Please check the examination details below before entering your candidate information

Candidate surname	Other names

Pearson Edexcel Level 3 GCE

Centre Number | Candidate Number

Time 1 hour 30 minutes

Paper reference **9FM0/01**

Further Mathematics

Advanced

PAPER 1: Core Pure Mathematics 1

October 2021 Shadow Set 1

You must have:
Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 9 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- Good luck with your examination.

Turn over

1. The transformation T_1 is a rotation through angle θ degrees anticlockwise about the origin.

The transformation T_2 is an enlargement, centre the origin, with scale factor c, where $c > 0$

The transformation T_1 followed by the transformation T_2 is represented by the matrix

$$\mathbf{T} = \begin{pmatrix} -5\sqrt{2} & 5\sqrt{2} \\ -5\sqrt{2} & -5\sqrt{2} \end{pmatrix}$$

(a) Determine

(i) the value of c,

(ii) the smallest value of θ

(4)

A triangle A has vertices at the points with coordinates $(0, k)$, $(0, -k)$, and $(3k, 0)$ where k is a positive constant.

The triangle A is transformed to the triangle A' by the transformation represented by $\mathbf{T}$.

The area of triangle A' is 75

(b) Find the value of k

(2)

(Total for Question 1 is 6 marks)

2. (a) Use the Maclaurin series expansions, given in the formula booklet, for sin x and arctan x to determine the series expansion of

$$\sin\left(\frac{x}{2}\right)\arctan\left(\frac{x}{2}\right)$$

in ascending powers of x, up to and including the term in x^6 stating the range of values of x for which the expansion is valid.

Give each term in simplest form.

(3)

(b) Use the answer to part (a) and calculus to find an approximation for

$$\int_1^{\sqrt{3}} \left(\frac{8}{x^3}\sin\left(\frac{x}{2}\right)\arctan\left(\frac{x}{2}\right)\right)\,\mathrm{d}x$$

giving your answer in the form $\ln x - \frac{b}{c}$ where a, b and c are integers and $\frac{b}{c}$ is in its simplest form.

(3)

(c) Use the integration function on your calculator to evaluate

$$\int_1^{\sqrt{3}} \left(\frac{8}{x^3}\sin\left(\frac{x}{2}\right)\arctan\left(\frac{x}{2}\right)\right)\,\mathrm{d}x$$

Give your answer to 5 decimal places.

(1)

(Total for Question 2 is 7 marks)

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3. The cubic equation

$$ax^3 + ax^2 + bx + c = 0$$

where a, b and c are constants, has roots α, β and γ

The cubic equation

$$y^3 - ky^2 + 6y + 28 = 0$$

where k is a constant, has roots $(3\alpha + 2)$, $(3\beta + 2)$ and $(3\gamma + 2)$

Without solving either cubic equation,

(i) show that $k = 3$

(ii) obtain a possible set of positive integer values for a, b and c.

(6)

(Total for Question 3 is 6 marks)

4. (i) A rotation about the x axis is represented by the matrix $\mathbf{R}$, where

$$\mathbf{R} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{pmatrix}.$$

Write down the inverse of $\mathbf{R}$ giving your answer in terms of $\cos\theta$ and $\sin\theta$

(1)

(ii) Given that

$$\begin{pmatrix} 8 & 7 & -12 \\ 2 & -2 & 2 \\ -4 & -1 & 6 \end{pmatrix}\begin{pmatrix} 1 & s & 1 \\ 2 & 0 & 2t \\ 1 & t & s \end{pmatrix} = \lambda\mathbf{I}$$

where s, t and λ are constants,

(a) determine

- the value of λ
- the value of s
- the value of t

(b) Hence deduce the inverse of the matrix $\begin{pmatrix} 1 & s & 1 \\ 2 & 0 & 2t \\ 1 & t & s \end{pmatrix}$

(3)

(iii) Given that

$$\mathbf{M} = \begin{pmatrix} 1 & 1 & 1 \\ \tan\theta & 1 & 0 \\ \tan 2\theta & -1 & 0 \end{pmatrix} \qquad \text{where } -\frac{\pi}{2} < \theta < \frac{\pi}{2}$$

determine the values of θ for which the matrix $\mathbf{M}$ is singular.

(4)

(Total for Question 4 is 8 marks)

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5. (i) Evaluate the improper integral

$$\int_{2}^{\infty} 3\mathrm{e}^{(4-2x)}\,\mathrm{d}x$$

(3)

(ii) A satellite orbits the Earth once every 40 minutes.

The height of the satellite, h km, above the surface of the Earth at time t minutes from the start of its first complete orbit is modelled by the equation

$$h = 380 - 2t + \frac{t^2}{20} - 15\sin\left(\frac{\pi}{10}t\right) \qquad 0 \leq t \leq 40\,.$$

(a) Verify that the height of the satellite above the surface of the Earth is the same at the end of its first orbit as it was at the start.

(1)

(b) Use calculus to show that, according to the model, the mean height of the satellite above the surface of the Earth during its first orbit is $\frac{1100}{3}$ km.

(3)

(Total for Question 5 is 7 marks)

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6. A population of rats becomes established on a small island.

When they were first discovered, in January 2020, scientists estimated that the population of rats was 1500.

