AQA Qualifications

Level 2 Certificate
Further Mathematics

Paper 1 83601
Mark scheme

83601
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Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts: alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Assessment Writer.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this Mark Scheme are available from aqa.org.uk

Glossary for Mark Schemes

GCSE examinations are marked in such a way as to award positive achievement wherever possible. Thus, for GCSE Mathematics papers, marks are awarded under various categories.

If a student uses a method which is not explicitly covered by the mark scheme the same principles of marking should be applied. Credit should be given to any valid methods. Examiners should seek advice from their senior examiner if in any doubt.

M	Method marks are awarded for a correct method which could lead to a correct answer.
M dep	A method mark dependent on a previous method mark being awarded.
A	Accuracy marks are awarded when following on from a correct method. It is not necessary to always see the method. This can be implied.
B	Marks awarded independent of method.
B dep	A mark that can only be awarded if a previous independent mark has been awarded.
ft	Follow through marks. Marks awarded following a mistake in an earlier step.
SC	Special case. Marks awarded within the scheme for a common misinterpretation which has some mathematical worth.
oe	Or equivalent. Accept answers that are equivalent. eg, accept 0.5 as well as $\frac{1}{2}$
[*a*, *b*]	Accept values between *a* and *b* inclusive.
3.14…	Accept answers which begin 3.14 eg 3.14, 3.142, 3.1416

Examiners should consistently apply the following principles

Diagrams

Diagrams that have working on them should be treated like normal responses. If a diagram has been written on but the correct response is within the answer space, the work within the answer space should be marked. Working on diagrams that contradicts work within the answer space is not to be considered as choice but as working, and is not, therefore, penalised.

Responses which appear to come from incorrect methods

Whenever there is doubt as to whether a candidate has used an incorrect method to obtain an answer, as a general principle, the benefit of doubt must be given to the candidate. In cases where there is no doubt that the answer has come from incorrect working then the candidate should be penalised.

Questions which ask candidates to show working

Instructions on marking will be given but usually marks are not awarded to candidates who show no working.

Questions which do not ask candidates to show working

As a general principle, a correct response is awarded full marks.

Misread or miscopy

Candidates often copy values from a question incorrectly. If the examiner thinks that the candidate has made a genuine misread, then only the accuracy marks (A or B marks), up to a maximum of 2 marks are penalised. The method marks can still be awarded.

Further work

Once the correct answer has been seen, further working may be ignored unless it goes on to contradict the correct answer.

Choice

When a choice of answers and/or methods is given, mark each attempt. If both methods are valid then M marks can be awarded but any incorrect answer or method would result in marks being lost.

Work not replaced

Erased or crossed out work that is still legible should be marked.

Work replaced

Erased or crossed out work that has been replaced is not awarded marks.

Premature approximation

Rounding off too early can lead to inaccuracy in the final answer. This should be penalised by 1 mark unless instructed otherwise.

Q	Answer	Mark	Comments
1	$(y =)\ x^3 - 10x^2$	B1	
	$3x^2 - 20x$	B2 ft	B1ft for each term ft their $x^3 - 10x^2$
	Additional Guidance		
	The given product must be expanded correctly to score the first B1, the product may be implied. Ignore correct simplification from incorrect differentiation, eg $3x - 20x = -17x$ Differentiating a second time to get $6x - 20$ scores B1 B1 Correct answer seen in working then $30x^2 - 20x$ on the answer line scores full marks ... ignore their transcription error. $y = x^2 - 10x^2$ B0, $dy/dx = 2x - 20x$ scores B1 $y = x^3 - 10$ B0, $dy/dx = 3x^2$ scores B2		

