Question	Scheme	Marks	AOs
1(a)	$\mathrm{f}(x)=\mathrm{e}^{2x}\cos x \Rightarrow \mathrm{f}'(x)=2\mathrm{e}^{2x}\cos x-\mathrm{e}^{2x}\sin x$	M1	1.1a
	$\mathrm{f}''(x)=4\mathrm{e}^{2x}\cos x-2\mathrm{e}^{2x}\sin x-\left(2\mathrm{e}^{2x}\sin x+\mathrm{e}^{2x}\cos x\right)$	M1 A1	1.1b 1.1b
	$\mathrm{f}''(x)=3\mathrm{e}^{2x}\cos x-4\mathrm{e}^{2x}\sin x=p\mathrm{e}^{2x}\cos x+q\left(2\mathrm{e}^{2x}\cos x-\mathrm{e}^{2x}\sin x\right)$ $\Rightarrow p=\ldots,q=\ldots$	M1	3.1a
	$\mathrm{f}''(x)=-5\mathrm{f}(x)+4\mathrm{f}'(x)$	A1	2.1
		(5)	
(b)	$\mathrm{f}(0)=1,\mathrm{f}'(0)=2,\mathrm{f}''(0)=3,\mathrm{f}'''(0)=2,\mathrm{f}''''(0)=-7,\mathrm{f}^{v}(0)=-38$	M1	1.1b
	$\mathrm{f}(x)=\mathrm{f}(0)+x\mathrm{f}'(0)+\dfrac{x^2}{2!}\mathrm{f}''(0)+\ldots$	M1	1.1b
	$\mathrm{f}(x)\approx 1+2x+\dfrac{3x^2}{2}+\dfrac{x^3}{3}-\dfrac{7x^4}{24}-\dfrac{19x^5}{60}$	A1	2.2a
		(3)	
			(8 marks)

Notes

(a)
M1: Realises the need to use the product rule and attempts the first derivative
M1: Applies the product rule again to find the second derivative
A1: Correct second derivative simplified or un-simplified
M1: Uses their derivatives in order to obtain values for p and q
A1: Completes the proof and obtains the correct values of p and q
(b)
M1: Attempts all 5 derivatives at $x = 0$ using the result from part (a)
M1: Uses the correct Maclaurin series including the factorials
A1: Correct expression

Question	Scheme	Marks	AOs
2(a)	$z=\cos\theta+\mathrm{i}\sin\theta \Rightarrow \frac{1}{z}=\cos\theta-\mathrm{i}\sin\theta$ $\Rightarrow\left(z+\frac{1}{z}\right)^5=(2\cos\theta)^5=32\cos^5\theta$	M1	2.1
	$\left(z+\frac{1}{z}\right)^5=z^5+\frac{1}{z^5}+5\left(z^3+\frac{1}{z^3}\right)+10\left(z+\frac{1}{z}\right)$	M1 A1	2.1 1.1b
	$=2\cos 5\theta+10\cos 3\theta+20\cos\theta$	M1	2.1
	$\cos^5\theta=\frac{1}{16}(\cos 5\theta+5\cos 3\theta+10\cos\theta)$ *	A1*	1.1b
		(5)	
(b)	$\cos\theta-\cos 5\theta=5\cos 3\theta\Rightarrow\cos\theta=5\cos 3\theta+\cos 5\theta=16\cos^5\theta-10\cos\theta$	M1	3.1a
	$16\cos^5\theta-11\cos\theta=0$	A1	1.1b
	$\cos\theta(16\cos^4\theta-11)=0\Rightarrow\cos\theta=0,\ \pm\sqrt[4]{\frac{11}{16}}$	M1	1.1b
	$\theta=3.57,\frac{3\pi}{2}(\text{or }4.71),5.86$	A1 A1	1.1b 1.1b
		(5)	
			(10 marks)

Notes

(a)

M1: Begins the proof by demonstrating that $\left(z+\frac{1}{z}\right)^5=32\cos^5\theta$

M1: Attempts to expand $\left(z+\frac{1}{z}\right)^5$ including the binomial coefficients

A1: Correct expansion

M1: Uses $z^n+\frac{1}{z^n}=2\cos n\theta$ to obtain an expression in terms of $\cos 5\theta$, $\cos 3\theta$ and $\cos\theta$

A1*: Concludes the argument by equating the two expressions leading to the printed answer with no errors

(b)

M1: Makes the connection with part (a) and reaches an equation in $\cos\theta$ only

A1: Correct equation

M1: Solves their equation for $\cos\theta$

A1: 2 correct solutions

A1: All 3 correct solutions. Ignore extra solutions outside the range but deduct this mark if there are extra answers in range.

