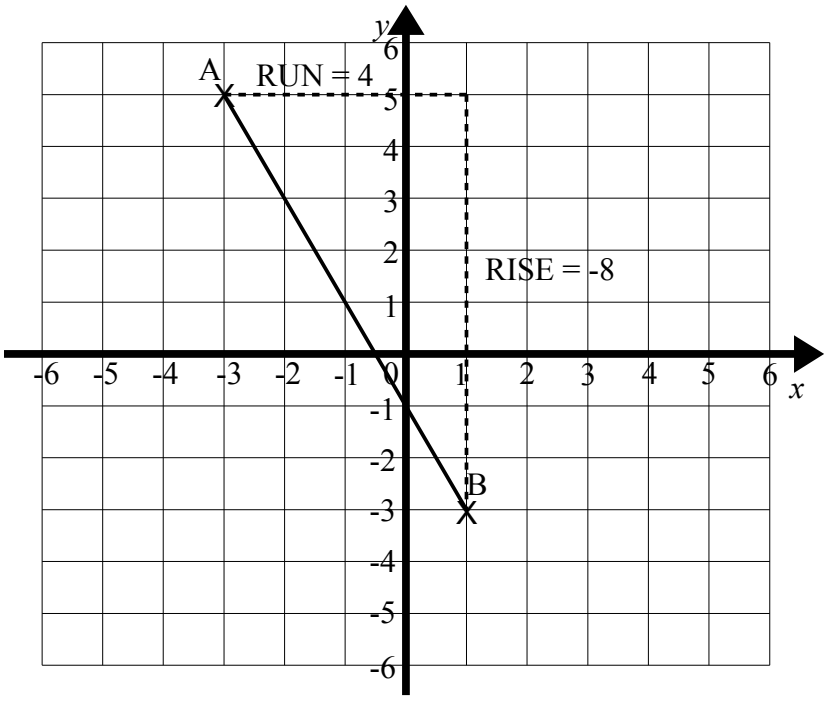
▷ A **straight line** can be drawn through any two points plotted on a graph.

Mathematically, a line can go on for ever, but we can slice it off if necessary to make a **line segment** instead.

▶ **SKILL: Find the midpoint, length and gradient of a line segment.**

The **midpoint** is the **average** of the co-ordinates of the end points.
The **length** is found using **Pythagoras** with the 'rise' and the 'run'.
The **gradient** (or the slope) is given by '**rise over run**'.
[**rise** is the **change in y** along the line, **run** is the **change in x**.]

Q: Find the midpoint, length and gradient of the line segment AB.



A: Point A is (-3, 5) and point B is (1, -3) so:

Midpoint = $\left(\frac{-3+1}{2},\ \frac{5+(-3)}{2}\right) = (-1,\ 1)$.

Length = $\sqrt{4^2+(-8)^2} = \sqrt{16+64} = \sqrt{80} = 8.94$.

Gradient = $\frac{rise}{run} = \frac{-8}{4} = -2$.

Remember:
- A line sloping **up** to the right has a **positive** gradient.
- A line sloping **down** to the right has a **negative** gradient.
- **Steep** lines have **big** gradients.
- **Shallow** lines have **small** gradients.

IGCSE GRAPHS

STRAIGHT LINE EQUATION: y = mx + c

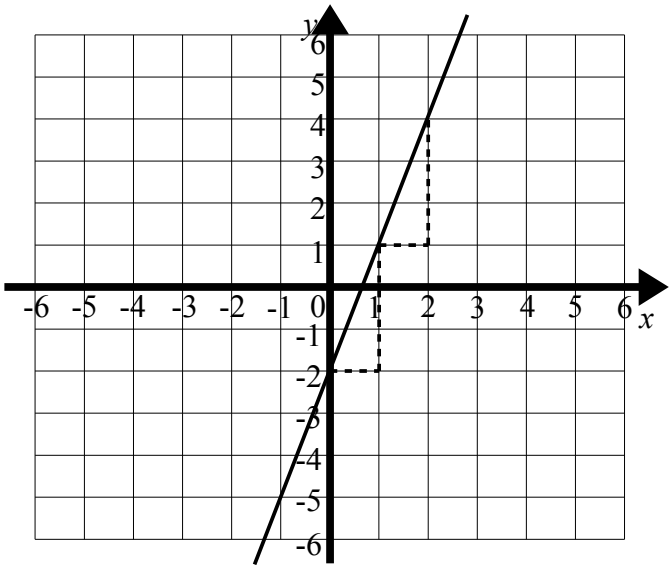
▷ The equation of a straight line can almost always be written as

y = mx + c where m is the gradient and c is the y-intercept.

The gradient is 'rise over run' (how far the graph goes up for every step to the right).
The y-intercept is where the graph cuts the y-axis.

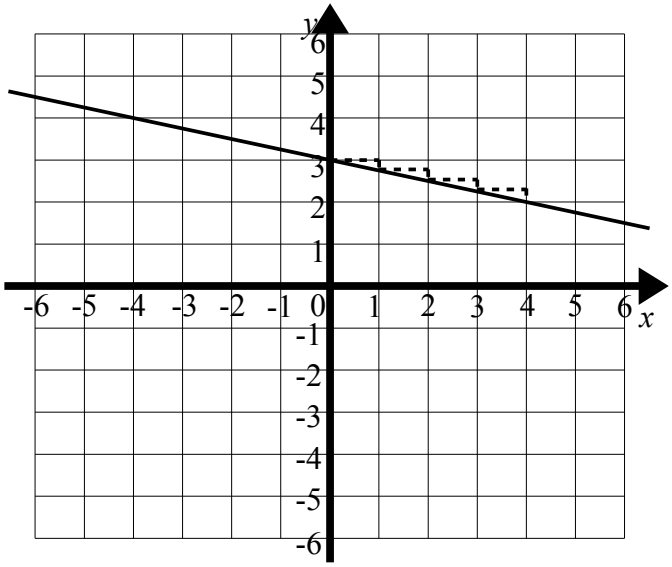
▶ **SKILL: Find the equation of a straight line.**

Q: Find the equation of this line.



A: The line cuts the y-axis at y = –2 so c = –2.
The gradient m = 3 because the 'stairs' go up in steps of 3. So **y = 3x – 2**.

Q: Find the equation of this line.



A: The line cuts the y-axis at y = 3 so c = 3.
The gradient m = –¼ because the 'stairs' go down in steps of ¼.
How can we tell it's ¼? Well, it takes 4 steps to go down a whole square.

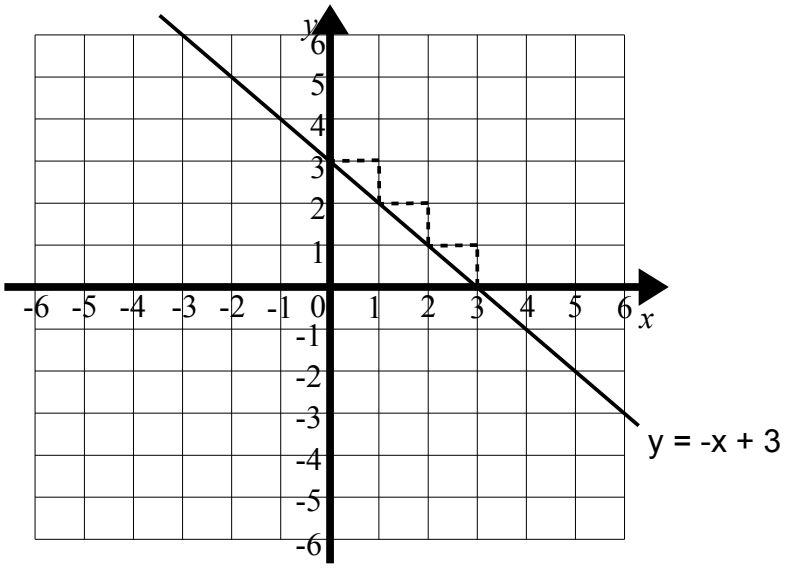
So **y = –¼x + 3**.

