A Level Core Pure Mathematics 2 Mock Paper Set 1 (9FM0/02) Mark Scheme

Question	Scheme	Marks	AOs
1(i)	$\alpha+\beta+\gamma=\frac{3}{2},\ \alpha\beta+\alpha\gamma+\beta\gamma=2,\ \alpha\beta\gamma=-\frac{7}{2}$	B1	3.1a
	$\frac{3}{\alpha}+\frac{3}{\beta}+\frac{3}{\gamma}=\frac{3(\alpha\beta+\alpha\gamma+\beta\gamma)}{\alpha\beta\gamma}$	M1	1.1b
	$=3(2)\div-\frac{7}{2}=-\frac{12}{7}$	A1ft	1.1b
		(3)	
(ii)	$(\alpha-2)(\beta-2)(\gamma-2)=(\alpha\beta-2\alpha-2\beta+4)(\gamma-2)$	M1	1.1b
	$=\alpha\beta\gamma-2(\alpha\beta+\alpha\gamma+\beta\gamma)+4(\alpha+\beta+\gamma)-8$	A1	1.1b
	$=-\frac{7}{2}-2(2)+4\left(\frac{3}{2}\right)-8=-\frac{19}{2}$	A1	1.1b
		(3)	
	Alternative		
	$2(x+2)^3-3(x+2)^2+4(x+2)+7=0$	M1	1.1b
	$=\ldots+16+\ldots-12+\ldots+8+7=19$	A1	1.1b
	$(\alpha-2)(\beta-2)(\gamma-2)=-\frac{19}{2}$	A1	1.1b
		(3)	
(iii)	$\alpha^2+\beta^2+\gamma^2=(\alpha+\beta+\gamma)^2-2(\alpha\beta+\alpha\gamma+\beta\gamma)$	M1	3.1a
	$=\left(\frac{3}{2}\right)^2-2(2)=-\frac{7}{4}$	A1ft	1.1b
		(2)	
			(8 marks)

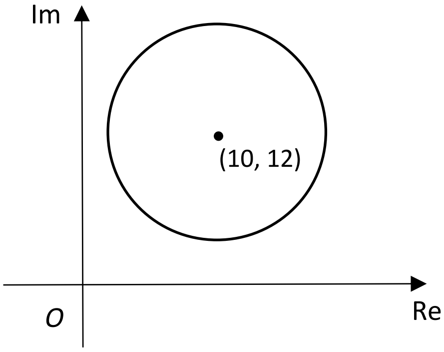
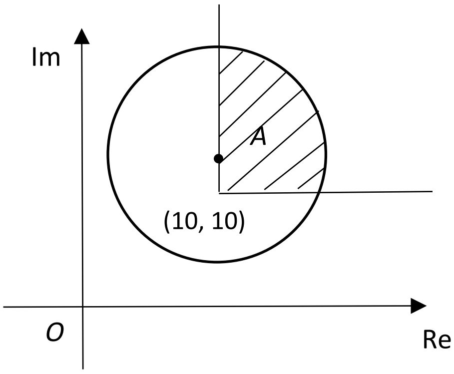
Notes

(i) B1: Identifies the correct values for all 3 expressions (can score anywhere)
M1: Uses a correct identity
A1ft: Correct value (follow through their $\frac{3}{2},2,-\frac{7}{2}$)
(ii) M1: Attempts to expand
A1: Correct expansion
A1: Correct value
Alternative:
M1: Substitutes (x + 2) for x in the given cubic
A1: Calculates the correct constant term
A1: Completes correctly by changing sign and dividing by 2
(iii) M1: Establishes the correct identity
A1ft: Correct value (follow through their $\frac{3}{2},2,-\frac{7}{2}$)

