TJP TOP TIPS

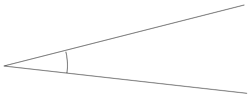
FOR

IGCSE SHAPE & SPACE

IGCSE SHAPE & SPACE

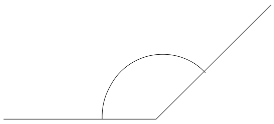
First, some important words; know what they mean (get someone to test you):

Acute angle: less than 90°.

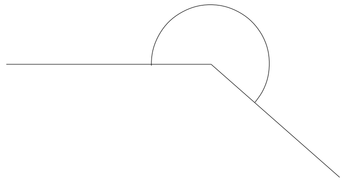


Right angle: exactly 90°.

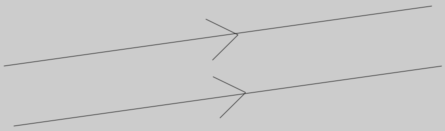
Obtuse angle: between 90° and 180°.



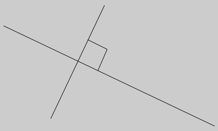
Reflex angle: greater than 180°.



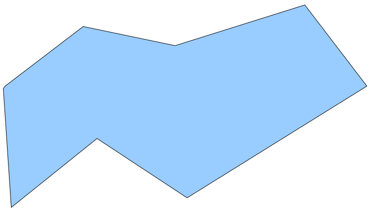
Parallel: two or more lines going in the **same direction** (with the same gradient).



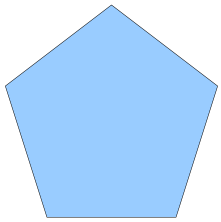
Perpendicular: two lines at **90°** to each other.



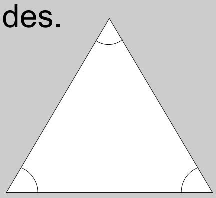
Polygon: a **closed 2-D shape** with **straight sides**.



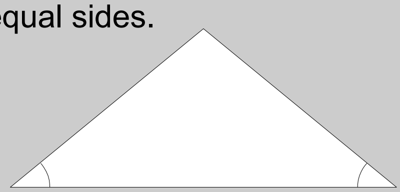
Regular Polygon: a polygon with **all angles equal** and all sides equal.



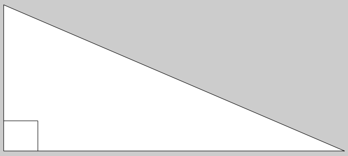
Equilateral triangle: **three equal angles** and three equal sides.



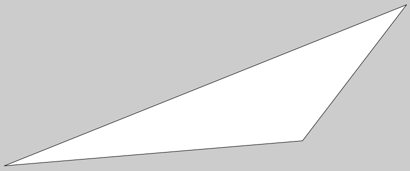
Isosceles triangle: two equal angles and two equal sides.



Right-angled triangle: one angle of **90°**.

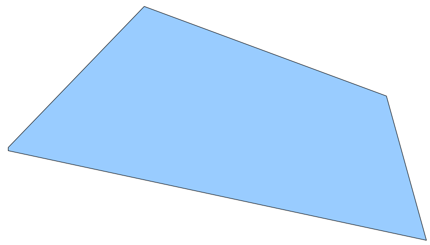


Scalene triangle: no special angles or sides.

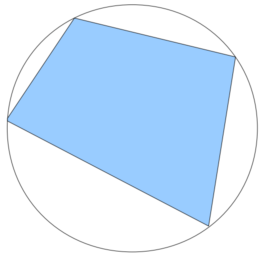


IGCSE SHAPE & SPACE

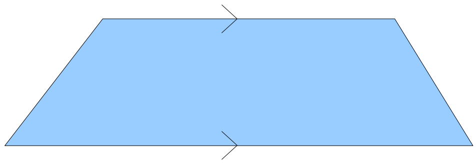
Quadrilateral: a **four-sided** 2-D shape.



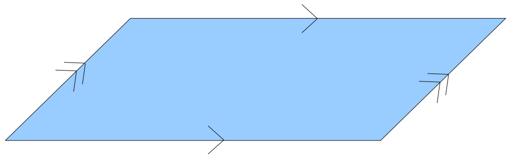
Cyclic quadrilateral: a four-sided shape whose **corners lie on a circle**.



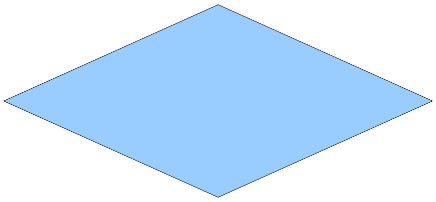
Trapezium: a shape with **one pair** of **parallel sides**.



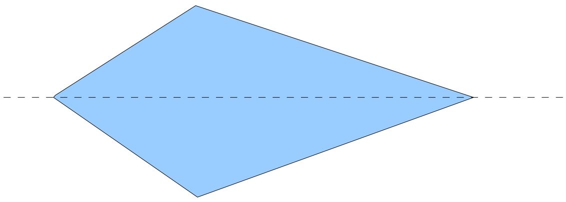
Parallelogram: a shape with **two pairs** of **parallel sides**.



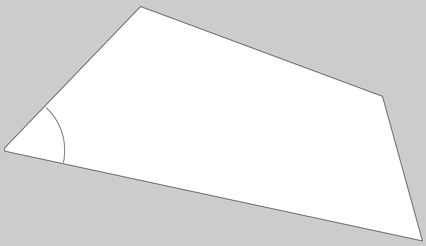
Rhombus: a parallelogram which has **all sides the same length** (a diamond).



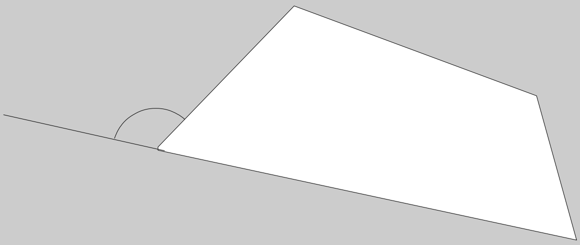
Kite: a shape with a **line of symmetry** passing through **two opposite corners**.



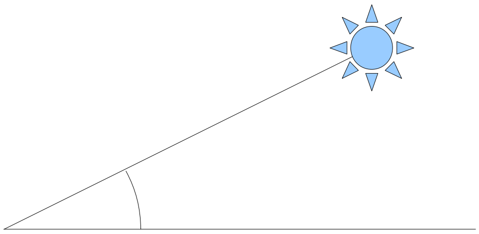
Interior angle: an angle **inside** a corner of a polygon.



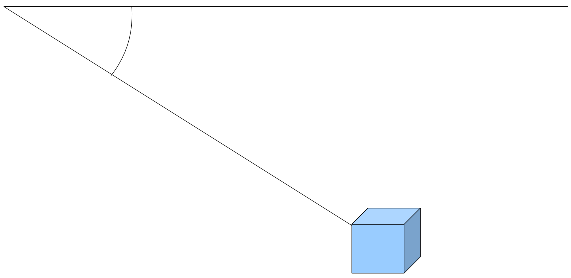
Exterior angle: the angle the **line turns through** at each corner (the 'turtle' angle).



Angle of elevation: the **angle above** the **horizontal**.



Angle of depression: the **angle below** the **horizontal**.



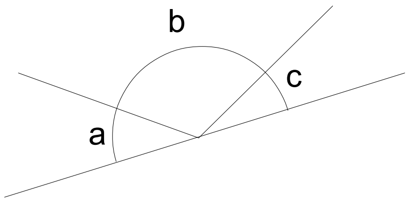
IGCSE SHAPE & SPACE

ANGLES

▷ **Angles** are measured in **degrees (°)** so that there are **360 degrees in a full circle**. Why 360? Well, it's probably to do with the number of days in a year, combined with the fact that lots of numbers go into 360 exactly (it has many factors).

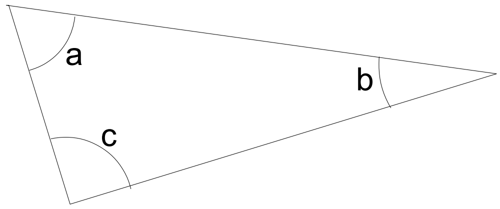
Here are some more **important angle facts**:

Angles on a straight line add up to 180°.



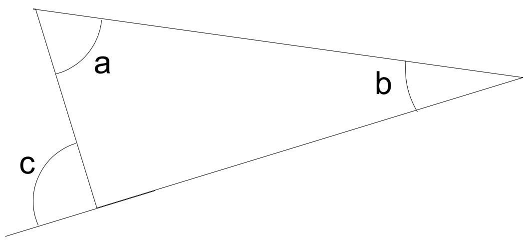
a + b + c = 180

Angles in a triangle (interior angles) add up to 180°.