Q	Answer	Mark	Comments
2	**Alternative method 1**		
	$a = 3$	B1	
	$4 - 8a = b$ or $4(1 - 2a) = b$	M1	oe eg $4 \times 1 + -2a \times 4 = b$
	$b = -20$	A1ft	ft from B0 M1
	Alternative method 2		
	$a = 3$	B1	
	$\begin{pmatrix} 4 - 8a \\ 4a \end{pmatrix}$	B1	Condone no brackets but do not condone a fraction
	$b = -20$	B1ft	ft from B0 B1
	Additional Guidance		
	alt 1 ... $a = 12$ B0, $b = -92$ M1 A1ft		

Q	Answer	Mark	Comments
3(a)			
	$2(3n) = 5n + 12$ or $3n = \frac{5n + 12}{2}$ or $3n = \frac{1}{2}(5n + 12)$ or $3n = 2.5n + 6$ or $0.5n = 6$	M1	oe
	12	A1	Accept 12th term
	Additional Guidance		
	Do not accept an embedded answer, they must state 12 Trial and improvement is 2 if correct, otherwise 0 $\frac{3n}{5n + 12} = \frac{1}{2}$ is M0		

Q	Answer	Mark	Comments
3(b)	$\frac{3}{5}$ or 0.6	B1	
	Additional Guidance		

Q	Answer	Mark	Comments

4(a)	(−5, 8)	B1	

<table>
<tr><td rowspan="3">4(b)</td><td>$\sqrt{10}$ or [3.1, 3.2]</td><td>B1</td><td></td></tr>
<tr><td colspan="3">Additional Guidance</td></tr>
<tr><td colspan="2"></td><td></td></tr>
</table>

Q	Answer	Mark	Comments
5	**Alternative method 1**		
	(Angle at circumference =) $x + 24$	M1	oe
	$x + 24 + 3x = 180$	M1 dep	oe
	39	A1	
	Alternative method 2		
	(Reflex angle at centre =) $6x$	M1	oe
	$2x + 48 + 6x = 360$	M1dep	oe
	39	A1	
	Alternative method 3		
	(Angle at circumference =) $180 - 3x$	M1	oe
	$2x + 48 = 2(180 - 3x)$	M1 dep	oe
	39	A1	
	Alternative method 4		
	(Reflex angle at centre =) $360 - (2x + 48)$	M1	oe
	$360 - (2x + 48) = 2(3x)$	M1 dep	oe
	39	A1	
	Additional Guidance		
	Look on the diagram for evidence of the 1st M1 39° seen on the answer line ... check to see that it has come from correct working Reasons are not necessary Here are two examples of equations coming from incorrect geometrical reasoning. $2(3x) = 2x + 48$ scores M0 M0 $2x + 48 + 6x = 180$ scores M1 M0 ... because the $6x$ in this equation implies the reflex angle at the centre, so it scores M1 in this particular case.		

<table>
<tr><th>Q</th><th>Answer</th><th>Mark</th><th>Comments</th></tr>
<tr><td rowspan="18">6</td><td colspan="3">Alternative method 1</td></tr>
<tr><td>$mx + 4 - 2x - 2p$ or $6x + 6$</td><td>M1</td><td>allow one error on the left hand side</td></tr>
<tr><td>$mx - 2x = 6x$ or $4 - 2p = 6$</td><td>M1</td><td>oe eg. $m - 2 = 6$
ft their expansion if 1st M1 earned</td></tr>
<tr><td>$m = 8$</td><td>A1</td><td></td></tr>
<tr><td>$p = -1$</td><td>A1</td><td></td></tr>
<tr><td colspan="3">Alternative method 2</td></tr>
<tr><td>$mx + 4 - 2x - 2p = 6x + 6$</td><td>M1</td><td rowspan="2">At most one error in total in the expansion and the simplifying
This must be in the form $ax + b = cx + d$</td></tr>
<tr><td>$mx - 2p = 8x + 2$</td><td>M1</td></tr>
<tr><td>$m = 8$</td><td>A1</td><td></td></tr>
<tr><td>$p = -1$</td><td>A1</td><td></td></tr>
<tr><td colspan="3">Alternative method 3</td></tr>
<tr><td>An equation obtained by substituting a value for x in the identity</td><td>M1</td><td rowspan="2">eg $x = 0$ $4 - 2p = 6$
$x = 1$ $m + 4 - 2 - 2p = 12$
$x = 2$ $2m + 4 - 4 - 2p = 18$</td></tr>
<tr><td>A second equation obtained by substituting a value for x in the identity</td><td>M1</td></tr>
<tr><td>$m = 8$</td><td>A1</td><td></td></tr>
<tr><td>$p = -1$</td><td>A1</td><td></td></tr>
<tr><td colspan="3">Additional Guidance</td></tr>
<tr><td colspan="2">$mx + 4 - 2x + 2p$ is one error in the left hand side
$mx + 4 - 2x - p$ is one error in the left hand side
$mx + 4 - 2x + p$ is two errors in the left hand side
In alt 2 ... allow at most one error in the expansion and the simplifying (the first two M marks) ... one error can then score M1 M1, two errors will score M1 M0
Also, in alt 3, allow at most one error in forming their two simultaneous equations</td><td></td></tr>
</table>