▶ **SKILL: Plot a straight line (using y = mx + c).**

TJP TOP TIP: Start at the y-intercept, then go right 1, up m (where m = gradient).
Always go right 1. If m is negative, you go down instead of up.

Q: Plot y = –x + 3

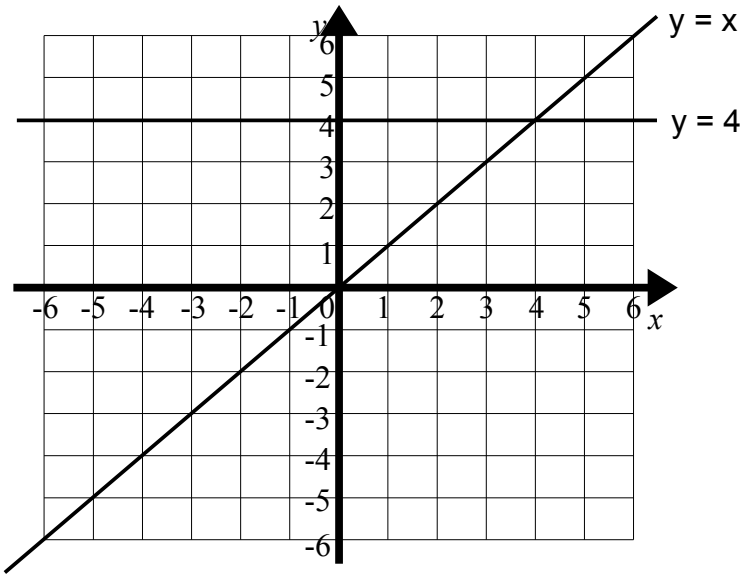
A: The y-intercept is 3, so start at 3 on the y-axis.
Then go right 1, down 1 (because there is –1 lot of x).



Q: Plot y = 4 and y = x on the same axes.

A: The line y = 4 has a y-intercept of 4 and zero gradient (no x at all).
So start at 4 on the y axis and go right 1, up 0 (in other words, horizontal).

The line y = x has a y-intercept of 0 and a gradient of 1 (1 lot of x).
So start at 0 in the y axis and go right 1, up 1.



IGCSE GRAPHS

STRAIGHT LINES: OTHER EQUATIONS

▷ We may come across two other forms of straight line equation.

A **vertical line** has no y-intercept and an infinite gradient, so it cannot be written as y = mx + c. Instead we write **x = c** where c is the **x-intercept**.

So x = 7 is a vertical line cutting the x-axis at 7.

Also, the form **ax + by = c** can be used to represent **any** straight line.
We can **either** rearrange it to get y = … **or else** use the trick shown below:

▶ **SKILL: Plot a straight line (using other equations).**

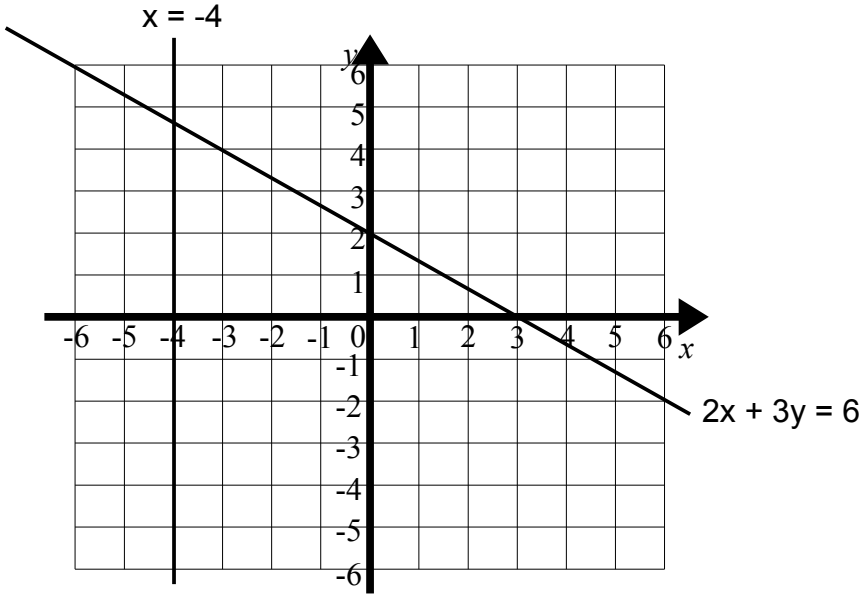
Q: Plot the lines x = –4 and 2x + 3y = 6 on the same axes.

A: First, x = –4 is a vertical line cutting the x axis at –4.

To plot 2x + 3y = 6, first set x = 0 and find y. y = 6÷3 = 2.
This gives the point (0, 2).

Now set y = 0 and find x. x = 6÷2 = 3.
This gives the point (3, 0).

Finally draw a straight line through these two points.



PARALLEL LINES

▷ **Parallel lines** have the **same gradient**.

Q: Find the line parallel to y = 2x – 7 that passes through the point (3, 2)

A: Our new line must have the same gradient, but we don't know its y-intercept.
y = 2x + c

Now find c by substituting the values x=3 and y=2 into the equation.
2 = 2×3 + c so c = 2 – 6 = – 4

Answer: **y = 2x – 4**

GRAPHING INEQUALITIES AND SHADING REGIONS

▷ If we are given an **inequality** such as y > 2x + 3, this corresponds to a **region** on a graph, not a line. Namely, all the points above the line y = 2x + 3.

First mark the **boundary line** by changing the inequality to an equation.
Then decide whether we want the region **above/below** the line or **left/right** of the line.

TJP TOP TIP: The '**more ink/less ink**' rule works again:
- A **<** or **>** inequality is plotted with a **dotted** line.
- A **≤** or **≥** inequality is plotted with a **solid** line.