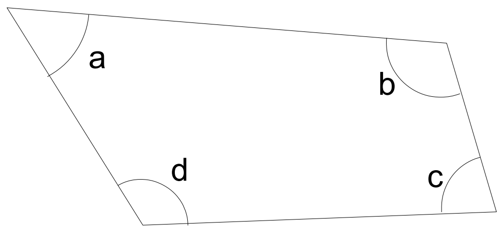
a + b + c = 180

Exterior angle in a triangle is the sum of the other two interior angles.



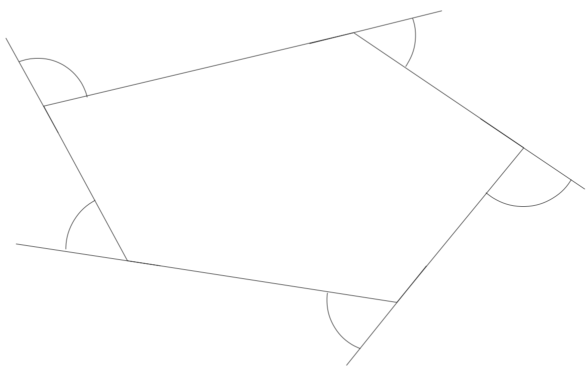
a + b = c

Angles in a quadrilateral add up to 360°.



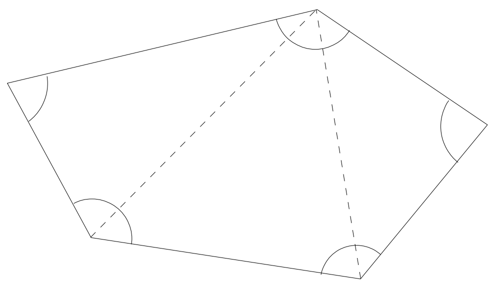
a + b + c + d = 360

Exterior angles in a polygon add up to 360°, always.



If you walk around the shape once, you turn through a total angle of 360°.

Interior angles in an n-sided polygon add up to 180(n – 2)°.

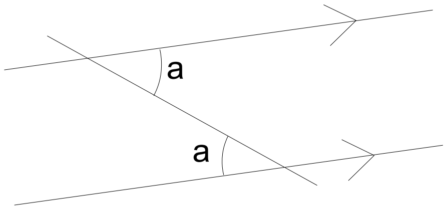


You can divide an n-sided polygon into n – 2 triangles, each 'worth' 180°.

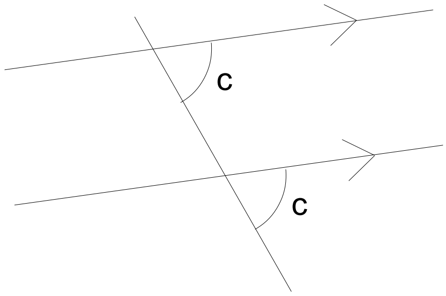
Exterior angle of a regular polygon with n sides is **360÷n°**.

Interior angle of a regular polygon with n sides is **180(n – 2)÷n°**.

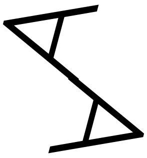
Alternate angles are equal.



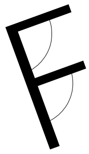
Corresponding angles are equal.



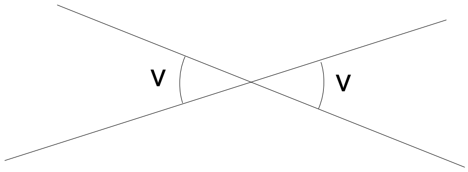
HB HELPFUL HINT: This is like an A and and A joined together, spelling **A**lternate **A**ngle.



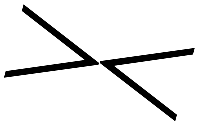
TJP TOP TIP: These are **'F'-ing Corresponding** Angles.



Vertically opposite angles are equal.



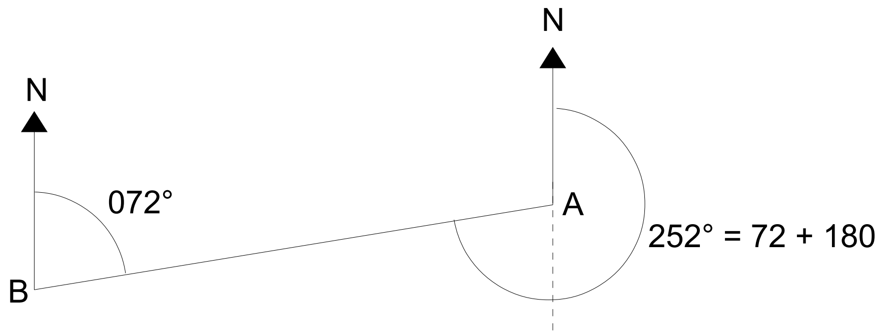
TJP TOP TIP: This is a V and a V (for **V**ertical), **O**pposite one another.



IGCSE INSIDER INFO: You **must** use the **proper names**: 'Z angle', 'F angle' and 'X angle' will score **zero marks**.

▷ A **Bearing** is simply an **angle** measured **clockwise** from **North**. Bearings are often used in navigation where a compass is used to find the direction of North.

The **bearing of A from B** means we are **at B, going towards A**.
The bearing of B from A is **180° different** from the bearing of A from B.



CONSTRUCTION

▷ You are expected to be able to **draw** and **measure**:

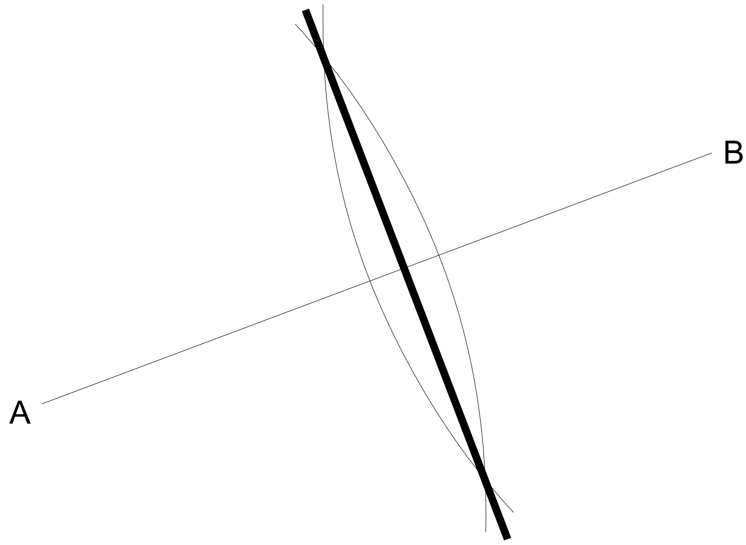
- a **straight line** to the nearest **mm**
- an **angle** to the nearest **degree**

so bring a **sharp pencil**, a **sharpener**, an **eraser**, a **ruler** and a **protractor**.

You will also need a **pair of compasses** for the following methods:

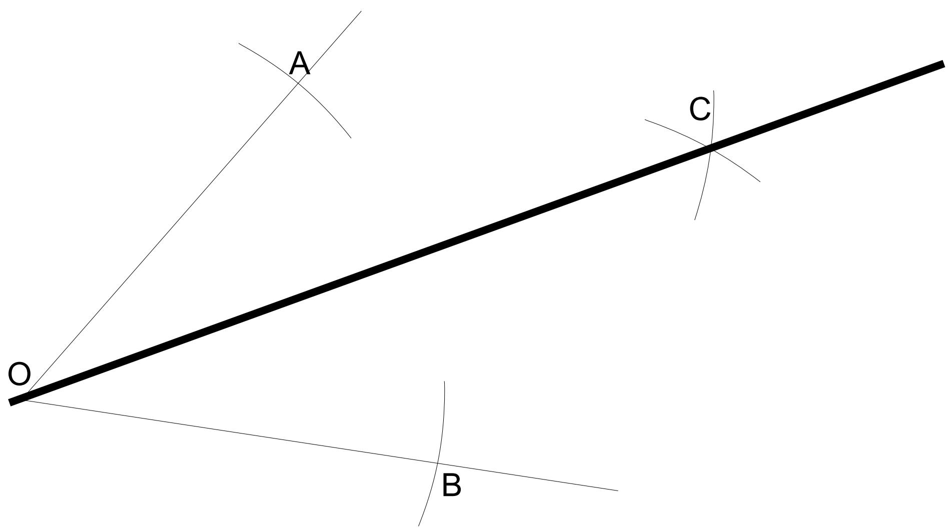
▶ **SKILL: Find the perpendicular bisector of a line segment AB** (without measuring).

- Set your compasses to a little over half way between the line ends A and B.
- Draw arcs with the compass point on A and then on B.
- Draw a straight line through the points where these arcs cross one another.