Q	Answer	Mark	Comments
7	**Alternative method 1**		
	$(x \pm a)(x \pm b)$	M1	$ab = 96$ or $a + b = -20$
	$(x - 8)(x - 12)$ or $(x =)$ 8 and $(x =)$ 12	A1	
	9, 10 and 11	A1	A0 if extra values seen
	Alternative method 2		
	$(x - 10)^2 - 100$ (+ 96) (< 0)	M1	oe eg $(x - 10)^2 - 4$ (< 0)
	$8 < x < 12$ or $(x =)$ 8 and $(x =)$ 12	A1	
	9, 10 and 11	A1	A0 if extra values seen
	Alternative method 3		
	$\dfrac{20 \pm \sqrt{\{(20)^2 - 4 \times 1 \times 96\}}}{2}$ or $\dfrac{20 \pm \sqrt{\{(-20)^2 - 4 \times 1 \times 96\}}}{2}$	M1	accept $(20)^2$ or $(-20)^2$ for b^2 in the discriminant
	8 and 12	A1	
	9, 10 and 11	A1	A0 if extra values seen
	Additional Guidance		
	9, 10 and 11 using Trial and Improvement - all correct is 3 marks, otherwise 0 marks. No working ... treat as Trial and Improvement For alt 3 ... substitution in the formula must be correct		

Q	Answer	Mark	Comments
8	$(-2)^3$ or -8 seen	B1	
	$-\sqrt{x}$ = (their -8) $-$ 3 or $-\sqrt{x} = -11$ or $\sqrt{x} = 11$	M1	
	121	A1	
	Additional Guidance		
	-2^3 (no brackets) is B0 unless -8 seen For M1 it must say $\sqrt{x}$ = ... or $-\sqrt{x}$ = ... Note: ... (their -8) cannot be -2 ... and it must be **correct** manipulation from their -8 eg $3 - \sqrt{x} = (-2)^3$ or $3 - \sqrt{x} = -8$ B1 $\sqrt{x} = -11$ M0 (error in manipulating terms) $x = 121$ A0 (correct answer from wrong working)		