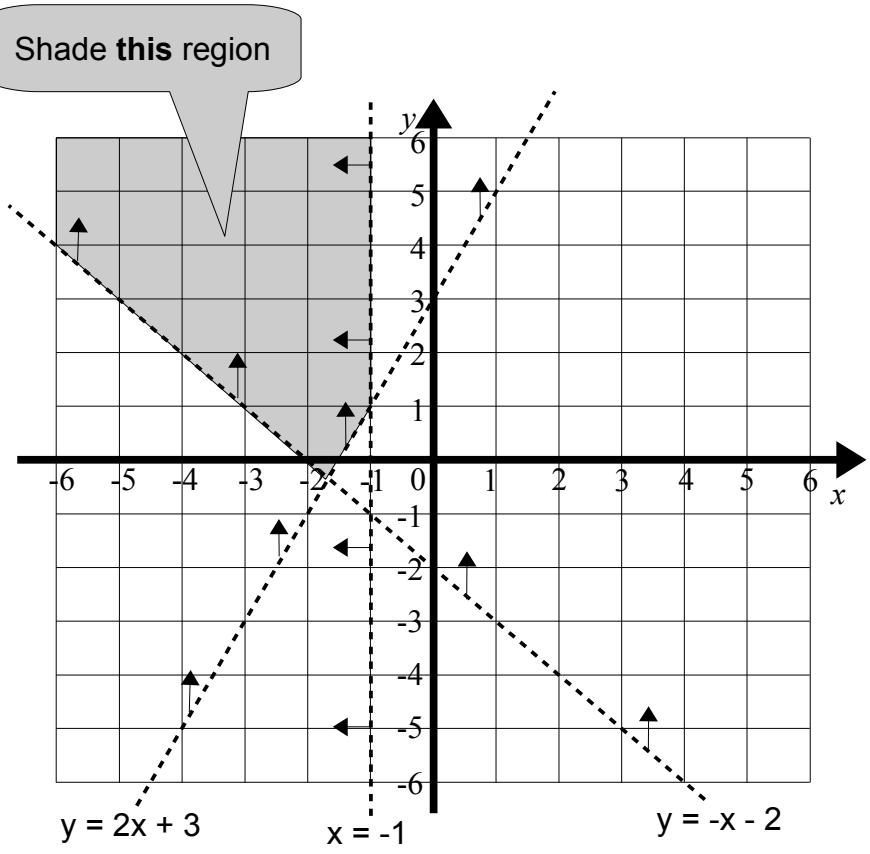
▶ **SKILL: Shade the region satisfying one or more inequalities.**

Q: Shade the region defined by y > 2x + 3, x < 1 and y > –x – 2.

A: First, mark the **dotted** lines y = 2x + 3, x = 1 and y = –x – 2 (less ink with < and >).
[See pages 5 and 6 for how to plot a straight line.]

Then decide if we want the region above/below or left/right.
'y > …' is above the line, 'x < …' is left of the line.

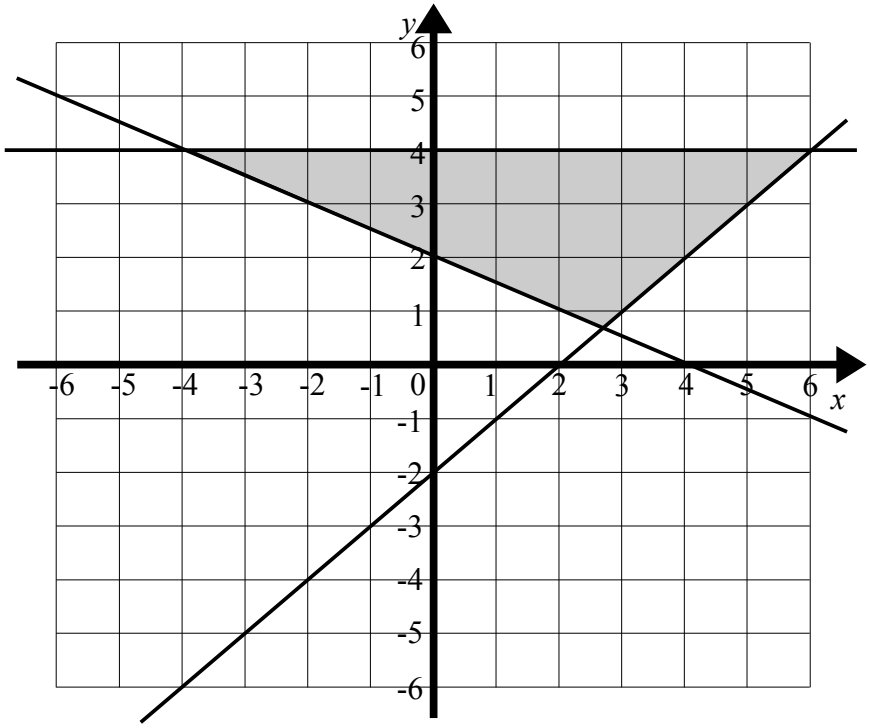
It is helpful to use little arrows to mark above/below or left/right.
You then want the region that all the arrows are pointing **into**.



Note: sometimes you may be asked to shade the **unwanted** region instead.

▶ **SKILL: Find the inequalities which define a given region.**

Q: List the inequalities which define the shaded region.



A: First, get the equations of the boundary lines:
[See page 4 for how to read off the equation of a straight line.]

Here, $y = 4$, $y = x - 2$ and $y = -\frac{1}{2}x + 2$

Now decide on the inequalities:

All the lines are solid, so use **≤** or **≥**.

Below the $y = 4$ line, so **$y \leq 4$**.

Above the $y = x - 2$ line, so **$y \geq x - 2$**.

Above the $y = -\frac{1}{2}x + 2$, so **$y \geq -\frac{1}{2}x + 2$**.

IGCSE GRAPHS

CONVERSION GRAPHS

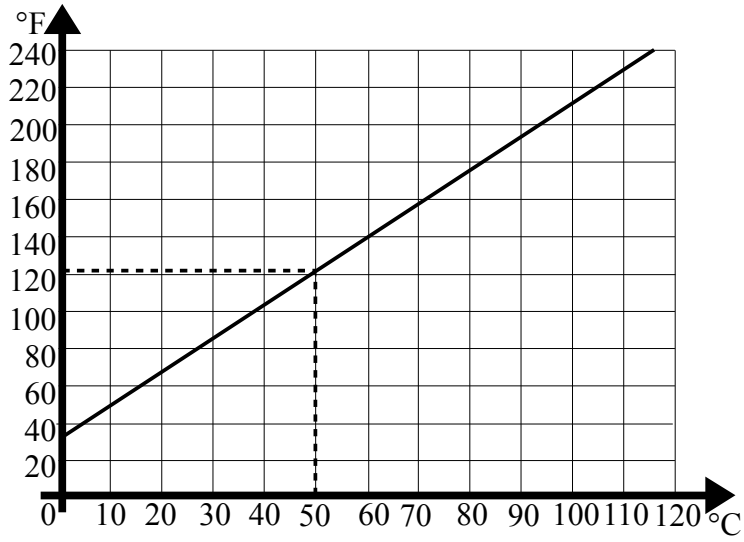
▷ You may get questions where you use a graph to **convert** one quantity into another. Common examples are currency exchange rates or converting units.

Simply read off the values from the graph, but make sure you use the right axis.

▶ **SKILL: Use a conversion graph.**

Q: Use this conversion graph to change 50°C to Fahrenheit.

A:



Answer is approximately **120°F**. [You are always allowed a little leeway.]

DISTANCE-TIME GRAPHS

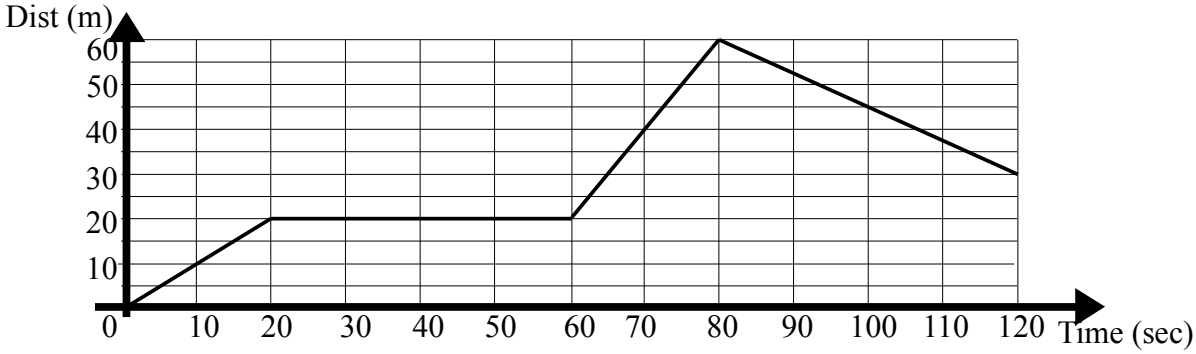
▷ **Distance-time graphs** always have **time** along the **horizontal** axis.
The **gradient** then gives the **velocity** because rise ÷ run = distance ÷ time.