By January 2021 the scientists estimated that the population of rats had increased by 800.

The scientists model the population of rats on the island using the differential equation

$$25\frac{\mathrm{d}^2 p}{\mathrm{d}t^2} - 20\frac{\mathrm{d}p}{\mathrm{d}t} + 4p = 0 \qquad t \geq 0$$

where p is the population of rats and t is the time, in years, since January 2020.

(a) Determine the particular solution to the differential equation.
Give any coefficients in your solution to a sensible degree of accuracy.

(7)

(b) Hence find, according to the model, an estimate for the population of rats on the island in January 2023.

(2)

(c) Suggest one reason why the model is unlikely to be accurate in the long term.

(1)

(Total for Question 6 is 10 marks)

7. The plane Π has equation

$$\mathbf{r} = \begin{pmatrix} 7 \\ 3 \\ 2 \end{pmatrix} + s\begin{pmatrix} 3 \\ 2 \\ -2 \end{pmatrix} + t\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$$

where s and t are scalar parameters.

(a) Show that vector $2\mathbf{i} - 2\mathbf{j} + \mathbf{k}$ is perpendicular to Π.

(2)

(b) Hence find a Cartesian equation of Π.

(2)

The line l has equation

$$\mathbf{r} = \begin{pmatrix} -4 \\ -6 \\ 6 \end{pmatrix} + \lambda\begin{pmatrix} 5 \\ 0 \\ -2 \end{pmatrix}$$

where λ is a scalar parameter.

The point P lies on l.

Given that the shortest distance between P and Π is 8

(c) determine the possible coordinates of P.

(4)

(d) state the co-ordinates of the point at which l and Π intersect.

(2)

(Total for Question 7 is 10 marks)

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8. Water and antifreeze are being mixed together in a tank.

The mixture is stirred continuously so that the water and antifreeze are instantly dispersed evenly throughout the tank.

Initially the tank holds a mixture of 8 litres of water and 2 litres of antifreeze, so that the concentration of antifreeze in the mixture is said to be 20%.

The concentration of antifreeze in the mixture is now increased by

- adding water to the tank at a rate of 0.1 litres per second
- adding antifreeze to the tank at a rate of 0.3 litres per second
- pumping mixture from the tank at a rate of 0.4 litres per second.

Let x litres be the amount of antifreeze in the tank at time t seconds after the mixture starts to be altered.

(a) Show that the change in the amount of antifreeze in the tank can be modelled by the differential equation

$$\frac{\mathrm{d}x}{\mathrm{d}t} = 0.3 - \frac{x}{k}$$

where k is a positive constant to be determined.

(2)

(b) By solving the differential equation, determine how long it will take for the concentration of antifreeze in the mixture to reach 40%, according to the model.

Give your answer to the nearest tenth of a second.

(6)

As t becomes large, the concentration of antifreeze in the mixture approaches c%, where c is a constant.

(c) Find the value of c

(2)

(Total for Question 8 is 10 marks)

9. (a) Use a hyperbolic substitution and calculus to show that

$$\int \frac{x^2}{\sqrt{x^2+4}}\,dx = \frac{x}{2}\sqrt{x^2+4} - 2\operatorname{arsinh}\left(\frac{x}{2}\right) + k$$

where k is an arbitrary constant.

(6)

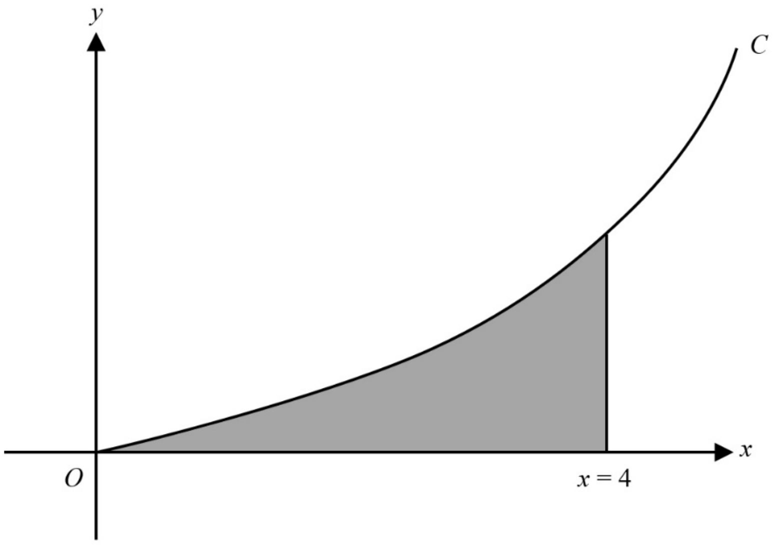

Figure 1

Figure 1 shows a sketch of part of the curve C with equation

$$y = x \operatorname{arsinh}\left(\frac{x}{2}\right) \qquad x \geq 0$$

The region shown shaded in Figure 1, is bounded by the curve C, the x-axis and the line with equation $x = 4$

(b) Using algebraic integration and the result from part (*a*), show that the area of R is given by

$$9\ln\left(2+\sqrt{5}\right) - 2\sqrt{5}$$

(5)

(Total for Question 9 is 11 marks)

TOTAL FOR PAPER IS 75 MARKS