Q	Answer	Mark	Comments
9	**Alternative method 1**		
	$x^2 - 5x - 5x + 25$ or $x^2 - 10x + 25$	M1	allow one error
	$x^3 - 5x^2 - 5x^2 + 25x - 5x^2 + 25x + 25x - 125$ or $x^3 - 10x^2 + 25x - 5x^2 + 50x - 125$	M1dep	oe ft their $x^2 - 5x - 5x + 25$ or $x^2 - 10x + 25$ and allow one error **only if** no errors made for the 1st M1
	$x^3 - 15x^2 + 75x - 125$	A1	
	Alternative method 2		
	$1x^3 + 3x^2(-5) + 3x(-5)^2 + 1(-5)^3$	M1	Using 1 3 3 1 ... coefficients from Pascal's triangle for the cubic expansion
	$x^3 - 15x^2 + 75x - 125$	M1	allow one error
	$x^3 - 15x^2 + 75x - 125$	A1	
	Additional Guidance		
	Penalise further work There must be three or four terms for the 1st M1 in alt 1 In alt 1 ... for M1 M1 they must make **at most one** error in the 1st two steps In the 2nd step, $-50x$ (instead of $+50x$) and $-10x$ (instead of $+50x$) are examples of **one** error In alt 2 ... $1x^3 + 3x^2(-5) + 3x(-5)^2 + 1(-5)^3 = x^3 - 15x^2 + 75x - 125$ written directly, with no intermediate steps needed or seen, automatically scores 3 marks		

Q	Answer	Mark	Comments
10	$x = 2^4$ or $x = 16$	B1	
	$\frac{1}{y^2} = 25$ or $y^2 = \frac{1}{25}$ or $25y^2 = 1$ or $y = \sqrt{\frac{1}{25}}$ or $y = \left(\frac{1}{25}\right)^{\frac{1}{2}}$	M1	
	$(y =)\ -\frac{1}{5}$ or $\pm\frac{1}{5}$ or $\frac{1}{5}$	A1	oe eg 0.2
	−80	A1	
	Additional Guidance		
	Condone 4^2 for the 1st M1 80 seen is likely to be 3 marks ... but check $(y =)\ -\frac{1}{5}$ or $\pm\frac{1}{5}$ or $\frac{1}{5}$ with no incorrect working seen implies M1 A1 $(y =)\ -\frac{1}{5}$ or $\pm\frac{1}{5}$ or $\frac{1}{5}$ from clearly incorrect working scores M0 A0		

Q	Answer	Mark	Comments
11	**Alternative method 1**		
	Correct method for finding the gradient of either *AB* **or** *BC*	M1	$\frac{1\frac{4}{5} - 3\frac{4}{5}}{2 - 1\frac{1}{5}}$ or $\frac{1.8 - 3.8}{2 - 1.2}$ or $\frac{3 - 1\frac{4}{5}}{5 - 2}$ or $\frac{3 - 1.8}{5 - 2}$
	Correct method for finding the gradient of *AB* **and** *BC*, **and** at least one of the gradients correct	M1	$\frac{-10}{4}$ or $\frac{-5}{2}$ or $-2\frac{1}{2}$ or -2.5 or $\frac{-2}{\frac{4}{5}}$ or $\frac{-2}{0.8}$ or $\frac{6}{15}$ or $\frac{2}{5}$ or $\frac{4}{10}$ or 0.4 or $\frac{1\frac{1}{5}}{3}$ or $\frac{1.2}{3}$
	product = −1 clearly shown or clearly showing that one is the negative reciprocal of the other	A1	
	Alternative method 2		
	$AB^2 = (2 - 1.2)^2 + (1.8 - 3.8)^2$ or $BC^2 = (5 - 2)^2 + (3 - 1.8)^2$ or $AC^2 = (5 - 1.2)^2 + (3 - 3.8)^2$	M1	oe
	$AB^2 = (0.8)^2 + (-2)^2$ and $BC^2 = (3)^2 + (1.2)^2$ and $AC^2 = (3.8)^2 + (-0.8)^2$	M1	oe all expressions simplified but not necessarily evaluated
	$AB^2 + BC^2 = 4.64 + 10.44 = 15.08$ and $AC^2 = 15.08$	A1	showing equal values is sufficient

Additional Guidance

The $\frac{y\text{-step}}{x\text{-step}}$ calculation, for the first M1, might be on the diagram

The expressions for the gradients can be either way round eg $\frac{3.8 - 1.8}{1.2 - 2} = \frac{2}{-0.8}$

Both gradients numerically correct but both with the wrong sign eg $\frac{10}{4}$ and $\frac{-6}{15}$

scores SC1

For the 2nd M mark we are looking for *m*/*n* where *m* and *n* are integers, mixed numbers, fractions, improper fractions or decimals eg $\frac{\frac{6}{5}}{3}$

The gradients for each line can be found by forming two simultaneous equations. It will be very rare, but check their working if you see this method.