Question	Scheme	Marks	AOs
2(a)(i)	$a\cos^2\frac{\pi}{3}=1\Rightarrow a=...$ or $a\cos^2\frac{\pi}{6}=3\Rightarrow a=...$	M1	3.4
	$a=4$	A1	2.2b
(ii)	$b\tan\frac{\pi}{3}-b\tan\frac{\pi}{6}=2\Rightarrow b=...$	M1	3.4
	$b=\sqrt{3}$	A1	2.2b
		(4)	
(b)	$V=\pi\int x^2\mathrm{d}y=\pi\int 16\cos^4\theta\times\sqrt{3}\sec^2\theta\,\mathrm{d}\theta$	M1	3.4
	$=16\pi\sqrt{3}\int\cos^2\theta\,\mathrm{d}\theta$	A1	1.1b
	$=16\pi\sqrt{3}\int\frac{\cos 2\theta+1}{2}\mathrm{d}\theta$	M1	3.1a
	$=8\pi\sqrt{3}\left[\frac{1}{2}\sin 2\theta+\theta\right]$	A1	1.1b
	$=8\pi\sqrt{3}\left[\frac{1}{2}\sin 2\theta+\theta\right]_{\frac{\pi}{6}}^{\frac{\pi}{3}}=8\pi\sqrt{3}\left(\frac{\sqrt{3}}{4}+\frac{\pi}{3}-\frac{\sqrt{3}}{4}-\frac{\pi}{6}\right)$	M1	3.4
	$=\frac{4\pi^2\sqrt{3}}{3}=22.8\text{ cm}^3$	A1	1.1b
		(6)	
			(10 marks)

Notes

(a)
M1: Interprets the information from the model and uses the parametric form of x to determine the value of a
A1: Correct value for a
M1: Interprets the information from the model and uses the parametric form of y to find b
A1: Correct value for b
(b)
M1: Uses the correct volume of revolution formula and the parametric equations for the model
A1: Correct simplified integral
M1: Uses a correct double angle identity on the integrand to achieve a suitable form for integration
A1: Correct integration
M1: Correct use of correct limits according to the model
A1: Correct volume (allow exact or awrt 22.8)

Question	Scheme	Marks	AOs
3(a)		M1	1.1b
		A1	1.1b
		(2)	
(b)		B1	1.1b
		B1ft	1.1b
		(2)	
(c)	$8^2-2^2=h^2 \Rightarrow h=...$	M1	3.1a
	$h=2\sqrt{15}$	A1	1.1b
	Triangle area $=\dfrac{1}{2}\times 2\times 2\sqrt{15}$	M1	2.1
	Sector area $=\dfrac{1}{2}\times 8^2\times\left(\pi-\tan^{-1}\left(\sqrt{15}\right)\right)$ or $\dfrac{1}{2}\times 8^2\times\left(\pi-\cos^{-1}\left(\dfrac{1}{4}\right)\right)$	M1	3.1a
	Total area $=\dfrac{1}{2}\times 8^2\times\left(\pi-\tan^{-1}\left(\sqrt{15}\right)\right)+\dfrac{1}{2}\times 2\times 2\sqrt{15}$ = 66.1	A1	1.1b
		(5)	
			(9 marks)

Notes

(a) M1: A circle drawn
A1: A circle entirely in the first quadrant with the centre marked at (10, 12)
(b) B1: Correct pair of rays added to their diagram
B1ft: Area between their rays and within the circle shaded
(c) M1: Correct strategy to find the base (or angle) of the triangular part
A1: Correct length or angle
M1: Correct method for the area of the triangle
M1: Correct strategy for the area of the sector
A1: Correct answer (awrt 66.1)

Question	Scheme	Marks	AOs
4(a)	$\begin{vmatrix} 1 & -3 & 2 \\ k & 1 & -1 \\ 6 & -5 & k-1 \end{vmatrix} = k-1-5+3\left(k\left(k-1\right)+6\right)+2\left(-5k-6\right)$	M1 A1	3.1a 1.1b
	$3k^2-12k=0 \Rightarrow k=\ldots$	M1	3.1a
	k = 0 or 4	A1	1.1b
		(4)	
(b)(i)	$\begin{pmatrix} 1 & -3 & 2 \\ 5 & 1 & -1 \\ 6 & -5 & 4 \end{pmatrix}^{-1} = \frac{1}{15}\begin{pmatrix} -1 & 2 & 1 \\ -26 & -8 & 11 \\ -31 & -13 & 16 \end{pmatrix}$	M1 A1	1.1b 1.1b
	$\frac{1}{15}\begin{pmatrix} -1 & 2 & 1 \\ -26 & -8 & 11 \\ -31 & -13 & 16 \end{pmatrix}\begin{pmatrix} -7 \\ -5 \\ 1 \end{pmatrix}=\ldots$	M1	1.1b
	$\left(-\frac{2}{15}, \frac{233}{15}, \frac{298}{15}\right)$	A1	1.1b
(b)(ii)	Three planes that meet at a point.	A1	2.2a
		(5)	
			(9 marks)