Question	Scheme	Marks	AOs
3	Area enclosed by curve = $\frac{1}{2}\int (7.5+1.5\cos 6\theta)^2\,d\theta$	M1	3.1a
	$(7.5+1.5\cos 6\theta)^2 = 56.25+22.5\cos 6\theta+2.25\cos^2 6\theta$ $=56.25+22.5\cos 6\theta+2.25\left(\frac{\cos 12\theta+1}{2}\right)$	M1	2.1
	$\frac{1}{2}\int (7.5+1.5\cos 6\theta)^2\,d\theta = \frac{459}{16}\theta+\frac{15}{8}\sin 6\theta+\frac{3}{64}\sin 12\theta(+c)$	A1ft	1.1b
	Area enclosed by curve = $\left[\frac{459}{16}\theta+\frac{15}{8}\sin 6\theta+\frac{3}{64}\sin 12\theta\right]_0^{2\pi}$	M1	3.1a
	$=\frac{459\pi}{8}(=180.24...)$	A1	1.1b
	Total shaded area = $\pi\times10^2-"\frac{459\pi}{8}"+\pi\times5^2$ = 314.15… – 180.24… + 78.53…	M1	3.1a
	$=\frac{541\pi}{8}\,\text{mm}^2=2.12\,\text{cm}^2$	A1	3.2a
		(7)	
			(7 marks)

Notes

M1: A correct strategy identified for finding the area enclosed by the polar curve using a correct formula

M1: Squares and uses $\cos^2 6\theta=\frac{\pm1\pm\cos 12\theta}{2}$ to obtain an expression in an integrable form

A1ft: Correct follow through integration

M1: Correct use of correct limits (e.g. may use $0\rightarrow2\pi$ or $2\times(0\rightarrow\pi)$ etc.)

A1: Correct area enclosed by the curve

M1: Fully correct strategy for obtaining the area to be painted

A1: Correct area

Question	Scheme	Marks	AOs
4(a)	$\dfrac{1}{(5r-2)(5r+3)} \equiv \dfrac{A}{5r-2}+\dfrac{B}{5r+3} \Rightarrow A=\ldots,\ B=\ldots,$ $\left(\text{NB } A=\dfrac{1}{5} \quad B=-\dfrac{1}{5}\right)$	M1	3.1a
	$\displaystyle\sum_{r=1}^{n}\frac{1}{(5r-2)(5r+3)}$ $\dfrac{1}{5}\left(\dfrac{1}{3}-\dfrac{1}{8}+\dfrac{1}{8}-\dfrac{1}{13}+\ldots+\dfrac{1}{5n-7}-\dfrac{1}{5n-2}+\dfrac{1}{5n-2}-\dfrac{1}{5n+3}\right)$	M1	2.1
	$=\dfrac{1}{5}\left(\dfrac{1}{3}-\dfrac{1}{5n+3}\right)$	A1	1.1b
	$=\dfrac{1}{5}\left(\dfrac{5n+3-3}{3(5n+3)}\right)$	M1	1.1b
	$=\dfrac{n}{3(5n+3)}$	A1	2.2a
		(5)	
(b)	$\displaystyle\sum_{r=10}^{50}\frac{1}{(5r-2)(5r+3)}=\mathrm{f}(50)-\mathrm{f}(9\text{ or }10)$	M1	1.1b
	$=\dfrac{50}{3(5\times50+3)}-\dfrac{9}{3(5\times9+3)}=\dfrac{41}{12144}$	A1	1.1b
		(2)	
		(7 marks)	