Remember the 

triangle [Dauntsey's School Triangle].

▶ **SKILL: Draw and interpret a distance-time graph.**

Q: Use this graph to find the total distance travelled and the maximum speed.



A: The total distance travelled is 60 (forwards) + 30 (back again) = **90 m**.
The maximum speed is between 60 and 80 seconds: speed = 40÷20 = **2 m/s**.

IGCSE GRAPHS

PLOTTING CURVES: TABLE OF VALUES

▷ You may be asked to plot a **curve** rather than a straight line.
Use a **table of values**; there is a clever trick to get your calculator to do this for you!

It is worth knowing the sort of graphs you can be asked to plot:

IGCSE INSIDER INFO: The syllabus says that all graphs to be plotted must be either $y = Ax^3+Bx^2+Cx+D$ where A, B, C, D are integers (and could be zero) or $y = Ax^3+Bx^2+Cx+D+\frac{E}{x}+\frac{F}{x^2}$ where a minimum of three of A, B, C, D, E, F are zero (so there will never be more than three terms in the function).

Also, x and y may be replaced by other letters.
Here are some examples listed in the syllabus itself:

$y = x^3$; $y = 3x^3-2x^2+5x-4$; $y = 2x^3-6x+2$; $V = 60w(60-w)$

$y = \frac{1}{x}$; $y = 2x^2+3x+\frac{1}{x}$; $y = \frac{3x^2-5}{x}$; $W = \frac{5}{d^2}$

▶ **SKILL: Fill in a table of values for plotting a curve.**

TJP TOP TIP: **Use your calculator to do all the hard work!**
(This works for the Casio FX-83 and FX-85; consult your manual for other models.)

- Press [MODE] and then [3].
- **Enter your function** using [ALPHA] [)] to enter x.
- **Start?** Enter the **first** x value in the table printed in the question.
- **End?** Enter the **last** x value in the table.
- **Step?** Enter the **gap** between the x values in the table.
- You will now see a table of x, y values on the screen. The y values are labelled f(x). Move up and down using the cursor keys, and **copy the y values into your table**. [**Ignore** the very left-hand column – these numbers are not required.]
- Press [MODE] and then [1] at the end to return to the normal calculator mode.

Q: Complete the following table of values when $y = 3x^3 + 2 – 18/x^2$

x	1	2	3	4	5
y					

A: Enter the function, set Start = 1, End = 5 and Step = 1.
Now read off the values and write them in the table:

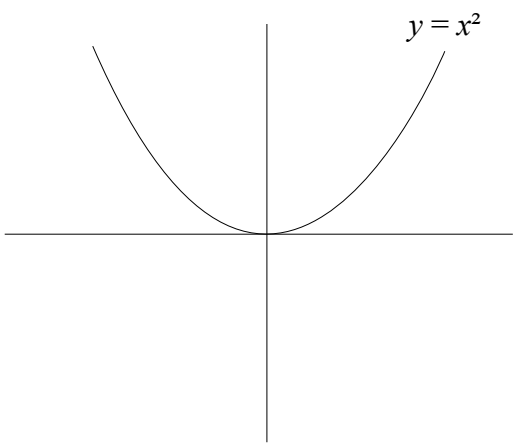
x	1	2	3	4	5
y	-13	21.5	81	192.875	376.28

TJP TOP TIP: Always place your hand **inside the curve** when you draw the curve.

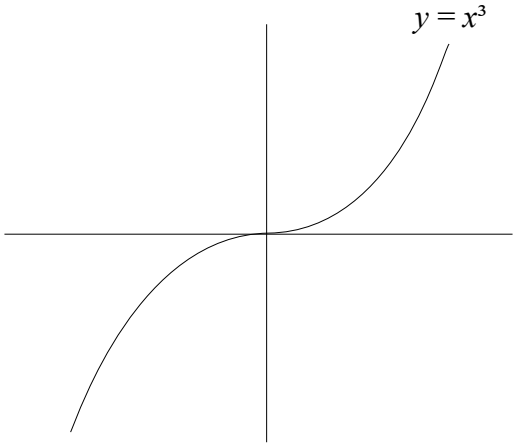
IGCSE GRAPHS

A GALLERY OF GRAPHS

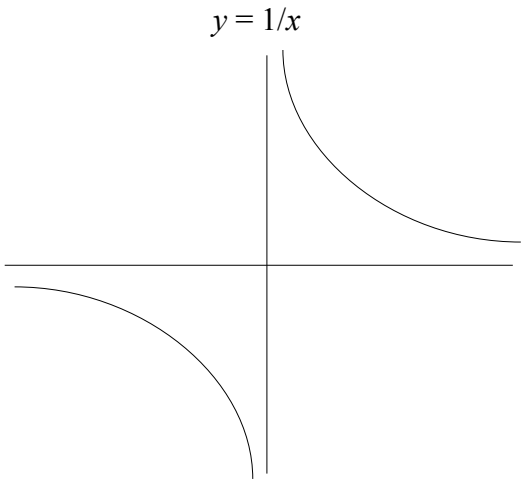
▷ You are expected to be able to sketch some common graphs – **learn the following**!



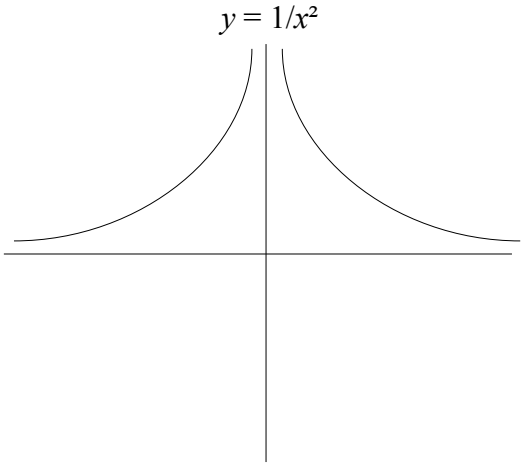
This is called a **parabola**.



This is a **cubic** graph; it goes flat at x=0.



This is a **reciprocal** graph.
It diverges to infinity at x=0.



This graph also diverges to infinity at x=0.

TJP TOP TIP: Note the symmetry of these graphs.
Graphs with **even** powers of x have **line symmetry** in the y axis.
Graphs with **odd** powers of x have **rotational symmetry** order 2 about the origin.