Notes

(a)
M1: Starts by attempting to find the determinant in terms of k
A1: Correct determinant
M1: Realises that the condition for non-uniqueness is a zero determinant and solves to find k
A1: Correct values
(b)
M1: Uses k = 5 and attempts the inverse matrix
A1: Correct inverse
M1: Multiplies their inverse by $(-7, -5, 1)^{\mathrm{T}}$
A1: Correct exact coordinates
A1: Deduces the correct interpretation

Question	Scheme	Marks	AOs
5(a)	$\displaystyle\int \frac{1}{\sqrt{x^2+2x+10}}\,\mathrm{d}x = \int \frac{1}{\sqrt{(x+1)^2+9}}\,\mathrm{d}x$	M1	3.1a
	$= k\sinh^{-1}\left(\dfrac{x+a}{b}\right)$	M1	1.1b
	$= \sinh^{-1}\left(\dfrac{x+1}{3}\right)(+c)$	A1	1.1b
		(3)	
(b)	$\displaystyle\int_2^{20} \frac{1}{\sqrt{x^2+2x+10}}\,\mathrm{d}x = \sinh^{-1}\left(\frac{20+1}{3}\right) - \sinh^{-1}\left(\frac{2+1}{3}\right)$ $= \ln\left(7+\sqrt{50}\right) - \ln\left(1+\sqrt{2}\right) = \ln\dfrac{7+\sqrt{50}}{1+\sqrt{2}}$	M1	1.1b
	$= \dfrac{1}{(20-2)}\ln\dfrac{7+\sqrt{50}}{1+\sqrt{2}}$	M1	2.1
	$\dfrac{1}{18}\ln\left(3+2\sqrt{2}\right)$ or e.g. $\dfrac{1}{9}\ln\left(1+\sqrt{2}\right)$	A1	2.2a
		(3)	
			(6 marks)

Notes

(a)
M1: Recognises the need to and attempts to complete the square
A1: Integrates to obtain an expression of the required form
A1: Correct answer with or without + *c*
(b)
M1: Correct use of limits and combines ln terms
M1: Correct applies the method for the mean value for their integration
A1: Deduces a correct expression

Question	Scheme	Marks	AOs
6	When $n=1$ lhs $=\begin{pmatrix}1&1&2\\0&1&1\\0&0&1\end{pmatrix}$ rhs $=\begin{pmatrix}1&1&\frac{1}{2}\left(1^2+3\times1\right)\\0&1&1\\0&0&1\end{pmatrix}=\begin{pmatrix}1&1&2\\0&1&1\\0&0&1\end{pmatrix}$ So the statement is true for n = 1	B1	2.2a
	Assume true for n = k so $\begin{pmatrix}1&1&2\\0&1&1\\0&0&1\end{pmatrix}^k=\begin{pmatrix}1&k&\frac{1}{2}\left(k^2+3k\right)\\0&1&k\\0&0&1\end{pmatrix}$	M1	2.4
	$\begin{pmatrix}1&1&2\\0&1&1\\0&0&1\end{pmatrix}^{k+1}=\begin{pmatrix}1&k&\frac{1}{2}\left(k^2+3k\right)\\0&1&k\\0&0&1\end{pmatrix}\begin{pmatrix}1&1&2\\0&1&1\\0&0&1\end{pmatrix}$	M1	1.1b
	$=\begin{pmatrix}1&k+1&2+k+\frac{1}{2}\left(k^2+3k\right)\\0&1&k+1\\0&0&1\end{pmatrix}$	A1	1.1b
	$2+k+\frac{1}{2}\left(k^2+3k\right)=2+\frac{5}{2}k+\frac{1}{2}k^2$ $=\frac{1}{2}\left(k^2+5k+4\right)=\frac{1}{2}\left(\left(k+1\right)^2+3\left(k+1\right)\right)$	A1	2.1
	If the statement is true for n = k then it has been shown true for n = k + 1 and as it is true for n = 1, the statement is true for all positive integers n.	A1	2.4
		(6)	
		(6 marks)	

