Please check the examination details below before entering your candidate information

Candidate surname	Other names

Pearson Edexcel Level 3 GCE

Centre Number | Candidate Number

Time 1 hour 30 minutes

Paper reference **9FM0/02**

Further Mathematics

Advanced

PAPER 2: Core Pure Mathematics 2

October 2021 Shadow Set 1

You must have:
Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 9 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- Good luck with your examination.

Turn over

1. Given that

$$z_1 = \sqrt{3}\left(\cos\left(\frac{5\pi}{12}\right) + \mathrm{i}\sin\left(\frac{5\pi}{12}\right)\right)$$

$$z_2 = 2\left(\cos\left(\frac{\pi}{4}\right) + \mathrm{i}\sin\left(\frac{\pi}{4}\right)\right)$$

(a) write down the exact value of

(i) $\left|\dfrac{z_1}{z_2}\right|$

(ii) $\arg\left(\dfrac{z_1}{z_2}\right)$

(2)

Given that $w = \dfrac{z_1}{z_2}$ and that arg $(w^n) = \pi$, where $n \in \mathbb{Z}^+$

(b) determine

(i) the smallest positive value of n

(ii) the corresponding value of $| w^n |$

(3)

(Total for Question 1 is 5 marks)

2.

$$\mathbf{M} = \begin{pmatrix} 2 & -3 \\ 0 & 4 \end{pmatrix}$$

The matrix **M** represents the linear transformation T.

Prove that, for the linear transformation T, the only invariant line whose gradient is not equal to zero passes through the origin.

(5)

(Total for Question 2 is 5 marks)

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3. $$\text{f}(x) = \arccos x \qquad -1 \leq x \leq 1$$

(a) Determine the first three non-zero terms, in ascending powers of x, of the Maclaurin series for $\text{f}(x)$, giving each coefficient in its simplest form.

(4)

(b) Substitute $x = \frac{\sqrt{2}}{2}$ into the answer to part (a) and hence find an approximate value for π

Give your answer in the form $\frac{a\sqrt{2}}{b}$ where a and b are integers to be determined.

(2)

(Total for Question 3 is 6 marks)

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4. In this question you may assume the results for

$$\sum_{r=1}^{n} r^3, \sum_{r=1}^{n} r^2 \text{ and } \sum_{r=1}^{n} r$$

The terms u_1, u_2, u_3, … of a sequence S_n are defined by the formula

$$u_n = (4n-1)^3$$

where n is a positive integer.

(a) Find a simplified expression for $\sum_{r=1}^{n} u_r$

(5)

(b) Find the smallest value of n for which $\sum_{r=1}^{n} u_r > 1\,500\,000$

(2)

(c) Find the two consecutive terms in S_n which have a difference of 28 828.

(3)

(Total for Question 4 is 10 marks)

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5. The curve C has equation

$$y = \arctan\left(\frac{1}{3}x\right) \qquad -3 \leq x \leq 3$$

(a) Show that C has no stationary points.

(2)

The point P is the point on C at which $x = \sqrt{3}$

The tangent to C at the point P crosses the x-axis at the point Q.

The normal to C at the point P crosses the x-axis at the point R.

(b) show that the area of the triangle PQR is $\frac{a}{b}\pi^2$ where a and b are integers to be determined.

(6)

(Total for Question 5 is 8 marks)

6.

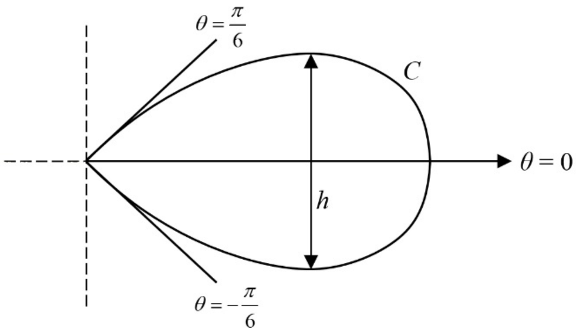

Figure 2

Figure 2 shows a sketch of the curve C with equation

$$r = 5\cos 3\theta \qquad -\frac{\pi}{6} \leq \theta \leq \frac{\pi}{6}$$

A blade is to be made in the form of a prism and C is proposed as the shape of its cross section, with all measurements being given in centimetres.

(a) Find the height of the blade, marked h on Figure 2, giving your answer to 3 significant figures.

(6)

The length of the blade is to be 30 cm.
The density of the material used to make the blade is 19 grams per cm^3

(b) Find the mass of the blade, in grams.