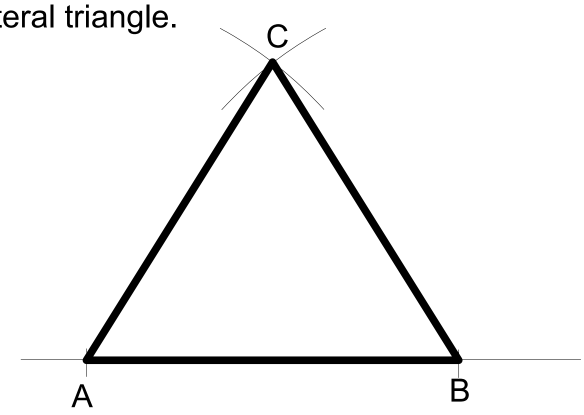
▶ **SKILL: Bisect an angle** (without a protractor).

- Set your compasses to a fixed radius, just shorter than the lines making the angle.
- Put the compass point on the angle corner O and draw an arc cutting each line at A, B.
- Then draw arcs with the compass point on A and B in turn to make a rhombus OACB.
- Draw a line through OC; this bisects the angle.

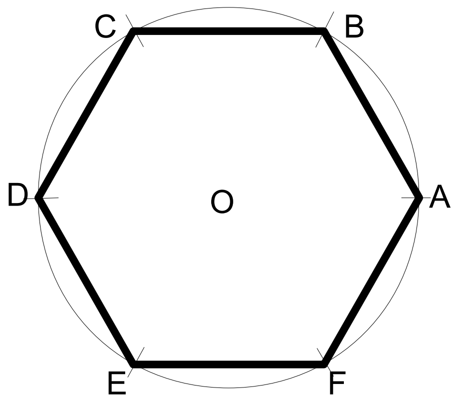


▶ **SKILL: Construct triangles and other 2-D shapes** (compasses and straightedge).

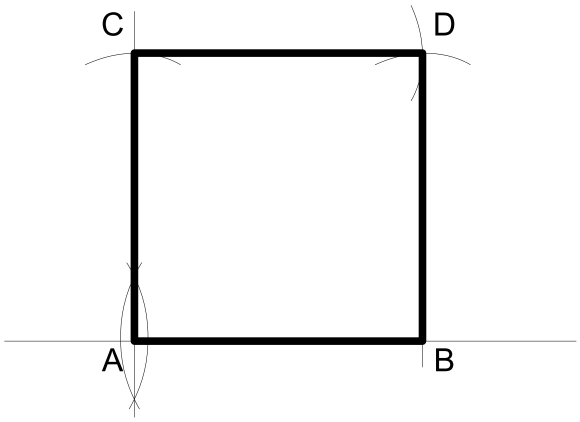
Equilateral Triangle:
- Given a baseline AB, set your compasses to this distance.
- Draw two arcs centred on points A and B so that the arcs cross at point C.
- ABC then forms an equilateral triangle.



Regular Hexagon:
- Set your compasses to the required side length.
- Draw a circle centre O.
- Use the compasses to mark off the side length all around the circumference.
- Join the marks to construct the hexagon ABCDEF.

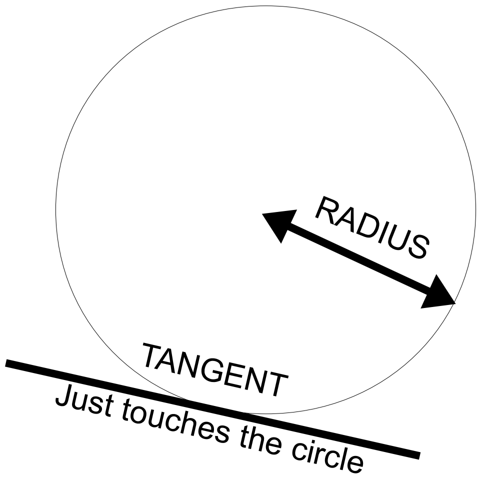


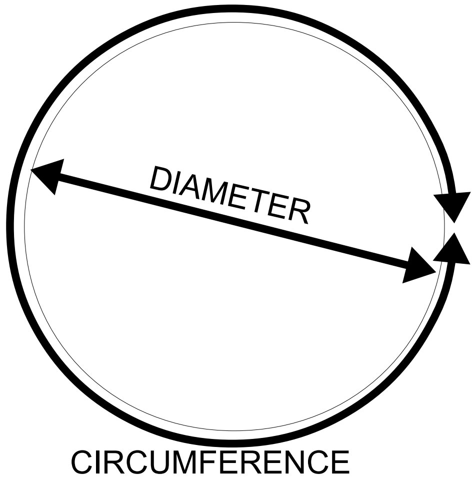
Square:
- Given a baseline AB, construct a perpendicular line at A (see previous page).
- Use your compasses to mark off the distance AB up this line to give C.
- Using this same radius, now draw arcs centred on C and B, crossing at D.



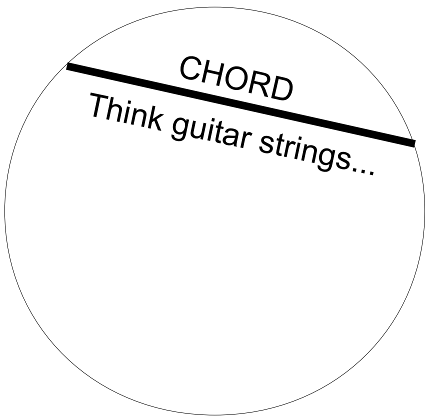
PARTS OF CIRCLES

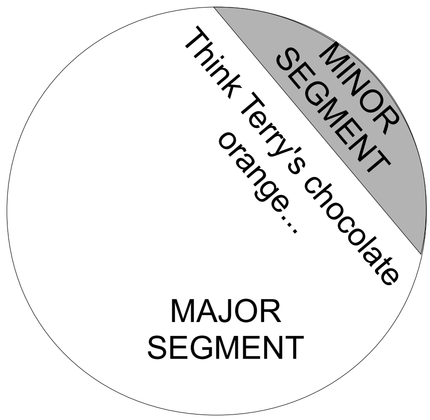
▷ **Learn** all the following words for parts of circles...

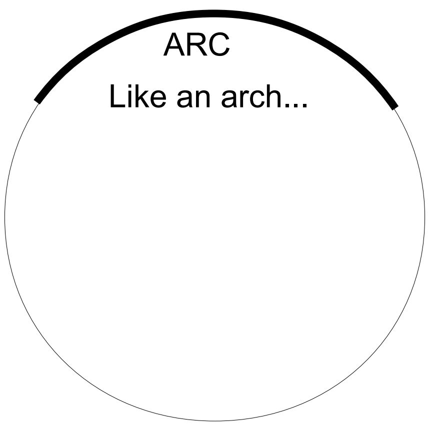


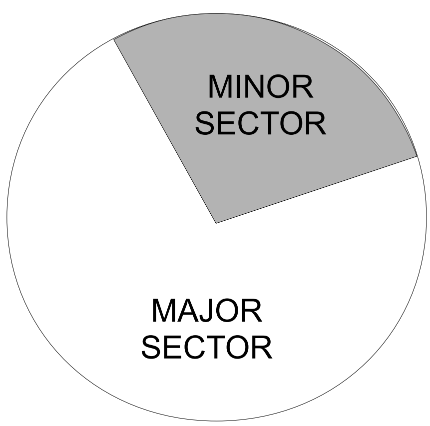


Radius < Diameter < Circumference, just like the length of the words...

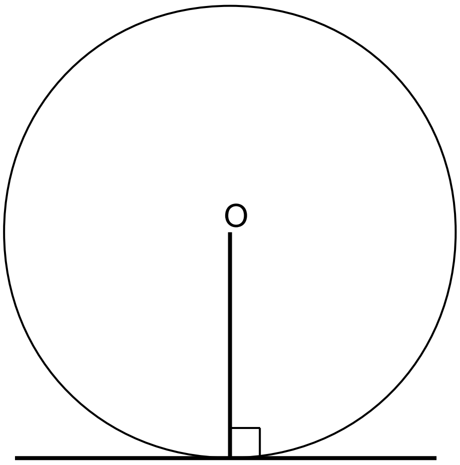




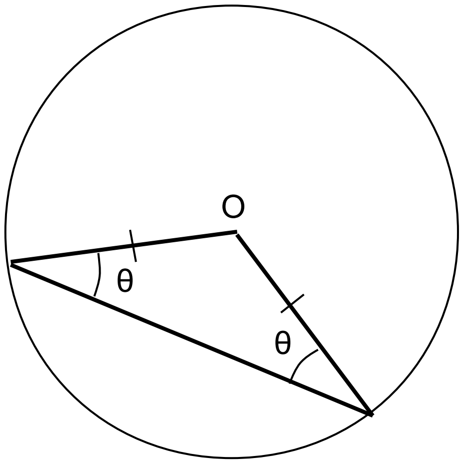




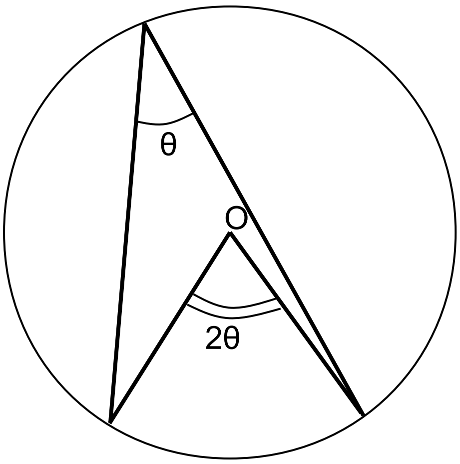
CIRCLE THEOREMS



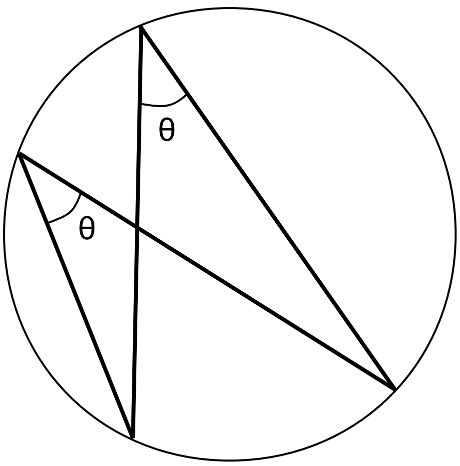
Radius meets tangent at 90°.