Q	Answer	Mark	Comments
12(a)	**Alternative method 1**		
	$(x + 3)^2 - 9\ (+ 2)$	M1	
	$h = 3$ and $k = -7$	A1	
	Alternative method 2		
	$x^2 + 2hx + h^2\ (+ k)$ or $2hx = 6x$ or $2h = 6$ or $h^2 + k = 2$	M1	
	$h = 3$ and $k = -7$	A1	
	Additional Guidance		
	$h = 3$ implies M1		

Q	Answer	Mark	Comments
12(b)	$(-3, -7)$	B1 ft	ft their h and k from part (a) only if $h \neq 0$ and $k \neq 0$
	Additional Guidance		
	for their h and k, the minimum point is $(-h, k)$		

Q	Answer	Mark	Comments
12(c)	$-3 \pm \sqrt{7}$	B1 ft	ft their h and k from part (a) only if $h \neq 0$ and $k \neq 0$
	Additional Guidance		
	For their h and k, the solutions are $-h \pm \sqrt{(-k)}$ If their k is > 0 then $\sqrt{(-k)}$ will be $\sqrt{}$ of a negative number ... condone Any use of the quadratic formula must be completely correct		

Q	Answer	Mark	Comments
13	$\sqrt{125} = 5\sqrt{5}$, $\sqrt{20} = 2\sqrt{5}$ and $\sqrt{80} = 4\sqrt{5}$	M1	allow one error any two of these correct seen anywhere in the working
	($\sqrt{x}$ =) $3\sqrt{5}$	A1	
	45	A1	
	Additional Guidance		

Q	Answer	Mark	Comments
14a	**Alternative method 1**		
	$(3)^3 - 8(3)^2 + 3a + 42 = 0$ or $27 - 72 + 3a + 42 = 0$	M1	Equating to zero might not be seen until later in the working.
	$3a = 3$	A1	$3a = 3$ implies $3a - 3 = 0$
	Alternative method 2		
	$(x^3 - 8x^2 + ax + 42) \div (x - 3)$ to give a quotient of $x^2 - 5x + (a - 15)$ and a remainder of $3a - 3$	M1	
	Remainder = 0 so $3a = 3$	A1	
	Alternative method 3		
	$x^3 - 8x^2 + ax + 42$ $= (x - 3)(x^2 + px - 14)$ Comparing x^2 coefficients gives $p = -5$	M1	
	Using $p = -5$ and comparing x coefficients gives $a = 1$	A1	
	Additional Guidance		
	In alt 1 ... assuming that $a = 1$ and showing that substituting $x = 3$ in the expression gives zero is only verifying the result ... and scores SC1 Similarly, assuming $a = 1$ and working as in alt 2 and alt 3 to verify the result.		SC1

Q	Answer	Mark	Comments
14b	**Alternative method 1**		
	$x^3 - 8x^2 + x + 42$ $\equiv (x - 3)(x^2 + kx - 14)$	M1	Sight of quadratic with x^2 and – 14 as the first and last terms
	$(x + 2)$ or $(x - 7)$	A1	
	$(x - 3)(x + 2)(x - 7)$	A1	any order
	Alternative method 2		
	Substitutes another value into the expression and tests for '= 0'	M1	their value correctly substituted eg. $2^3 - 8(2)^2 + 2 + 42$ (= 20) $\neq 0$
	$(x + 2)$ or $(x - 7)$	A1	
	$(x - 3)(x + 2)(x - 7)$	A1	any order
	Alternative method 3		
	Long division of polynomials getting as far as $x^2 - 5x$	M1	$(x^3 - 8x^2 + x + 42) \div (x - 3) = x^2 - 5x - 14$
	$(x + 2)$ or $(x - 7)$	A1	
	$(x - 3)(x + 2)(x - 7)$	A1	any order
	Additional Guidance		
	An answer of $(x + 2)(x - 7)$ ie $(x - 3)$ missing ... implies M1 A1 An answer of $(x - 3)(x - 2)(x + 7)$ scores SC1 ... sign errors in two factors Ignore 'solutions' ie $x = 3, -2$ and 7		

