Please check the examination details below before entering your candidate information

Candidate surname	Other names

Pearson Edexcel Level 3 GCE

Centre Number	Candidate Number

Monday 11 May 2020

Morning (Time: 1 hour 30 minutes) | Paper Reference **9FM0/01**

Further Mathematics

Advanced

Paper 1: Core Pure Mathematics 1 **Shadow Set 1**

You must have:
Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 7 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over

1

$$f(z) = 5z^3 + pz^2 + 270z + q$$

where p and q are real constants.

Given that $5 - 3\sqrt{3}\mathrm{i}$ is a root of the equation $f(z) = 0$

(a) show all the roots of $f(z) = 0$ on a single Argand diagram,

(7)

(b) find the value of p and the value of q.

(3)

(Total for Question 1 is 10 marks)

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2 (a) Explain why $\displaystyle\int_3^{\infty} \frac{9}{x(3x-5)}\,\mathrm{d}x$ is an improper integral.

(1)

(b) Prove that

$$\int_3^{\infty} \frac{9}{x(3x-5)}\,\mathrm{d}x = a \ln b$$

where a and b are rational numbers to be determined.

(6)

(Total for Question 2 is 7 marks)

3

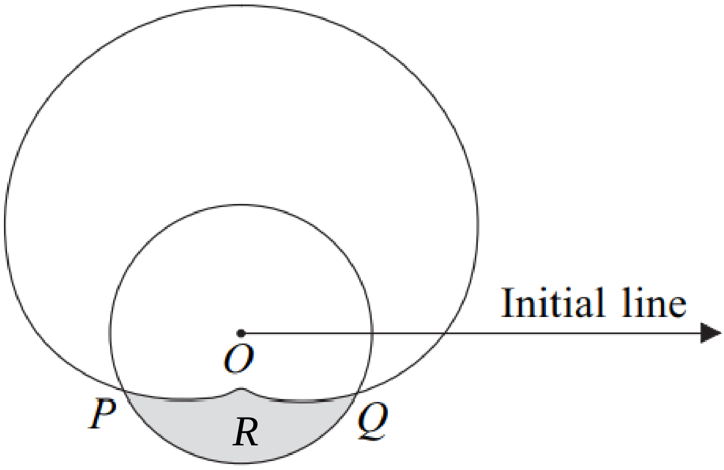

Figure 1

Figure 1 shows a sketch of a curve and a circle with polar equations

Curve: $r = 3 + 2\sin\theta \qquad 0 \leqslant \theta < 2\pi$

Circle: $r = 2 \qquad 0 \leqslant \theta < 2\pi$

The region R lies inside the circle and outside the curve and is shown shaded in Figure 1.

Show that the area of R is

$$\frac{1}{6}\left(m\sqrt{3} + n\pi\right)$$

where m and n are integers to be determined.

(9)

(Total for Question 3 is 9 marks)

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4 The plane Π_1 has equation

$$\mathbf{r} = \mathbf{i} - 5\mathbf{j} + 4\mathbf{k} + \lambda(3\mathbf{i} + \mathbf{j} - 2\mathbf{k}) + \mu(-\mathbf{i} + 2\mathbf{j} + \mathbf{k})$$

where λ and μ are scalar parameters.

(*a*) Find a Cartesian equation for Π_1

(4)

The line l has equation

$$\frac{x+1}{3} = \frac{2-y}{2} = \frac{z-1}{4}$$

(*b*) Find the coordinates of the point of intersection of l with Π_1

(3)

The plane Π_2 has equation

$$\mathbf{r.}\,(3\mathbf{i} - \mathbf{j} + 2\mathbf{k}) = 3$$

(*c*) Find, to the nearest degree, the acute angle between Π_1 and Π_2

(2)

(Total for Question 4 is 9 marks)

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5 Two enzymes, X and Y, are involved in a biological reaction. The amounts in grams of these enzymes, t minutes after the reaction starts, are x and y respectively and are modelled by the differential equations

$$\frac{\mathrm{d}x}{\mathrm{d}t} = -7x + 10y - 20$$

$$\frac{\mathrm{d}y}{\mathrm{d}t} = -4x + 5y - 3$$

(*a*) Show that

$$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} + 2\frac{\mathrm{d}x}{\mathrm{d}t} + 5x = 70$$

(3)

(*b*) Find, according to the model, a general solution for the amount in grams of enzyme X present at time t minutes.

(6)

(*c*) Find, according to the model, a general solution for the amount in grams of enzyme Y present at time t minutes.

(3)

Given that $x = 6$ and $y = 7.2$ when $t = 0$

(*d*) find

(i) the particular solution for x,

(ii) the particular solution for y.

(4)

A scientist thinks that the biological reaction will have stopped after 5 minutes.

(*e*) Explain whether this is supported by the model.

(1)

(Total for Question 5 is 17 marks)

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6 (i) Prove by induction that for $n \in \mathbb{Z}^+$

$$\sum_{r=1}^{n} (2r-1)^2 = \frac{1}{3}n(2n-1)(2n+1)$$

(6)

(ii) Prove by induction that for all positive **odd** integers n

$$\mathrm{f}(n) = 3^n + 7^n$$

is divisible by 10

(6)

(Total for Question 6 is 12 marks)

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7 A sample of insects is being studied.

The number of insects, N, in hundreds, is modelled by the differential equation

$$(2+t)\frac{\mathrm{d}N}{\mathrm{d}t}+N=\frac{1}{\sqrt{t}}(2+t)$$

where t is the time in hours after the start of the study.

Initially, there are exactly 700 insects in the container.

(a) Determine, according to the model, the number of insects in the container 6 hours after the start of the study.

(6)

(b) Find, according to the model, the rate of change of the number of insects in the container 36 hours after the start of the study.

(4)

(c) State a limitation of the model.

(1)

(Total for Question 7 is 11 marks)

TOTAL FOR PAPER IS 75 MARKS