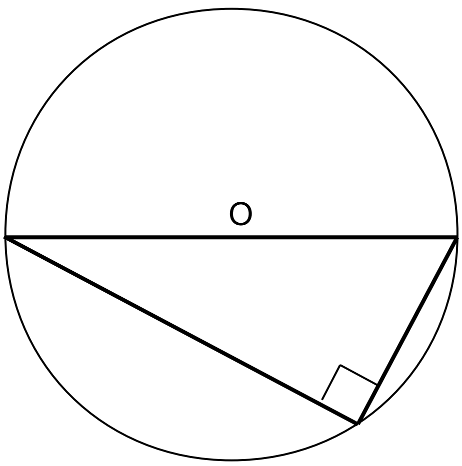
Isosceles triangle (with two corners on the circumference and one at the centre).



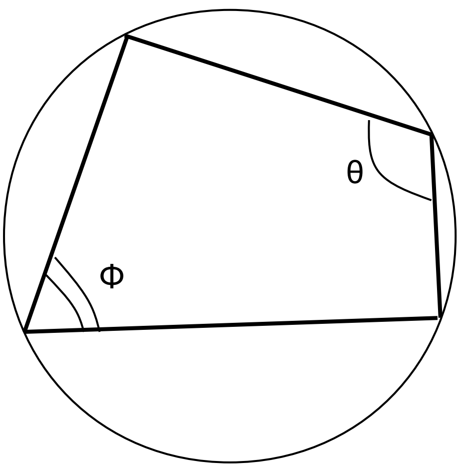
Angle at the centre is twice the angle at the circumference.



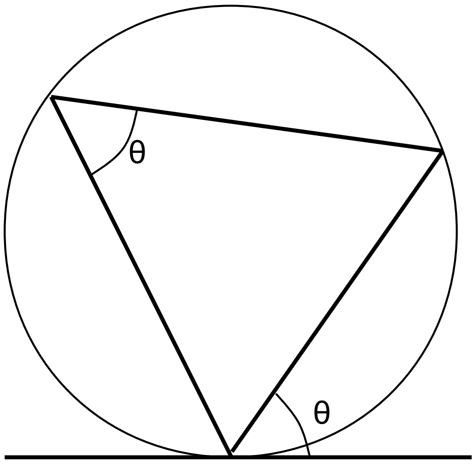
Angles in the same segment are equal.

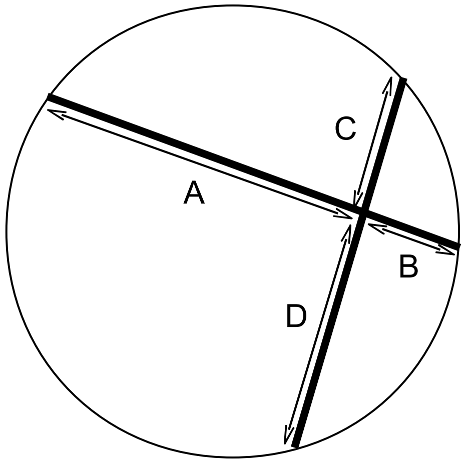


Angle in a semicircle = 90°.



Opposite angles in a cyclic quadrilateral add up to 180°. $\theta + \phi = 180°$.

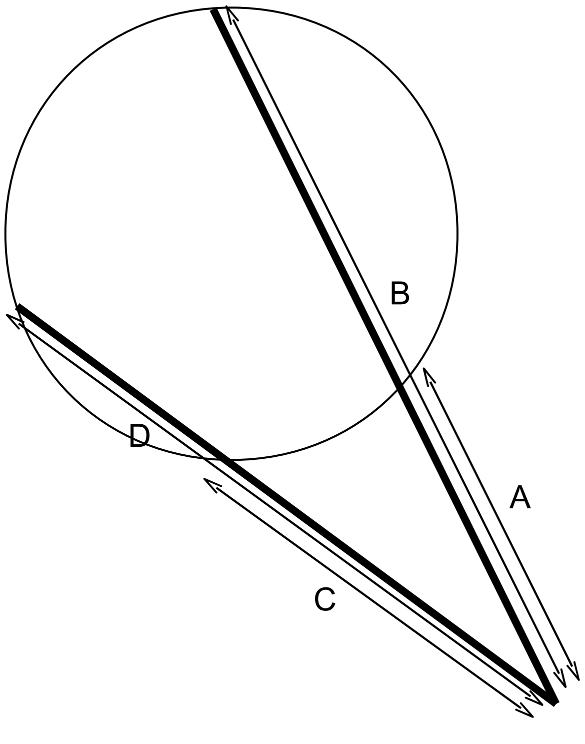


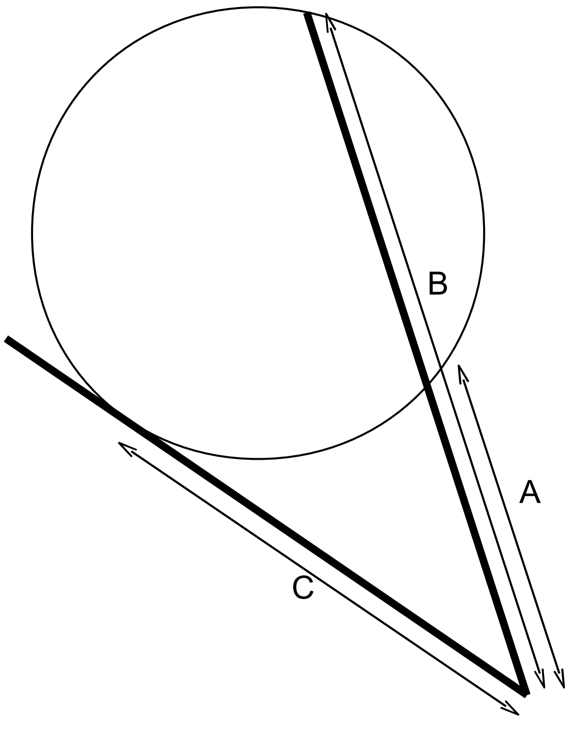


Alternate segment theorem.
Marked angles are equal.

Intersecting Chords (I). A×B = C×D

HB HELPFUL HINT: The lines make a × shape, so we × the lengths together.