Q	Answer	Mark	Comments
15	$\frac{6}{(\sqrt{7}+2)} \times \frac{(\sqrt{7}-2)}{(\sqrt{7}-2)}$	M1	$\frac{6}{(\sqrt{7}+2)} \times \frac{(2-\sqrt{7})}{(2-\sqrt{7})}$
	Denominator = 3 or −3	A1	
	$2(\sqrt{7}-2)$ or $2\sqrt{7}-4$	A1	
	Additional Guidance		
	Correct answer seen, then error in factorising, do **not** penalise $\frac{6}{(\sqrt{7}+2)} \times (\sqrt{7}-2)$ followed by $\frac{6\sqrt{7}-12}{3}$ implies M1 A1 (they have 'recovered')		

Q	Answer	Mark	Comments
16	**Alternative method 1**		
	Sketch of right-angled triangle with √11 on 'opposite' and 6 on hypotenuse	M1	accept O = √11 and H = 6 without a sketch for M1
	Adjacent = $\sqrt{(6^2 - (\sqrt{11})^2)}$	M1	
	Adjacent = 5	A1	
	$-\,^5/_6$	A1	
	Alternative method 2		
	$(\sin^2\theta =)\ \frac{11}{36}$	B1	
	their $^{11}/_{36}$ + $\cos^2\theta = 1$	M1	oe
	$\cos^2\theta = {}^{25}/_{36}$	A1	
	$-\,^5/_6$	A1	
	Additional guidance		
	An answer of $^5/_6$ implies M1 M1 A1 A0 ... unless from clearly incorrect working. In alt 2, their $^{11}/_{36}$ must not involve a trig function eg $\sin^2(^{11}/_{36}) + \cos^2\theta = 1$ scores B1 M0 A0 A0		

Q	Answer	Mark	Comments
17	$\frac{dy}{dx} = x^2 - 2x - 3$	M1	
	$x^2 - 2x - 3 = 0$	M1	ft their $x^2 - 2x - 3$ ... they **must** equate to 0
	$(x - 3)(x + 1)$ or $x = 3$	A1	
	$0 = \frac{1}{3} \times 3^3 - 3^2 - 9 + k$	M1dep	oe ft their 3, which must be a **positive** value ... it must be the greater of their two values
	$(k =)$ 9	A1	
	Additional Guidance		
	They **must** start this question by differentiating ... any other attempted solutions will score zero marks. The 2nd M mark can be implied by them equating their factors to zero or by seeing an answer of $x = 3$ Substituting $x = 3$ in $y = \frac{1}{3}x^3 - x^2 - 3x + k$ implies the 1st A1 (M1 M1 already earned) The 3rd M mark is dependent on **both** of the first two M marks Look out for $k = 9$ coming from using $x = -3$ when $y = 0$ ... they cannot use a negative value for x at this stage		

Q	Answer	Mark	Comments
18	$(x^2 - 9)(x^2 + 9)$ or $(x + 3)(x^3 - 3x^2 + 9x - 27)$ or $(x - 3)(x^3 + 3x^2 + 9x + 27)$	M1	
	$(x + 3)(x - 3)(x^2 + 9)$	A1	Do not award A1 if further working
	Additional Guidance		