Notes

(a)
M1: A complete strategy to find A and B e.g. partial fractions
M1: Starts the process of differences to identify the relevant fractions at the start and end
A1: Correct fractions that do not cancel
M1: Attempt common denominator
A1: Correct answer
(b)
M1: Uses the answer to part (a) to calculate f(50) – f(9 or 10)
A1: Correct exact answer

Question	Scheme	Marks	AOs
5(a)	$(t+4)\frac{\mathrm{d}v}{\mathrm{d}t}+5v=10(t+4)\Rightarrow\frac{\mathrm{d}v}{\mathrm{d}t}+\frac{5v}{(t+4)}=10$	M1	1.1b
	$\text{IF}=\mathrm{e}^{\int\frac{5}{t+4}\mathrm{d}t}=(t+4)^5\Rightarrow v(t+4)^5=\int 10(t+4)^5\mathrm{d}t$	M1	3.1b
	$v(t+4)^5=\frac{5}{3}(t+4)^6+c$	A1	1.1b
	$t=0,\ v=0\Rightarrow c=-\frac{20480}{3}$	M1	3.4
	$t=3\Rightarrow v=\frac{5}{3}\times 7-\frac{20480}{3\times 7^5}$	M1	3.4
	$v=11.3\left(\text{ ms}^{-1}\right)$	A1	1.1b
		(6)	
(b)	For large values of t, the velocity increases	B1	1.1b
		(1)	
(c)	E.g. • The raindrop may hit an obstacle as it falls • The raindrop is unlikely to be at rest initially • The raindrop may be affected by the wind as it falls • The raindrop will eventually hit the ground	B1	3.5b
		(1)	
			(8 marks)

Notes

(a)
M1: Divides through by $(t + 4)$
M1: Uses the model to find the integrating factor and attempts the solution of the differential equation
A1: Correct solution
M1: Interprets the initial conditions to find the constant of integration
M1: Uses their solution to the problem to find the velocity after 3 seconds
A1: Correct value
(b)
B1: Makes a sensible comment regarding the motion of the raindrop e.g. as t increases so does v
(c)
B1: States a limitation of the model – see scheme for examples

Question	Scheme	Marks	AOs
6	When n = 1, $3^n - 2^n = 1$ When n = 2, $3^n - 2^n = 9 - 4 = 5$ So the result is true for n = 1 and n = 2	B1	2.2a
	Assume true for $n = k$ and $n = k + 1$ so $u_k = 3^k - 2^k$ and $u_{k+1} = 3^{k+1} - 2^{k+1}$	M1	2.4
	$u_{k+2} = 5\left(3^{k+1} - 2^{k+1}\right) - 6\left(3^k - 2^k\right)$	M1	1.1b
	$u_{k+2} = 5\times3^{k+1} - 5\times2^{k+1} - 2\times3^{k+1} + 3\times2^{k+1}$	A1	1.1b
	$= 3\times3^{k+1} - 2\times2^{k+1}$ $= 3^{k+2} - 2^{k+2}$	A1	2.1
	If the statement is true for $n = k$ and $n = k + 1$then it has been shown true for $n = k + 2$ and as it is true for n = 1 and n = 2, the statement is true for all positive integers n.	A1	2.4
		(6)	
			(6 marks)

Notes

B1: Shows the statement is true for n = 1 and n = 2
M1: Makes a statement that assumes the result is true for $n = k$ and $n = k + 1$
M1: Substitutes the assumption statements into the given result
A1: Correct expression that has been processed correctly to be in terms of 3^{k+1} and 2^{k+1}
A1: Obtains $3^{k+2} - 2^{k+2}$ with no errors and all working shown
A1: Correct complete conclusion that may be part of a narrative throughout the proof