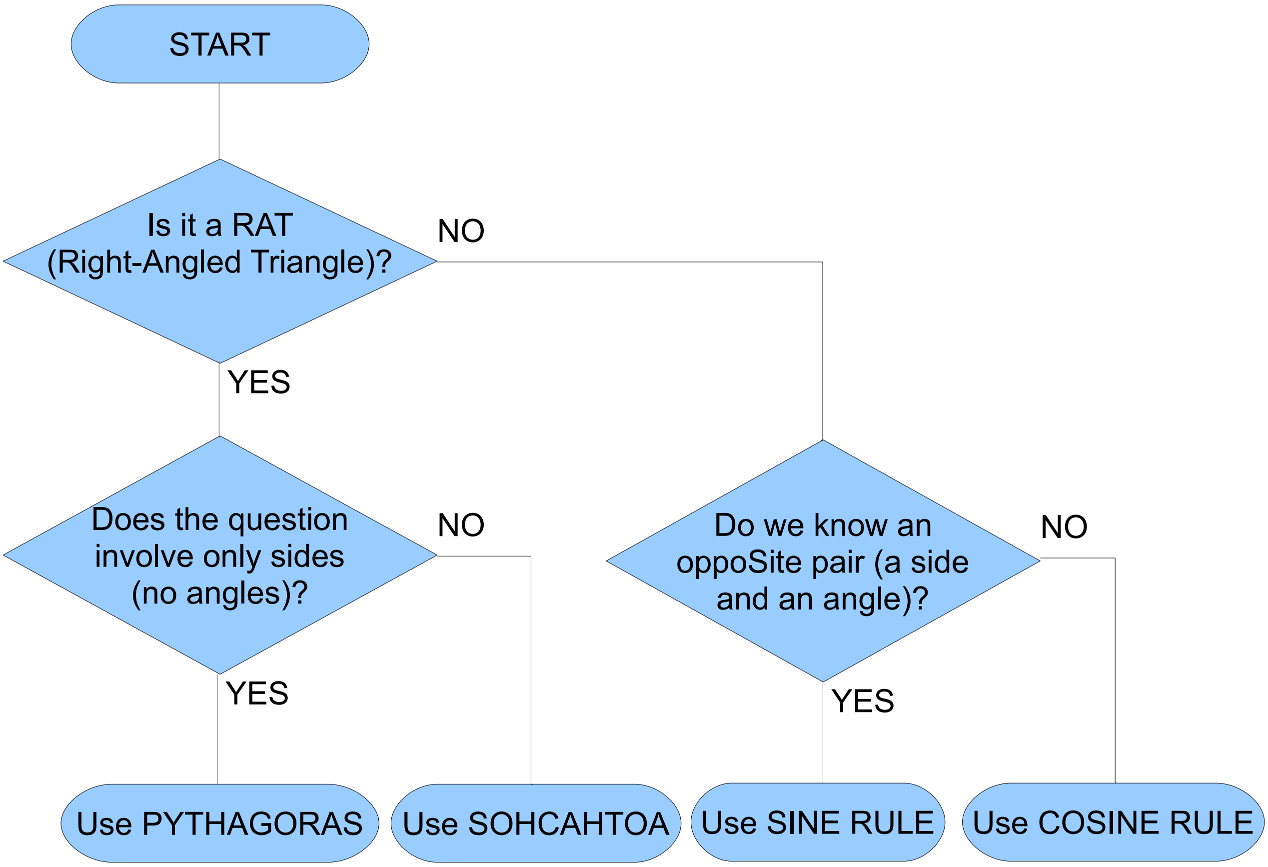
Intersecting Chords (II). A×B = C×D

Intersecting Chords (III). A×B = C²

IGCSE INSIDER INFO: You have to give a **reason** for each step when finding the unknown angles in a question. Use the **official names** above, not 'Star Trek', etc.

FORMULAE FOR TRIANGLES

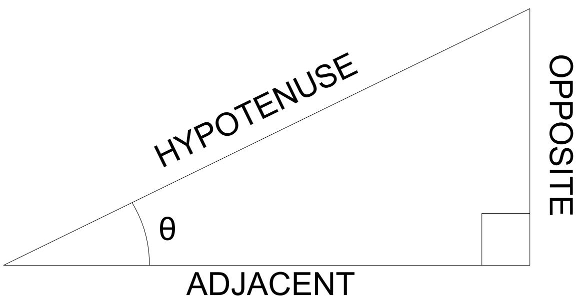
Which formula do I use to find angles or sides? Here's a flow chart to help...



All these formulae (except for one) are given inside the front cover of your IGCSE paper.

ANATOMY OF A RAT

▷ Learn these special names for the sides of a right-angled triangle.



The **Hypotenuse** is always opposite the right angle (Pythagoras and SOHCAHTOA) and it is the **longest side**.

The **Opposite** is opposite the given angle (SOHCAHTOA).

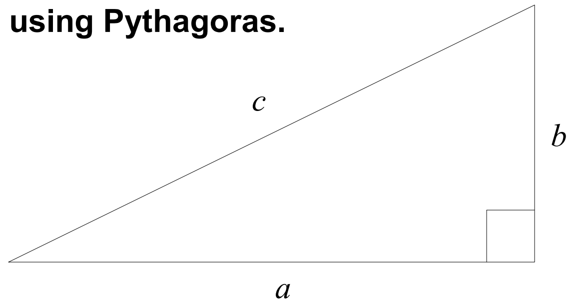
The **Adjacent** is next to (adjacent to) the given angle (SOHCAHTOA).

PYTHAGORAS

▷ If we know two of the three sides of a RAT (Right-Angled Triangle), we can find the missing side using **Pythagoras' Theorem**.

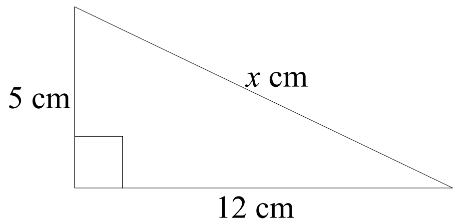
▶ **SKILL: Find an unknown side of a RAT using Pythagoras.**

Use $a^2+b^2 = c^2$
where c^2 is the hypotenuse.



TJP TOP TIP: Go ahead and **relabel** the triangle if you like, to get c in the right place on the hypotenuse. Just remember to **change back** to the correct letter at the end.

Q: Find the length of side x in this triangle.



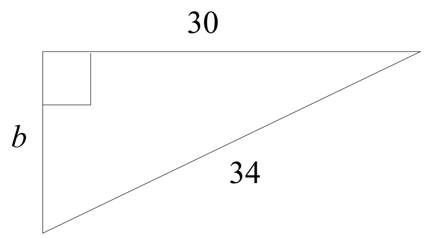
A: Use $a^2+b^2 = c^2$ substituting to get

$5^2+12^2 = x^2$

$25+144 = 169 = x^2$

So $x = \sqrt{169} = 13$; the side has length 13 cm.

Q: Find the value of b.



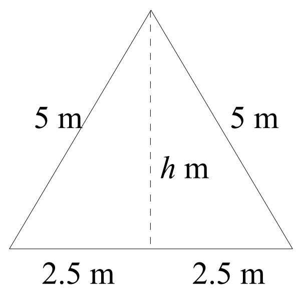
A: $b^2+30^2 = 34^2$

$b^2+900 = 1156$

$b^2 = 1156-900 = 256$

So $b = \sqrt{256} = 16$.

Q: Find the height of an equilateral triangle of side 5 m.



A: Hint: split it into two RATs!

$2.5^2+h^2 = 5^2$

$6.25+h^2 = 25$

$h^2 = 25-6.25 = 18.75$

So $h = \sqrt{18.75} = 4.33$. The height is 4.33 m (3 sig figs).

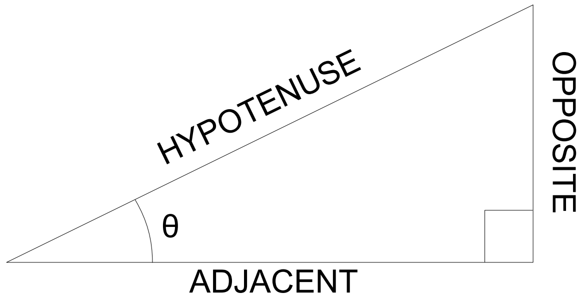
TRIGONOMETRY (SOHCAHTOA)

▷ **Trigonometry** gives the connection between the sides and the angles of a RAT.

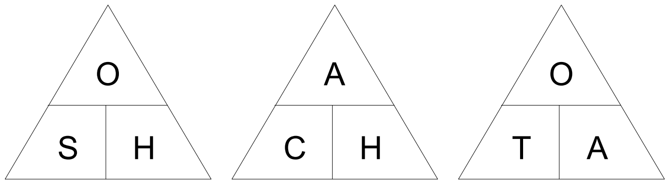
$$\sin\theta = \frac{Opp}{Hyp}$$

$$\cos\theta = \frac{Adj}{Hyp}$$

$$\tan\theta = \frac{Opp}{Adj}$$



▶ **Learn these triangles**
(they spell SOHCAHTOA)

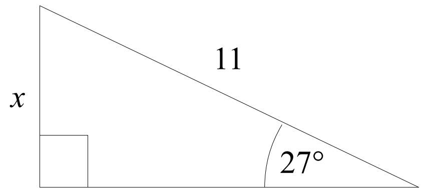


Decide which SOHCAHTOA triangle to use by seeing which side does **not** feature in the question (as either a number or an unknown letter).

Then cover up the letter you are trying to find; the remaining two letters give the formula you need. For example, to find H in the SOH triangle, we get O ÷ S.
Two letters on the bottom line are multiplied; a letter on top is divided by one underneath.

▶ **SKILL: Find an unknown side of a RAT using SOHCAHTOA.**

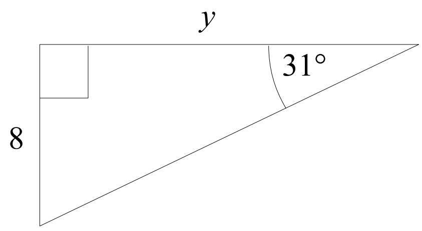
Q: Find the length x.



A: The Adjacent side does not feature in this question, so use **SOH** (no Adj).