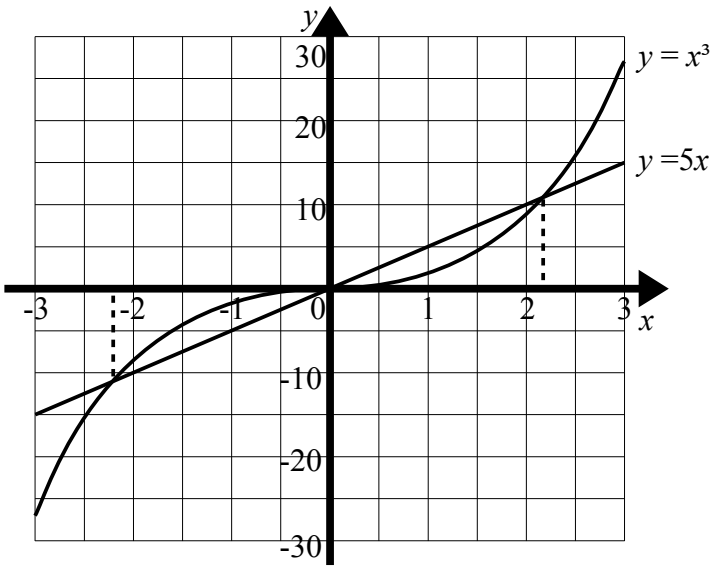
INTERSECTION OF TWO GRAPHS

▷ As well as solving two equations simultaneously, we may be asked to solve them **graphically**. This means plotting the two lines/curves and reading off where they cross.

▶ **SKILL: Solve two equations graphically.**

Q: Use the graphs of $y = x^3$ and $y = 5x$ to solve $x^3 = 5x$.

A:



The graphs cross at x = –2.3, x = 0 and x = 2.3. [You are allowed a margin for error.]

▷ Sometimes we need to **find a straight line** to plot in order to solve an equation graphically.

TJP TOP TIP: **Subtract** the **equation to be solved** from the **curve that's plotted**.

This is explained in the example below.

▶ **SKILL: Choose which straight line to plot to solve an equation graphically.**

Q: What straight line should be plotted with $y = x^2+4x-7$ if we want to solve $x^2+2x+3 = 0$.

A: Subtract the equation to solve from the curve are given.

$y = x^2+4x-7$
minus
$0 = x^2+2x+3$
equals
$y = 2x-10$

So we should plot the straight line $y = 2x-10$.

IGCSE GRAPHS

GRADIENTS OF CURVES

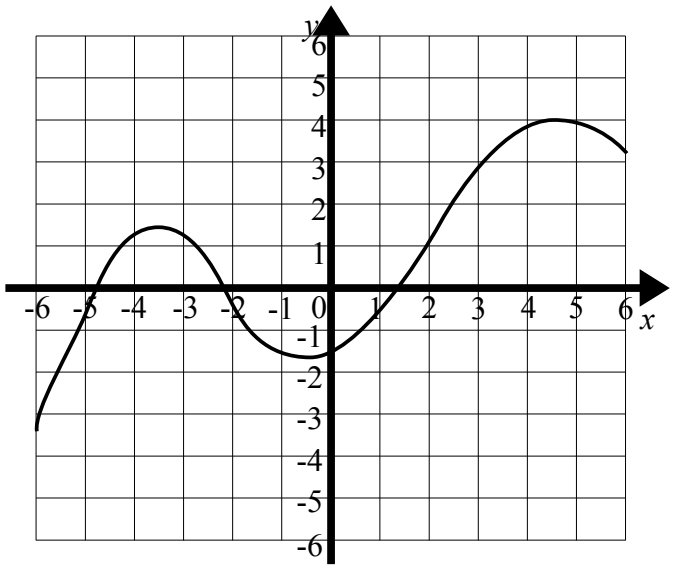
▷ To find the **gradient of a curve at a point from its graph**:

- draw the **tangent,**
- work out **rise ÷ run**.

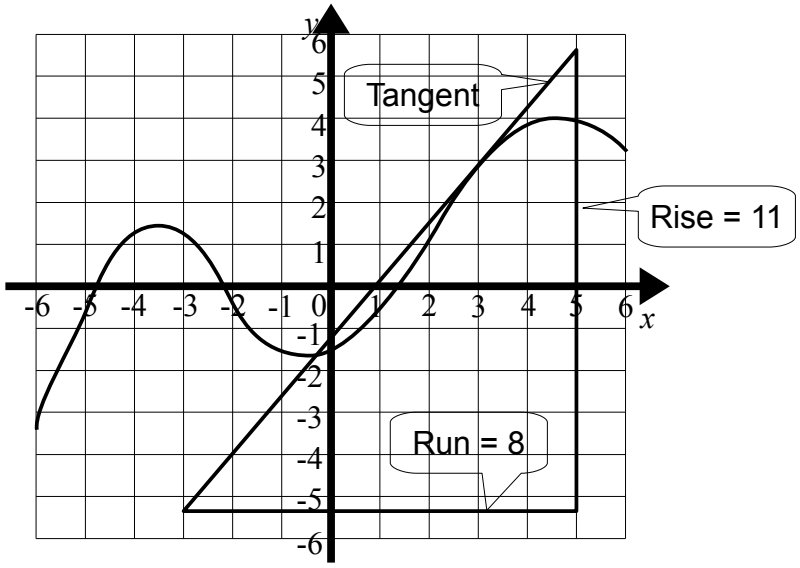
Warning: first make sure that you aren't supposed to use **dy/dx** to find the gradient.

▶ **SKILL: Find the gradient of a curve at a given point.**

Q: Find the gradient of this curve at x = 3.



A: Draw the tangent touching the curve at x = 3, then work out its rise ÷ run.



Gradient = Rise ÷ Run = 11 ÷ 8 ≈ **1.4**. [Anything close to the right answer will be fine.]

TJP TOP TIP: Make the rise ÷ run triangle **as big as possible** for maximum accuracy. Remember: it's a **negative** gradient if the tangent slopes **down** to the right.

IGCSE GRAPHS

FUNCTIONS WITH GRAPHS

▷ A **function** question may often use a **graph** instead of a formula. All you need to know is that the **domain** (what goes in) is on the **x axis** and the **range** is on the **y axis**.

▶ **SKILL: Use a graph to answer questions on functions.**

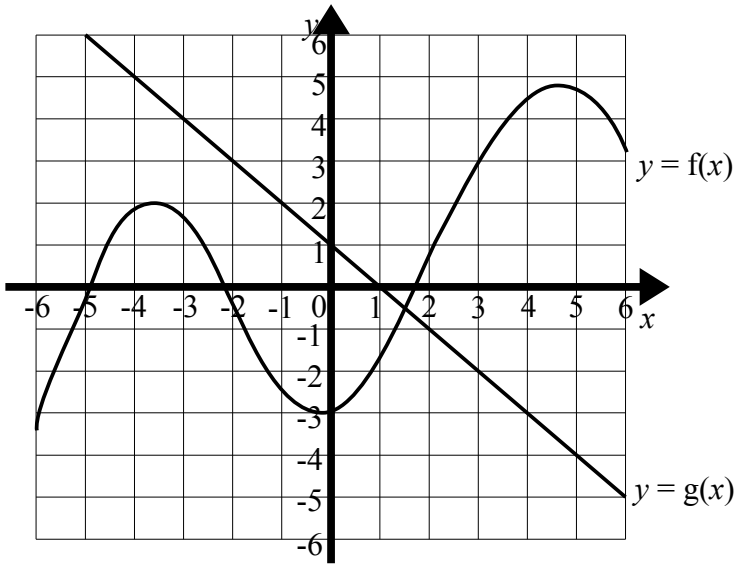
Q: Here is a graph of f(x) and g(x). Use it to find the following:

(a) f(3)

(b) fg(1)

(c) solve f(x) = 2

(d) solve f(x) = g(x)



A: (a) f(3) = **3**.