(6)

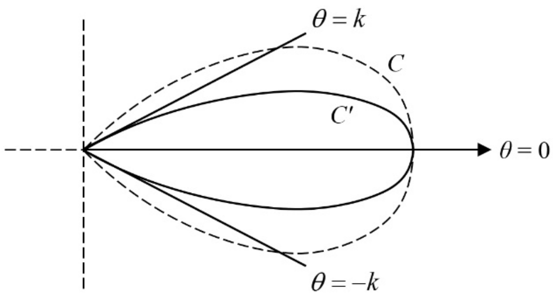

Figure 3

In a revised proposal, the cross section of the blade is to be made narrower, as shown by the sketch in Figure 3, in which the revised cross section is labelled C'. The horizontal measurement of the blade is to be unchanged. The revised cross section has equation

$$r = a\cos b\theta \qquad -k \leq \theta \leq k$$

where a, b and k are positive constants.

(c) Suggest a suitable set of values for the constants a, b and k.

(2)

(Total for Question 6 is 14 marks)

7. **Solutions based entirely on graphical or numerical methods are not acceptable.**

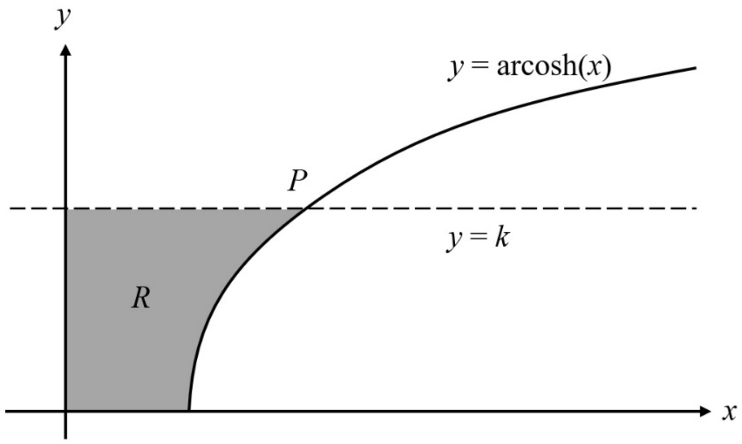

Figure 1

Figure 1 shows a sketch of part of the curve with equation

$$y = \text{arcosh}\, x \qquad x \geq 0$$

and the straight line with equation $y = k$

The line and the curve intersect at the point P.

Given that $k = \frac{1}{2}\ln 10$

(a) show that the x co-ordinate of P is $\frac{11\sqrt{10}}{20}$

(3)

The finite region R, shown shaded in Figure 1, is bounded by the curve with equation $y = \text{arcosh}\, x$, the y-axis and the line with equation $y = k$

The region R is rotated through 2π radians about the y-axis.

(b) Use calculus to show that the volume of the solid generated is $\frac{\pi}{80}\left(99 + 20\ln 10\right)$

(6)

(Total for Question 7 is 9 marks)

8. (i) The point P is one vertex of an equilateral triangle in an Argand diagram. The centre of the equilateral triangle is at the origin.

Given that P represents the complex number $\sqrt{6}+\sqrt{2}+\left(\sqrt{6}-\sqrt{2}\right)\mathrm{i}$, determine the complex numbers that represent the other vertices of the equilateral triangle, giving your answers in the form $r\mathrm{e}^{\mathrm{i}\theta}$

(5)

(ii) (a) On a single Argand diagram, shade the region, R, that satisfies both

$$|z-3|\leq 4 \qquad \text{and} \qquad -\frac{1}{6}\pi \leq \arg(z+1) \leq \frac{1}{6}\pi$$

(2)

(b) Show that the area of R is

$$\frac{8}{3}\left(3\sqrt{3}+2\pi\right)$$

(3)

(Total for Question 8 is 10 marks)

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9. (a) Given that $|z|<1$, write down the sum of the infinite series

$$1+z+z^2+z^3+\ldots$$

(1)

(b) Given that $z=\frac{1}{2}(\cos\theta+\mathrm{i}\sin\theta)$,

(i) use the answer to part (a), and de Moivre's theorem or otherwise, to prove that

$$\frac{1}{2}\cos\theta+\frac{1}{4}\cos 2\theta+\frac{1}{8}\cos 3\theta+\ldots=\frac{2\cos\theta-1}{5-4\cos\theta}$$

(5)

(ii) find the real part of z in the case when $\frac{1}{2}\cos\theta+\frac{1}{4}\cos 2\theta+\frac{1}{8}\cos 3\theta+\ldots=-\frac{2}{7}$.

(2)

(Total for Question 9 is 8 marks)

TOTAL FOR PAPER IS 75 MARKS