We are finding the Opposite side, so from the SOH triangle, Opp = Sin × Hyp.
$x = \sin(27)\times 11 = 4.99$ (3 sig figs).

Q: Find the length y.



A: The Hypotenuse does not feature in this question, so use **TOA** (no Hyp).

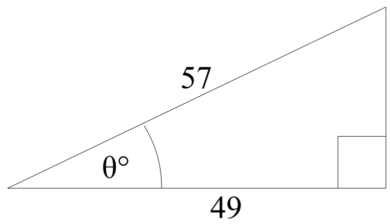
We are finding the Adjacent side, so from the TOA triangle, Adj = Opp ÷ Tan
$y = 8 \div \tan(31) = 13.3$ (3 sig figs).

▶ **SKILL: Find an unknown angle of a RAT using SOHCAHTOA.**

TJP TOP TIP: Always use the **SHIFT** button to find an **angle**.
This gives the inverse sin, cos or tan, written as $\sin^{-1}$, $\cos^{-1}$, $\tan^{-1}$.

If you get **error** on your calculator, you have got your numbers the wrong way round.

Q: Find the angle θ in this triangle.



A: The Opposite does not feature in this question, so use **CAH** (no Opp).

We are finding Cos, so from the CAH triangle, Cos = Adj ÷ Hyp

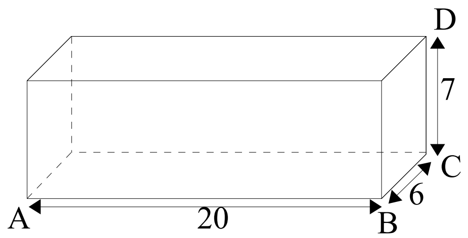
$\theta = \cos^{-1}(49 \div 57) = 30.7°$ (1 dp).

THREE-DIMENSIONAL PROBLEMS

▷ We may get a **3-D question** which we have to break down into 2-D RATs in order to find a mystery length or angle.

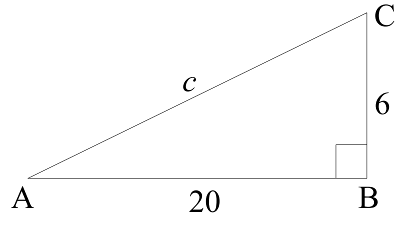
▶ **SKILL: Use Pythagoras and Trigonometry to solve a 3-D problem.**

Q: Find the long diagonal AD of this cuboid, and also angle DAC.



A: Find diagonal AC first, using this triangle:

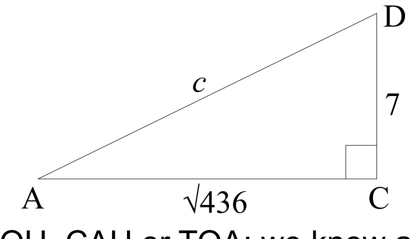
$AC^2 = 20^2 + 6^2 = 436$



Then find AD using this triangle:

$AD^2 = 436 + 7^2 = 485$

$AD = \sqrt{485} = 22.0$ (3 sig figs)



To find DAC (the angle at A), we could use SOH, CAH or TOA; we know all the sides. Here we'll use TOA.

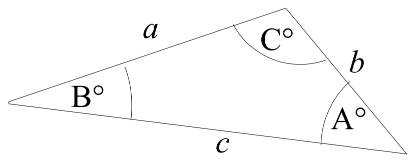
$DAC = \tan^{-1}(7 \div \sqrt{436}) = 18.5°$ (1 dp).

SINE RULE

▷ If we **know** the values of an **opposite pair** (a side and the angle opposite it) in a non right-angled triangle (**non-RAT**), we use the **Sine Rule**.

TJP TOP TIP: OppoSite pair ⇨ **S**ine Rule.

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

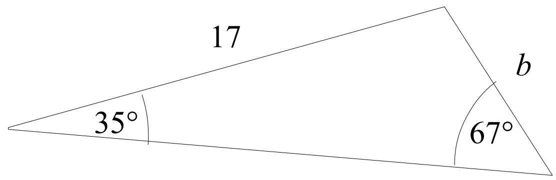


IGCSE INSIDER INFO: You can flip the printed formula upside-down to get the unknown quantity on the top line – this makes life much easier.

Note: little letters are sides, CAPITAL letters are angles, and a is opposite A, b opposite B, etc.

▶ **SKILL: Use the Sine Rule to find an unknown side.**

Q: Find the value of side b in this triangle.

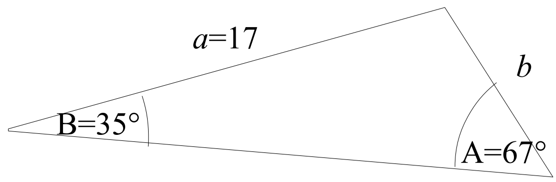


A: First, label the sides and angles with a, A and B (we don't need c and C here).

$$\frac{17}{\sin 67} = \frac{b}{\sin 35}$$

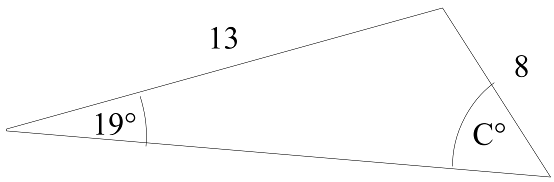
$$\frac{17}{\sin 67} \times \sin 35 = b$$

$b = 10.6$ (3 sig figs)



▶ **SKILL: Use the Sine Rule to find an unknown angle.**

Q: Find the value of angle C in this triangle.



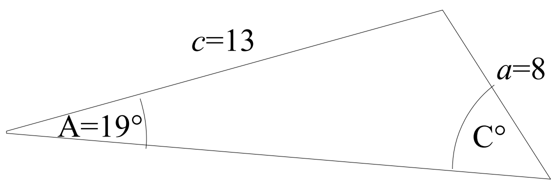
A: Label the other sides and angles involved in the question.
Then use the **flipped** version of the formula to get the angle on the top.
[Note: we could have used b and B instead of a and A.]

$$\frac{\sin 19}{8} = \frac{\sin C}{13}$$

$$\frac{\sin 19}{8} \times 13 = \sin C$$

$\sin C = 0.529048$

$C = \sin^{-1}(0.529048) = 31.9°$ (1 dp).

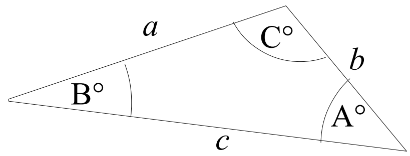


COSINE RULE

▷ If we **don't know** the values of an **opposite pair** (a side and the angle opposite it) in a non right-angled triangle (**non-RAT**), we use the **Cosine Rule**.

TJP TOP TIP: **No** Sine Rule ⇨ **Co**sine Rule.

$$a^2 = b^2 + c^2 - 2bc\cos A$$



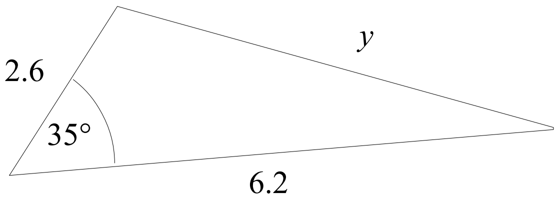
Be careful with BIDMAS when working out this formula. Don't press '=' until the end...

And remember to square root!

▶ **SKILL: Use the Cosine Rule to find an unknown side.**

TJP TOP TIP: Relabel the triangle so that the side you are finding is called a. 'Cos that's what is on the left hand side of the formula...

Q: Find side y.



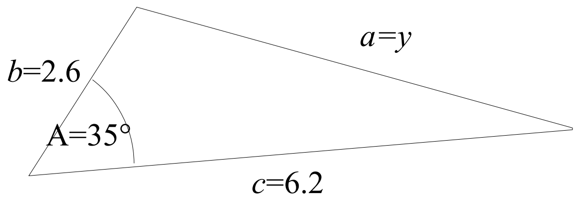
A: Relabel the triangle with $a = y$, and put angle A opposite it.
It doesn't matter where you put b and c, by the way...

$$a^2 = 2.6^2 + 6.2^2 - 2 \times 2.6 \times 6.2 \cos 35$$

$$a^2 = 6.76 + 38.44 - 32.24 \cos 35$$

$$a^2 = 18.79$$

$a = y = 4.33$ (3 sig figs).

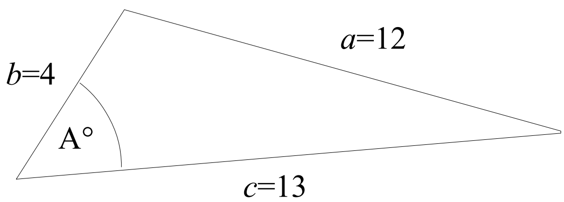


▶ **SKILL: Use the Cosine Rule to find an unknown angle.**

IGCSE INSIDER INFO: **LEARN THIS FORMULA!** $\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$

It's **not** given in the front of the exam paper.