(b) fg(1) = f(g(1)) = f(0) = **-3**.

(c) f(x) = 2 at **x = -3.5** and **x = 2.5**.

(d) f(x) = g(x) where the graphs cross, at **x = 1.5**.

TRANSFORMATIONS

▷ A **transformation** is a way of changing the position, shape or size of an object.

There are many different types of transformations, and most of them correspond to the different ways you can alter a graphics object on a computer screen.

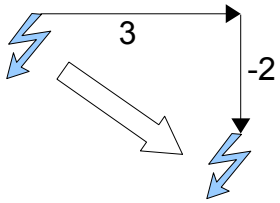
However, we only need to know **four transformations** for IGCSE. These are:

- **Translation**: a move or a shift
- **Enlargement**: a grow or a shrink
- **Reflection**: a flip
- **Rotation**: a spin

For each of these transformations, we need to have certain pieces of information.

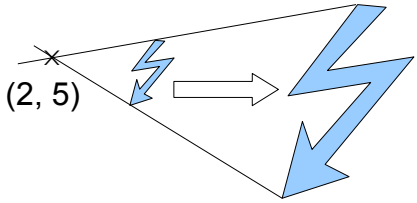
- **Translation**: the **movement** in the x and y directions as a **column vector**.

  Translate by $\begin{pmatrix} 3 \\ -2 \end{pmatrix}$ means 'move right 3 and down 2'.



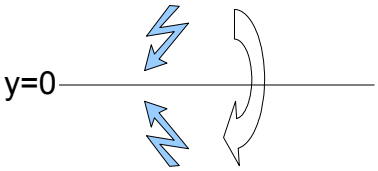
- **Enlargement**: the co-ordinates of the **centre of enlargement**, and the **scale factor**.

  Enlarge by factor 3, centre (2, 5) means 'make it three times bigger, keeping the point (2, 5) fixed'.



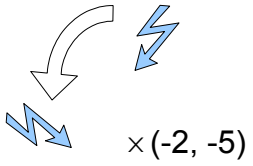
- **Reflection**: the **line of reflection** (or its equation).

  Reflect in the line y = 0 means 'use the x-axis (y=0) as a mirror line'.



- **Rotation**: the co-ordinates of the **centre of reflection**, and the **angle**.

  Rotate by +90° about (-2, -5) means 'turn the object through 90° anticlockwise about the point (-2, -5).



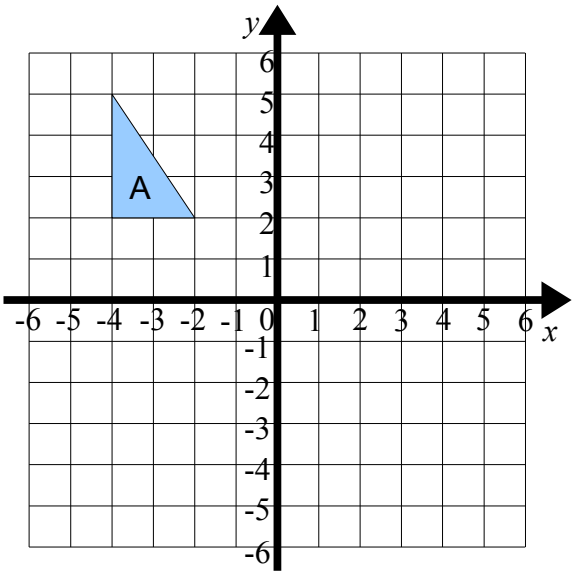
Note: a **positive** angle is taken to be **anticlockwise**, and a **negative** angle is **clockwise**.

IGCSE GRAPHS

TRANSLATIONS

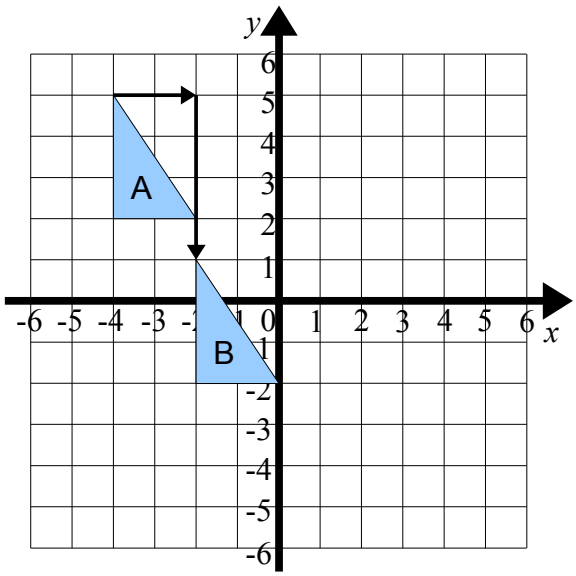
▶ **SKILL: Translate a shape by a given amount.**

Q: Translate the shape A by $\begin{pmatrix} 2 \\ -4 \end{pmatrix}$ and label the new shape B.



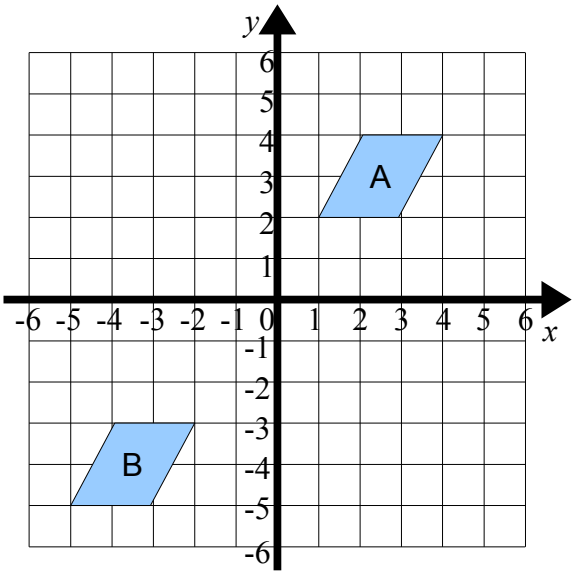
A: Move it 2 right and 4 down.

Count between corresponding corners!

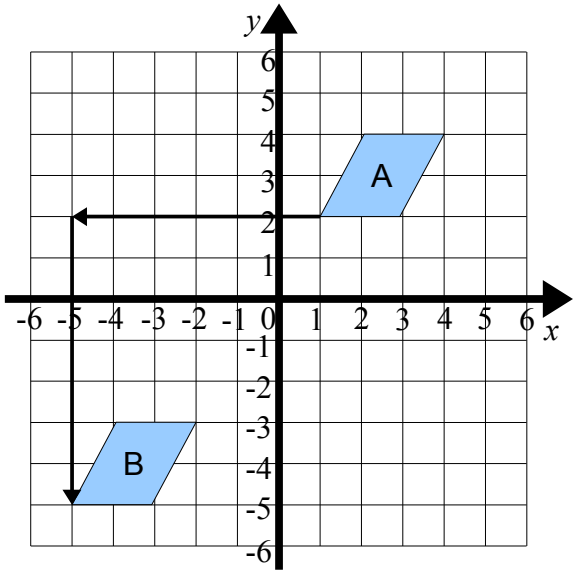


▶ **SKILL: Find a translation vector.**

Q: Find the translation that moves A onto B.



A: Pick a matching corner on both shapes and count the squares.