Question	Scheme	Marks	AOs
7(a)	$\mathbf{r} = 8\mathbf{i} + 2\mathbf{j} + 10\mathbf{k} + k(2\mathbf{i} - 3\mathbf{j} + 4\mathbf{k})$ or $(8\mathbf{i} + 2\mathbf{j} + 10\mathbf{k}).(2\mathbf{i} - 3\mathbf{j} + 4\mathbf{k}) = 16 - 6 + 40$	M1	1.1b
	$(8\mathbf{i} + 2\mathbf{j} + 10\mathbf{k} + k(2\mathbf{i} - 3\mathbf{j} + 4\mathbf{k})).(2\mathbf{i} - 3\mathbf{j} + 4\mathbf{k}) = -8 \Rightarrow k = -2$ $\Rightarrow d = \lvert 2(2\mathbf{i} - 3\mathbf{j} + 4\mathbf{k}) \rvert = \sqrt{116}$ or $2\sqrt{29}$ Or $d = \dfrac{(8\mathbf{i} + 2\mathbf{j} + 10\mathbf{k}).(2\mathbf{i} - 3\mathbf{j} + 4\mathbf{k}) + 8}{\sqrt{2^2 + 3^2 + 4^2}} = \dfrac{58}{\sqrt{29}}$	M1 A1	3.1a 1.1b
		(3)	
(b)	$(4\mathbf{i} + \mathbf{j} - 7\mathbf{k}).(\mathbf{i} + 3\mathbf{j} + \mathbf{k}) = 4 + 3 - 7 = 0$ $(4\mathbf{i} + \mathbf{j} - 7\mathbf{k}).(2\mathbf{i} - \mathbf{j} + \mathbf{k}) = 8 - 1 - 7 = 0$	M1	1.1b
	As $4\mathbf{i} + \mathbf{j} - 7\mathbf{k}$ is perpendicular to both direction vectors of Π_2 then it must be perpendicular to Π_2	A1	2.2a
		(2)	
(c)	$(4\mathbf{i} + \mathbf{j} - 7\mathbf{k}).(2\mathbf{i} - 3\mathbf{j} + 4\mathbf{k}) = 8 - 3 - 28 = -23$	M1	1.1b
	$\sqrt{4^2 + 1^2 + 7^2}\sqrt{2^2 + 3^2 + 4^2}\cos\theta = -23$ $\Rightarrow \cos\theta = \dfrac{-23}{\sqrt{66}\sqrt{29}}$	M1	2.1
	$\theta = 58°$	A1	1.1b
		(3)	
(d)	$4x + y - 7z = 0$ and $2x - 3y + 4z = -8$		
	$x = 0 \to \left(0, \frac{56}{17}, \frac{8}{17}\right)$, $y = 0 \to \left(-\frac{28}{15}, 0, -\frac{16}{15}\right)$, $z = 0 \to \left(-\frac{4}{7}, \frac{16}{7}, 0\right)$ $\Rightarrow dir = 17\mathbf{i} + 30\mathbf{j} + 14\mathbf{k}$	M1 A1	3.1a 1.1b
	$\mathbf{r} = \dfrac{56}{17}\mathbf{j} + \dfrac{8}{17}\mathbf{k} + \lambda(17\mathbf{i} + 30\mathbf{j} + 14\mathbf{k})$	M1 A1	1.1b 2.5
		(4)	
		(12 marks)	

Notes

(a)
M1: Starts by attempting to find an appropriate scalar product or finding the parametric equation of the perpendicular line
M1: A complete strategy to establish the required distance
A1: Correct exact answer (allow any exact form)
(b)
M1: Attempts both scalar products
A1: Makes a correct deduction
(c)
M1: Calculates the scalar product between the normal vectors
M1: Applies the scalar product formula with their – 23 to find a value for $\cos\theta$
A1: Correct answer
(d)

M1: Attempts to find the direction e.g. by finding 2 points on the line or uses vector product A1: Correct direction of required line M1: Uses their direction and a point on the line to form a vector equation for the line A1: Correct equation using correct notation