Q: Find angle A.



A: $\cos A = \dfrac{4^2 + 13^2 - 12^2}{2 \times 4 \times 13}$

$$\cos A = \frac{41}{104} = 0.39423$$

$$A = \cos^{-1}(0.39423) = 66.8\,°.$$

AREA OF A TRIANGLE

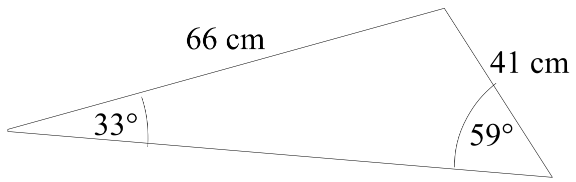
▷ The **area of a triangle** is ½ base × height, but if we have a non right-angled triangle the height may not be obvious.

Fortunately, we are given a formula to work out the area. Area = $A = \frac{1}{2}ab\sin C$

TJP TOP TIP: Relabel the triangle so that the angle is called C, sandwiched between sides a and b.

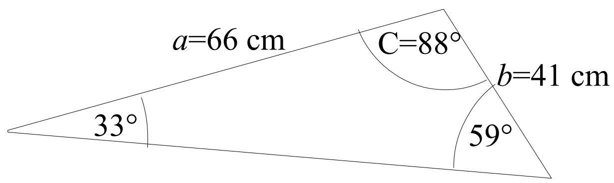
▶ **SKILL: Find the area of a non right-angled triangle.**

Q: Calculate the area of this triangle.



A: A sneaky question; we first have to fill in the missing angle at the top, which is sandwiched between the two given sides.

This angle is 180 – 33 – 59 = 88°.

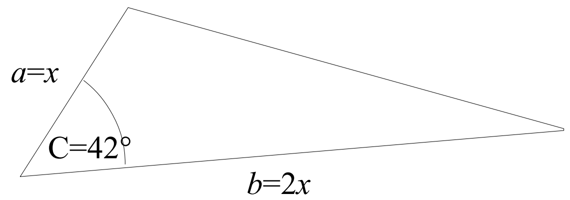


$A = \frac{1}{2}\times 66\times 41\times\sin 88$

$A = 1352.18$

So the area is 1350 cm² (3 sig fig).

Q: If this triangle has an area of 132 cm², find the value of x.



A: Substitute into the area formula:

$A = \frac{1}{2}x\times 2x\sin 42 = 132$

$x^2\sin 42 = 132$

$x^2 = \frac{132}{\sin 42} = 197.27$

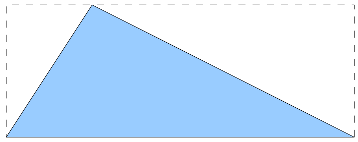
$x = 14.0$ cm (3 sig figs).

IGCSE SHAPE & SPACE

PERIMETER AND AREA

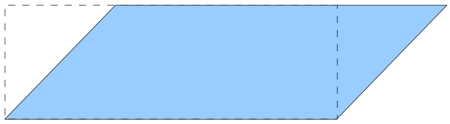
▷ The **perimeter** of a shape is the distance all the way around it (imagine walking round it). A circle's perimeter is called the **circumference**; it gets its own special name.

Triangle
- Perimeter = sum of the sides
- Area = ½ base × vertical height

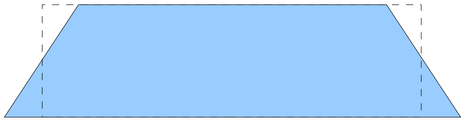


Rectangle
- Perimeter = 2 × base + 2 × height
- Area = base × height

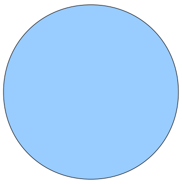
Parallelogram
- Perimeter = 2 × base + 2 × slope height
- Area = base × vertical height



Trapezium
- Perimeter = sum of the sides
- Area = mean width × vertical height



Circle
- Circumference = 2 π r
- Area = π r²



TJP TOP TIP: Area is squarier!

▶ **SKILL: Find the perimeter of a sector of a circle.**

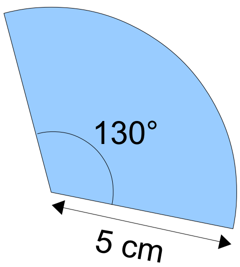
Hint: length of arc = fraction of circle × circumference

Perimeter = radius + radius + arc

= 5 + 5 + (130/360) × 2 π × 5

= 21.3446

= **21.3 cm** (3 sig figs)



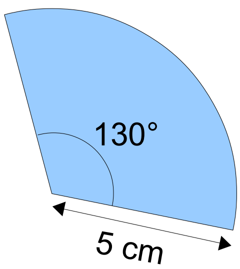
▶ **SKILL: Find the area of a sector of a circle.**

Hint: area of sector = fraction of circle × area of circle

Area = (130/360) × π × 5²

= 28.3616

= **28.4 cm²** (3 sig figs)

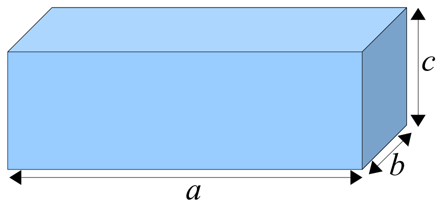


SURFACE AREA AND VOLUME

Face – a **flat side** of a 3-D shape.
Edge – a **line joining two corners** of a 3-D shape.
Vertex – a **corner** of a 3-D shape.

▷ The **surface area** is the **total area** of the **outside** of a 3-D shape.
If you are asked for the **curved surface area**, just leave out any flat faces.
[The formulae you are given include only the **curved** surface area for cones/cylinders.]

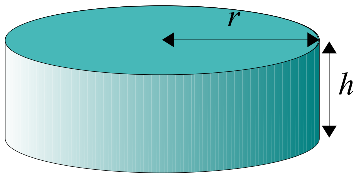
Cuboid
- Surface area = $2(ab+bc+ca)$
- Volume = abc



Prism
- Surface area = sum of rectangles + 2 ends
- Volume = area of end × length

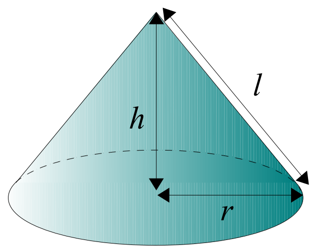
Cylinder
- Surface area = $2\pi rh+2\pi r^2$
- Volume = $\pi r^2 h$

Note: h may be the length if the cylinder is lying on its side.

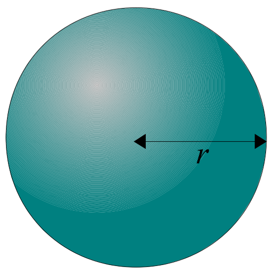


Cone
- Surface area = $\pi rl+\pi r^2$
- Volume = $\frac{1}{3}\pi r^2 h$

Note: l, h, r obey $l^2 = h^2+r^2$ where l is the slant height and h is the vertical height.



Sphere
- Surface area = $4\pi r^2$
- Volume = $\frac{4}{3}\pi r^3$



To find the volume or curved surface area of a **hemisphere** (a half-sphere), simply work it out for a whole sphere and then **halve your answer**.

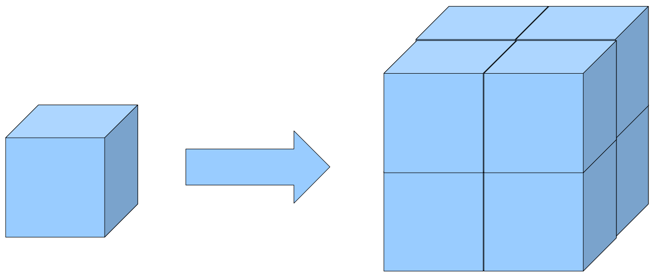
SIMILARITY AND ENLARGEMENT

▷ Shapes are **similar** if one is an **enlargement** of the other (so that all its lengths are multiplied by the same amount). The **scale factor = new length ÷ old length**.

Shapes are **congruent** if they are the **same shape and size** (so that you could fit one on top of the other).

If we enlarge a shape on a special 3-D photocopier, what happens to the lengths, the areas and the volumes?

Here is a clue: set the enlargement to double all lengths, and then see what happens.



The area of each face is multiplied by $4 = 2^2$ (area is squarier)

The volume of the cube is multiplied by $8 = 2^3$ (volume is 3-D)

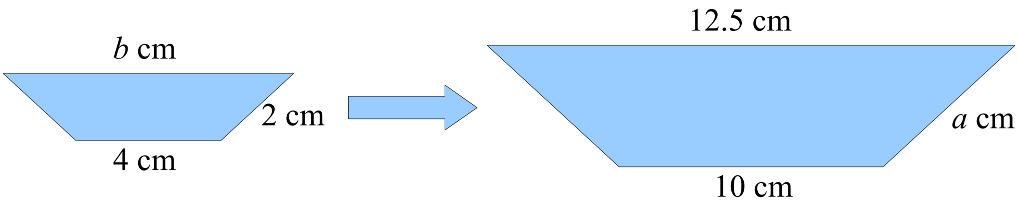
But the angles in the corners stay at 90° (don't change the angles!)