Q	Answer	Mark	Comments
19(a)	**Alternative method 1**		
	$x^2 + x^2 = (3\sqrt{2})^2$	M1	where x is the side of the square It must say $(3\sqrt{2})^2$ ... $x^2 + x^2 = 18$ is M0
	$2x^2 = 18$	A1	oe
	Alternative method 2		
	$\sin 45° = \dfrac{x}{3\sqrt{2}}$ or $\cos 45° = \dfrac{x}{3\sqrt{2}}$	M1	where x is the side of the square allow use of the sine rule
	$\dfrac{1}{\sqrt{2}} = \dfrac{x}{3\sqrt{2}}$	A1	oe
	Additional Guidance		
	In alt 1 ... Ignore further work after $2x^2 = 18$ $1 : 1 : \sqrt{2}$ scaled up by a factor of 3 to give $3 : 3 : 3\sqrt{2}$ is 'verify' so scores SC1 or $3^2 + 3^2 = 9 + 9 = 18$ and $(3\sqrt{2})^2 = 9 \times 2 = 18$ is also 'verify', so SC1		SC1

Q	Answer	Mark	Comments
19b	**Alternative method 1**		
	$\tan 60 = \frac{3}{DE}$ or $DE = \frac{3}{\sqrt{3}}$ or $1 : \sqrt{3} : 2$ triangle	M1	oe
	$DE = \sqrt{3}$	A1	oe eg $\frac{3}{\sqrt{3}}$
	$AE = 2\sqrt{3}$	A1	oe eg $\frac{6}{\sqrt{3}}$ or $\sqrt{12}$
	$3 \times 3 + \sqrt{3} + 2\sqrt{3}$	A1	oe
	Alternative method 2		
	$\sin 60 = \frac{3}{AE}$ or $AE = \frac{3}{(\sqrt{3}/2)}$	M1	oe
	$AE = 2\sqrt{3}$	A1	oe eg $\frac{6}{\sqrt{3}}$ or $\sqrt{12}$
	$DE = \sqrt{3}$	A1	oe eg $\frac{3}{\sqrt{3}}$
	$3 \times 3 + \sqrt{3} + 2\sqrt{3}$	A1	oe
	Additional Guidance		
	They can leave *DE* and *AE* in unsimplified form for the first two A marks but must then simplify their expression for the perimeter to clearly show the required result for the final A mark.		

Q	Answer	Mark	Comments
20	**Alternative method 1**		
	$(3n)^2 + (2n)^2 - 2 \times 3n \times 2n \times \cos P$	M1	oe Condone missing brackets for first M1
	$w^2 = 9n^2 + 4n^2 - 12n^2 \times \frac{1}{3}$	A1	
	$w^2 = 9n^2$	A1	
	$w = 3n$ so the triangle is isosceles or $QP = QR$	A1	
	Alternative method 2		
	$\frac{(3n)^2 + (2n)^2 - w^2}{2 \times 3n \times 2n}$	M1	oe Condone missing brackets for first M1
	$\frac{1}{3} = \frac{9n^2 + 4n^2 - w^2}{12n^2}$	A1	
	$w^2 = 9n^2$	A1	
	$w = 3n$ so the triangle is isosceles or $QP = QR$	A1	
	Alternative method 3		
	Drop a perpendicular from Q to PR	M1	let this be QS
	$\cos P = \frac{PS}{3n}$ or $\frac{1}{3} = \frac{PS}{3n}$	M1	
	$PS = n$	A1	
	PR is bisected by the perpendicular from Q hence ΔQPR is isosceles	A1	oe
	Additional Guidance		
	The final A1 is for a statement saying that two sides have been shown to be equal. Substituting $w = 3n$ in either version of the cosine rule and verifying that $\cos P = \frac{1}{3}$ scores SC2 alt 3 ... if they drop a perpendicular from Q to PR then **assume** that $PS = n$ and then verify that $\cos P = {}^1/_3$ ... eg $PS = n$, $QP = 3n$, $\cos P = \frac{PS}{QP} = \frac{n}{3n} = \frac{1}{3}$ ... they score SC2	SC2	