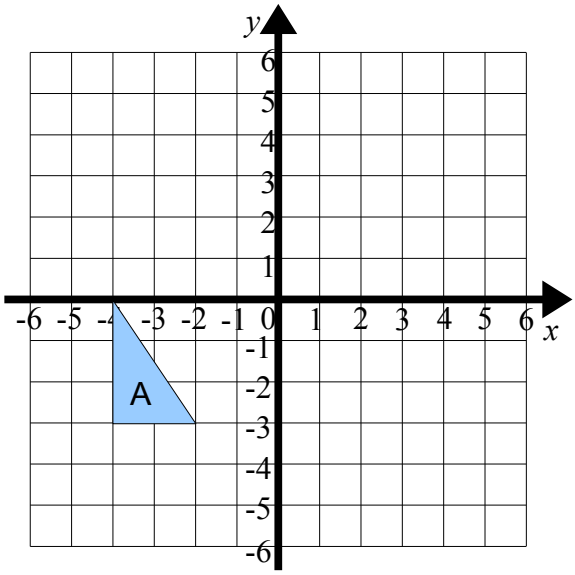


Translation of $\begin{pmatrix} -6 \\ -7 \end{pmatrix}$.

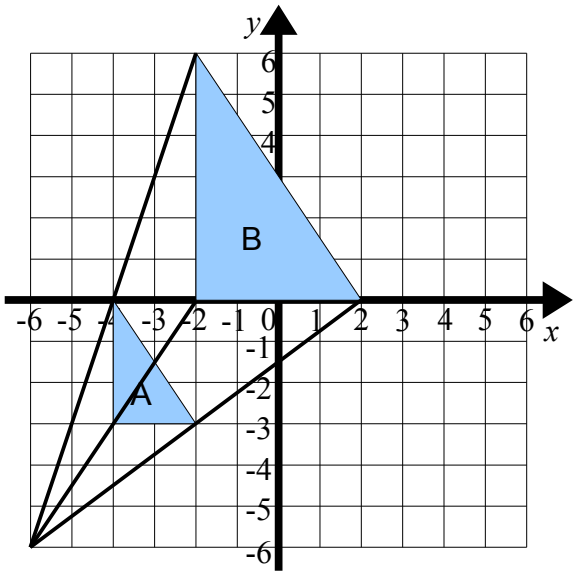
ENLARGEMENTS

▶ **SKILL: Enlarge a shape by a given scale factor from a given centre.**

Q: Enlarge shape A by factor 2, centre (-6, -6) and label the new shape B.

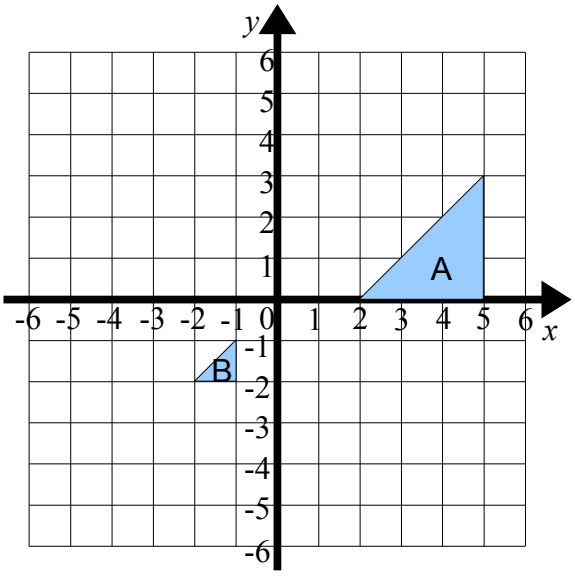


A: Draw guidelines out from (-6, -6) through the corners of A, and double the lengths. Count squares to be more accurate: for top corner, right 2 up 6 becomes right 4 up 12.

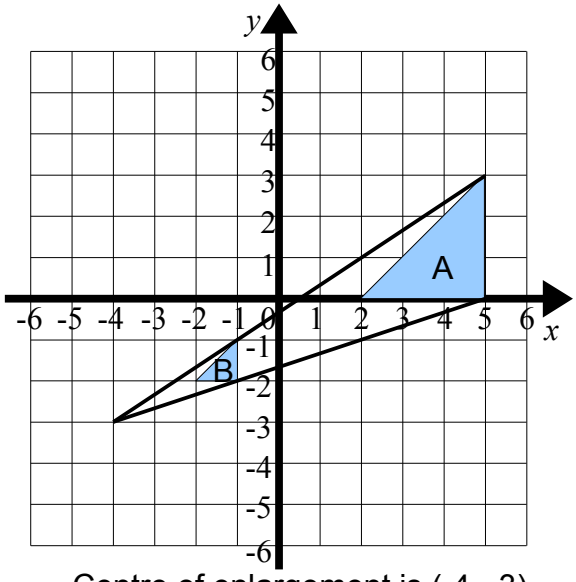


▶ **SKILL: Identify an enlargement (scale factor and centre).**

Q: Find the enlargement that turns shape A into shape B.



A: Draw straight lines through matching points on the two shapes.
Scale factor = new height ÷ old height.

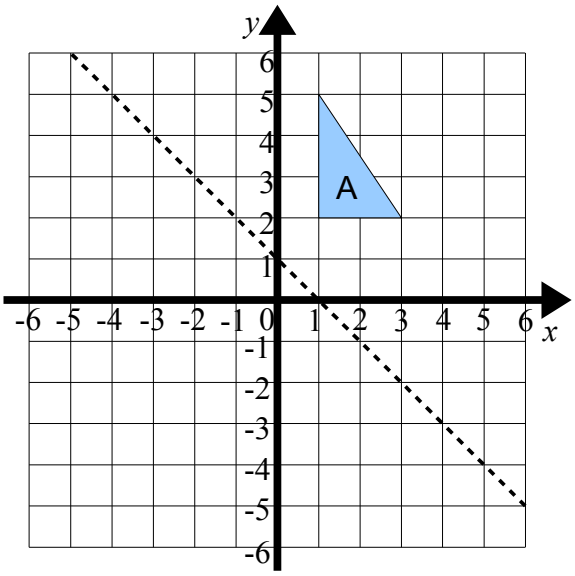


Centre of enlargement is (-4, -3),
Scale factor is ⅓.

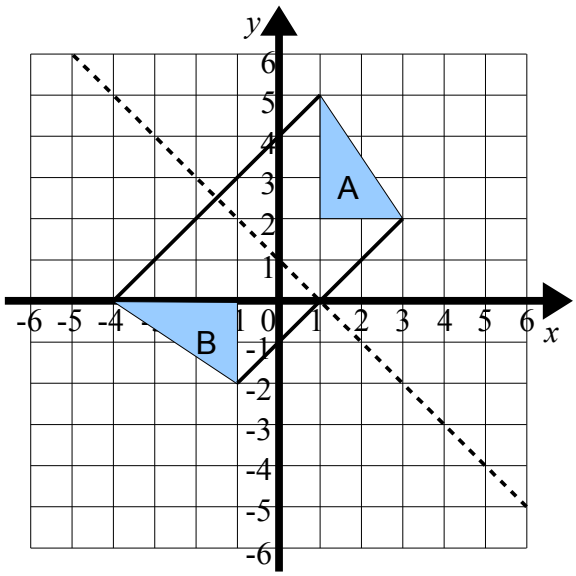
REFLECTIONS

▶ **SKILL: Reflect a shape in a given line.**

Q: Reflect shape A in the dotted line and label the new shape B.