In general use LAV to work out scale factors.
(This stands for Length Area Volume)
Always start by finding the scale factor N.

L: ×N
A: $\times N^2$
V: $\times N^3$

▶ **SKILL: Solve a problem involving similar shapes.**

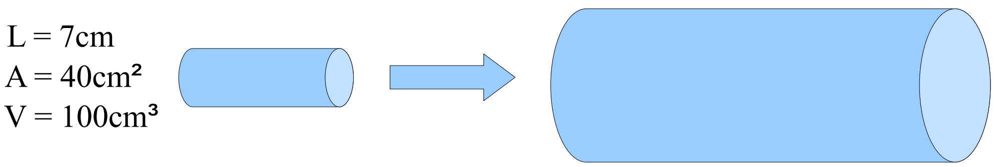
Q: Two shapes are similar; find the missing lengths a and b.



A: Use two matching numbers to get the scale factor = 10 ÷ 4 = 2.5.

Now use it to find a = 2 × 2.5 = **5 cm** and b = 12.5 ÷ 2.5 = **5 cm**.

Q: A cylinder is enlarged by scale factor 3. Find the new length, area and volume.



A: New L = 7 × 3 = **21 cm**
New A = $40 \times 3^2 = 40 \times 9$ = **360 cm²**
New V = $100 \times 3^3 = 100 \times 27$ = **2700 cm³**

IGCSE SHAPE & SPACE

METRIC UNITS

▷ You need to know all these common units of length and volume.

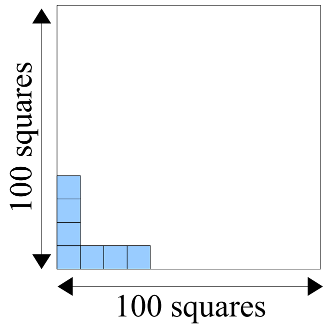
10 mm = 1 cm	1000 ml = 1 litre
1000 mm = 1 metre	1 ml = 1 cm³
100 cm = 1 metre	1 litre = 1000 cm³
1000 m = 1 km	

milli = 1 thousandth	centi = 1 hundredth	kilo = one thousand

▶ **SKILL: Convert between area/volume measures.**

Q: Convert 3 m² to cm².

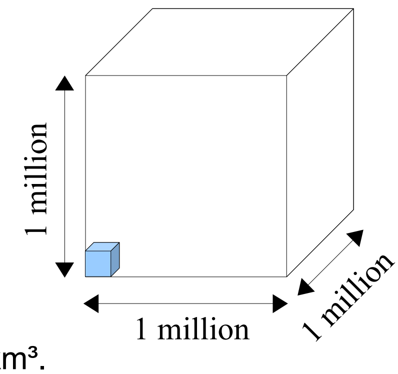
A: Each square metre has 100×100 = 100^2 square centimetres in it (not just 100).



So the answer is 3×100^2 = **30,000 cm²**.

Q: How many cubic millimetres are there in one cubic kilometre?

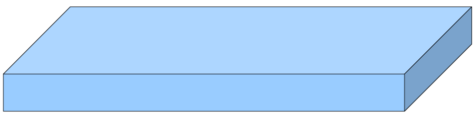
A: Along one edge of this giant cube, there are 1000×1000 = 10^6 millimetres in a kilometre.



So there are $10^6 \times 10^6 \times 10^6$ = **10^{18} mm³** in one km³.

Q: How many litres of fruit juice would it take to fill a swimming pool 10m by 50m by 3m?

A: The pool has a volume of 10 × 50 × 3 = 1500 m³.



This converts to 1500×100^3 = 1,500,000,000 cm³

Since 1 litre = 1000 cm³, the pool will hold **1,500,000 litres** of juice.

VECTORS

▷ A **scalar** has **size** (an ordinary number), e.g. time, mass, speed, distance.

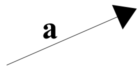
A **vector** has **size and direction**, e.g. force, weight, velocity, displacement.

The **size (or length) of a vector** is called its **magnitude** or **modulus**.

A vector quantity is shown by a bold letter **a** (in print), or underlined $\underline{a}$ if handwritten.
The same letter 'a' written normally means the **modulus** or length of the vector **a**.

If we have a vector going from A to B, we can write this as $\overrightarrow{AB}$.

We can show the direction of a vector in several different ways:

- by drawing a **diagram**: 

- by giving an **angle or bearing**: 10km on a bearing of 327°.
- by giving the x and y amounts in brackets (a **column vector**): $\boldsymbol{c} = \begin{pmatrix} 8 \\ -6 \end{pmatrix}$.

If you **multiply a vector by a scalar** (ordinary number) you just make the vector longer or shorter. For example, doubling a vector makes it twice as long (in the same direction).

The **resultant** is the result of **adding** or **subtracting** two or more **vectors**.

Use **Pythagoras** to find the **modulus** of a column vector.

▶ **SKILL: Solve problems involving column vectors.**

Q: If $\boldsymbol{a} = \begin{pmatrix} 2 \\ 5 \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} -7 \\ 7 \end{pmatrix}$, find (i) $3\boldsymbol{a}$ (ii) $\boldsymbol{a}+\boldsymbol{b}$ (iii) the modulus of $\boldsymbol{a}$.

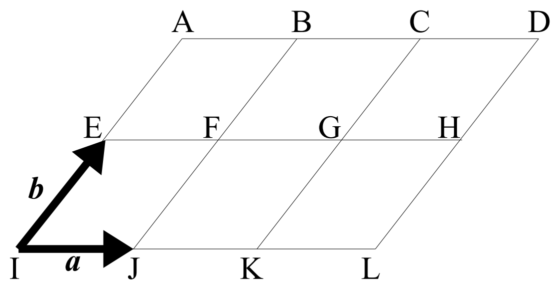
A: (i) $3\boldsymbol{a} = 3\begin{pmatrix} 2 \\ 5 \end{pmatrix} = \begin{pmatrix} 6 \\ 15 \end{pmatrix}$ (ii) $\boldsymbol{a}+\boldsymbol{b} = \begin{pmatrix} 2 \\ 5 \end{pmatrix}+\begin{pmatrix} -7 \\ 7 \end{pmatrix} = \begin{pmatrix} -5 \\ 12 \end{pmatrix}$

(iii) modulus of $\boldsymbol{a} = \sqrt{2^2+5^2} = \sqrt{29} = 5.39$ (3 sig figs)

▶ **SKILL: Solve geometrical vector questions.**

Q: Use the grid below to write vector expressions for (i) $\overrightarrow{AC}$ (ii) $\overrightarrow{HG}$ (iii) $\overrightarrow{KF}$ (iv) $\overrightarrow{DI}$

Hint: you can only move in the $\boldsymbol{a}$ and $\boldsymbol{b}$ directions, and backwards is negative.

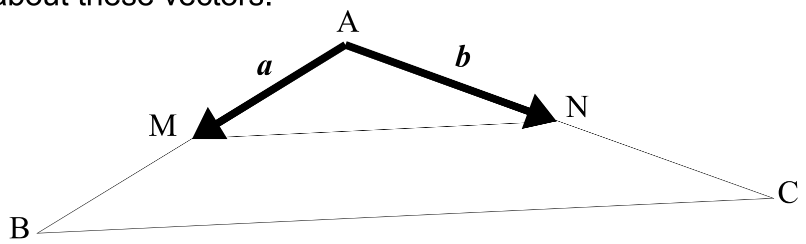


A: (i) $\overrightarrow{AC} = 2\boldsymbol{a}$ (ii) $\overrightarrow{HG} = -\boldsymbol{a}$ (iii) $\overrightarrow{KF} = -\boldsymbol{a}+\boldsymbol{b}$ (iv) $\overrightarrow{DI} = -3\boldsymbol{a}-2\boldsymbol{b}$

TJP TOP TIP: Two vectors are **parallel** if one is a **multiple** of the other.

▶ **SKILL: Use vectors to prove geometrical results.**

Q: If M is the midpoint of AB and N is the midpoint of AC, find $\overrightarrow{MN}$ and $\overrightarrow{BC}$ and hence state two facts about these vectors.



A: $\overrightarrow{MN} = -\boldsymbol{a}+\boldsymbol{b}$ and $\overrightarrow{BC} = -2\boldsymbol{a}+2\boldsymbol{b}$.

Therefore $\overrightarrow{BC} = 2\overrightarrow{MN}$, so:

(i) $\overrightarrow{BC}$ is twice as long as $\overrightarrow{MN}$,

(ii) $\overrightarrow{BC}$ is parallel to $\overrightarrow{MN}$.

IGCSE SHAPE & SPACE

CONTENTS