Question	Scheme	Marks	AOs
8(a)	$y=\frac{\mathrm{d}x}{\mathrm{d}t}+5x-51\Rightarrow\frac{\mathrm{d}y}{\mathrm{d}t}=\frac{\mathrm{d}^2x}{\mathrm{d}t^2}+5\frac{\mathrm{d}x}{\mathrm{d}t}$	B1	2.1
	$\Rightarrow\frac{\mathrm{d}^2x}{\mathrm{d}t^2}+5\frac{\mathrm{d}x}{\mathrm{d}t}=12x-6\left(\frac{\mathrm{d}x}{\mathrm{d}t}+5x-51\right)$	M1	2.1
	$\Rightarrow\frac{\mathrm{d}^2x}{\mathrm{d}t^2}+11\frac{\mathrm{d}x}{\mathrm{d}t}+18x=306*$	A1*	1.1b
		(3)	
(b)	$m^2+11m+18=0\Rightarrow m=...$	M1	3.4
	$m=-2,-9$	A1	1.1b
	$x=A\mathrm{e}^{\alpha t}+B\mathrm{e}^{\beta t}$	M1	3.4
	$x=A\mathrm{e}^{-9t}+B\mathrm{e}^{-2t}$	A1	1.1b
	PI: Try $x=k\Rightarrow18k=306$ $\Rightarrow k=17$	M1	3.4
	$GS:x=A\mathrm{e}^{-9t}+B\mathrm{e}^{-2t}+17$	A1ft	1.1b
		(6)	
(c)	$y=\frac{\mathrm{d}x}{\mathrm{d}t}+5x-51\Rightarrow y=-9A\mathrm{e}^{-9t}-2B\mathrm{e}^{-2t}+5A\mathrm{e}^{-9t}+5B\mathrm{e}^{-2t}+85-51$	M1	3.4
	$y=3B\mathrm{e}^{-2t}-4A\mathrm{e}^{-9t}+34$	A1	1.1b
		(2)	
(d)	$0=A+B+17,\ 0=3B-4A+34\Rightarrow A=...,B=...$ (NB $A=-\frac{17}{7},B=-\frac{102}{7}$)	M1	3.3
	$x=17-\frac{17}{7}\mathrm{e}^{-9t}-\frac{102}{7}\mathrm{e}^{-2t},y=34+\frac{68}{7}\mathrm{e}^{-9t}-\frac{306}{7}\mathrm{e}^{-2t}$	A1	1.1b
	$\frac{\mathrm{d}x}{\mathrm{d}t}=\frac{\mathrm{d}y}{\mathrm{d}t}\Rightarrow\frac{153}{7}\mathrm{e}^{-9t}+\frac{204}{7}\mathrm{e}^{-2t}=-\frac{612}{7}\mathrm{e}^{-9t}+\frac{612}{7}\mathrm{e}^{-2t}\Rightarrow\mathrm{e}^{k}=\alpha$	M1	3.1b
	$\mathrm{e}^{7t}=\frac{15}{8}\Rightarrow7t=\ln\left(\frac{15}{8}\right)\Rightarrow t=\frac{1}{7}\ln\left(\frac{15}{8}\right)$	M1	1.1b
	= 5.39 minutes	A1	3.2a
		(5)	
(e)	E.g. • The model suggests that, in the long term, the amount of antibiotic in the blood (and/or the body tissue) will remain constant and this is unlikely	B1	3.5a
		(1)	
			(17 marks)

Notes

(a)
B1: Differentiates the first equation with respect to *t* correctly
M1: Proceeds to the printed answer by substituting into the second equation
A1*: Achieves the printed answer with no errors

(b)
M1: Uses the model to form and solve the Auxiliary Equation
A1: Correct roots of the AE
M1: Uses the model to form the Complementary Function
A1: Correct CF
M1: Chooses the correct form of the PI according to the model and uses a complete method to find the PI
A1ft: Combines their CF and PI to give x in terms of t
(c)
M1: Uses the model and their answer to part (b) to give y in terms of t
A1: Correct equation
(d)
M1: Realises the need to use the initial conditions to establish the values of their constants
A1: Correct particular solutions for x and y
M1: Differentiates both expressions, sets them equal and proceeds to reach an equation of the form $\mathrm{e}^k = \alpha$
M1: Correct use of logarithms to reach $t = \ldots$
A1: Correct value
(e)
B1: Suggests a suitable evaluation of the model