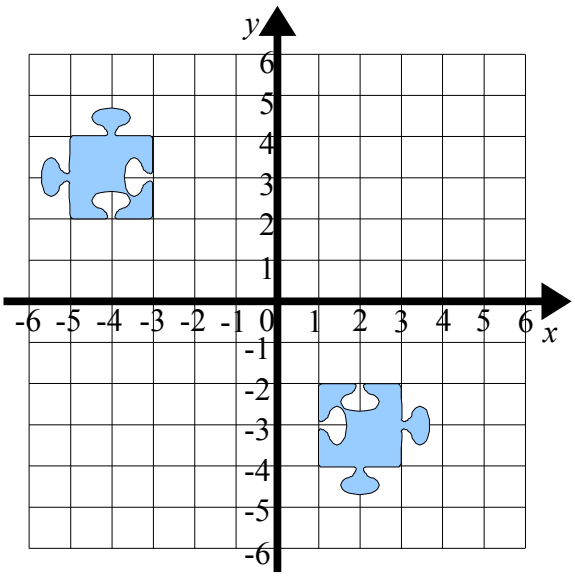


A: Count (diagonally) to the mirror line, then count the same distance beyond.

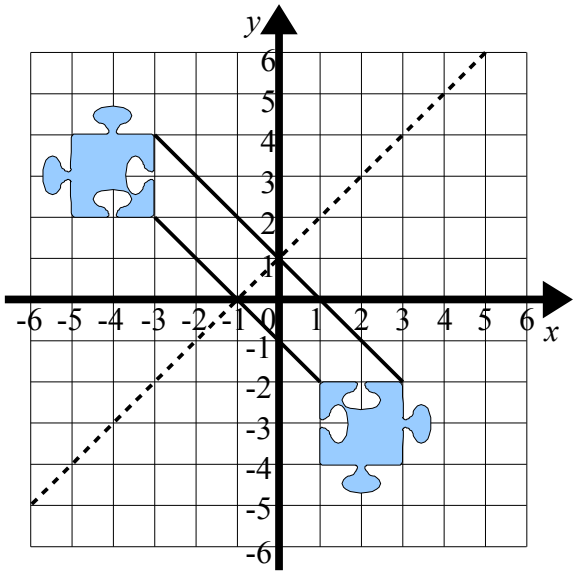


▶ **SKILL: Find a line of reflection.**

Q: Give the equation of the line of reflection.



A: Join matching points on the two shapes; the mirror line is exactly half way between the shapes.

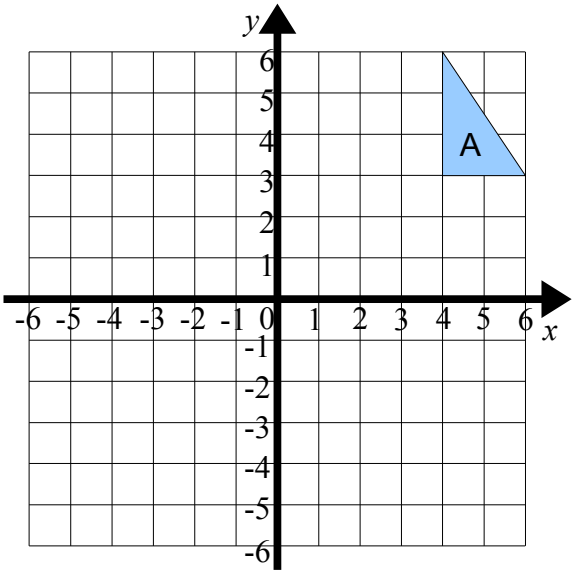


The line of reflection is y = x + 1.

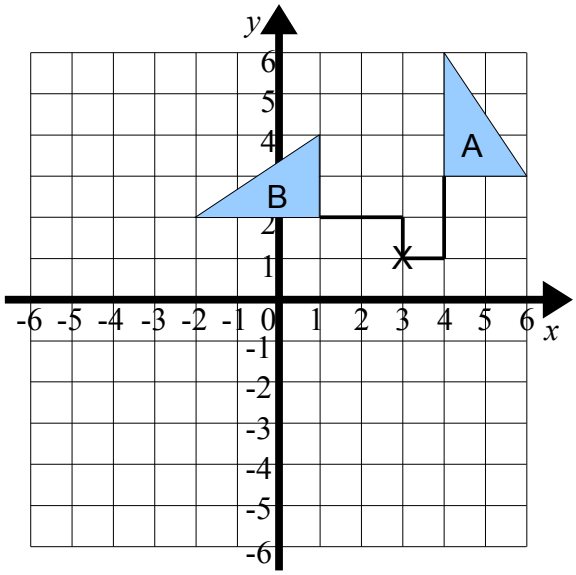
ROTATIONS

▶ **SKILL: Rotate a shape by a given angle around a given point.**

Q: Rotate A by 90° about (3,1) and label the new shape B.



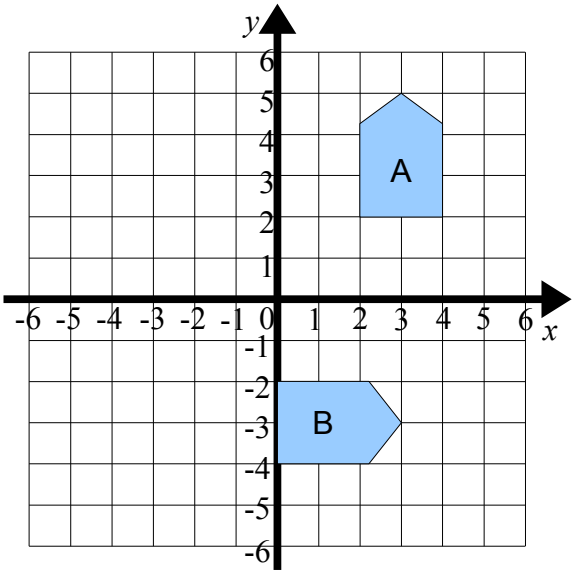
A: Draw an 'L' from the centre of rotation to the shape, and rotate the L.



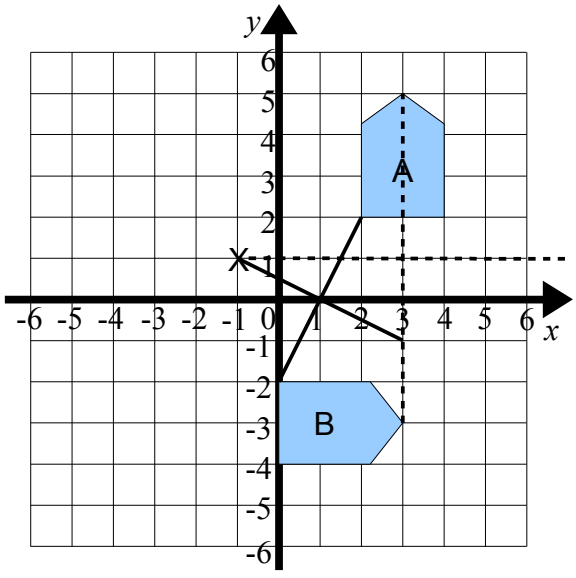
IGCSE INSIDER INFO: Ask for **tracing paper** in the exam to help you do rotations.

▶ **SKILL: Identify a rotation (angle and centre).**

Q: Find the angle and centre of rotation that moves shape A onto shape B.



A: Join matching points with a '+' shape. These focus to the centre of rotation.



Rotation angle is –90° (90° clockwise), Centre of rotation is (-1, 1).

IGCSE GRAPHS

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