Notes

B1: Shows the statement is true for n = 1
M1: Makes a statement that assumes the result is true for n = k
M1: Attempts to multiply the correct matrices
A1: Correct matrix in terms of k
A1: Correct matrix in terms of k + 1 including sufficient explanation for the element at the top right hand corner
A1: Correct complete conclusion

Question	Scheme	Marks	AOs
7(a)	$4\cosh^3 x - 3\cosh x \equiv 4\left(\dfrac{e^x + e^{-x}}{2}\right)^3 - 3\left(\dfrac{e^x + e^{-x}}{2}\right)$	M1	1.2
	$\equiv 4\left(\dfrac{e^{3x} + 3e^x + 3e^{-x} + e^{-3x}}{8}\right) - 3\left(\dfrac{e^x + e^{-x}}{2}\right)$	M1	1.1b
	$= \dfrac{e^{3x}}{2} + \dfrac{3e^x}{2} + \dfrac{3e^{-x}}{2} + \dfrac{e^{-3x}}{2} - \dfrac{3e^x}{2} - \dfrac{3e^{-x}}{2} \equiv \dfrac{e^{3x} + e^{-3x}}{2} \equiv \cosh 3x$ *	A1*	2.1
		(3)	
(b)	$\cosh 3x = 9\cosh x \Rightarrow 4\cosh^3 x - 3\cosh x = 9\cosh x$ $\cosh^2 x = 3$	M1	3.1a
	$\cosh x = \sqrt{3} \Rightarrow x = \ln\left(\sqrt{3} + \sqrt{\left(\sqrt{3}\right)^2 - 1}\right)$	M1	1.1b
	$x = \ln\left(\sqrt{3} + \sqrt{2}\right)$ **or** $x = \ln\left(\sqrt{3} - \sqrt{2}\right)$	A1	1.1b
	$x = \ln\left(\sqrt{3} + \sqrt{2}\right)$ **and** $x = \ln\left(\sqrt{3} - \sqrt{2}\right)$ With no "solutions" being found by attempts to solve $\cosh x = 0$ or $\cosh x = -\sqrt{3}$	A1	2.3
		(4)	
			(7 marks)

Notes

(a)
M1: Recalls the definition of cosh$x$ in terms of exponentials and substitutes
M1: Expands the cubed bracket correctly
A1*: Correct proof with no errors
(b)
M1: Uses the result from part (a) and collects terms to make progress in solving the equation
M1: Recalls the definition of cosh in terms of e or uses the definition of $\cosh^{-1}x$
A1: One correct solution
A1: Both correct solutions and no others from $\cosh x = 0$ or $\cosh x = -\sqrt{3}$

Question	Scheme	Marks	AOs
8(a)	Because $\dfrac{2}{\sqrt[3]{2-x}}$ is undefined at x = 2 and the limits of the integration are either side of this discontinuity	B1	2.4
		(1)	
(b)	$\displaystyle\int \frac{1}{\sqrt[3]{2-x}}\,\mathrm{d}x = -3(2-x)^{\frac{2}{3}}\ (+c)$	M1 A1	2.1 1.1b
	$\displaystyle\int_0^5 \frac{2}{\sqrt[3]{2-x}}\,\mathrm{d}x = \int_0^2 \frac{2}{\sqrt[3]{2-x}}\,\mathrm{d}x + \int_2^5 \frac{2}{\sqrt[3]{2-x}}\,\mathrm{d}x$	M1	3.1a
	$\displaystyle= \lim_{a\to 2^-}\int_0^a \frac{2}{\sqrt[3]{2-x}}\,\mathrm{d}x + \lim_{b\to 2^+}\int_b^5 \frac{2}{\sqrt[3]{2-x}}\,\mathrm{d}x$ $\displaystyle= \lim_{a\to 2^-}\left[-3(2-x)^{\frac{2}{3}}\right]_0^a + \lim_{b\to 2^+}\left[-3(2-x)^{\frac{2}{3}}\right]_b^5$ $\displaystyle= -3\left(\lim_{a\to 2^-}\left((2-a)^{\frac{2}{3}} - (2-0)^{\frac{2}{3}}\right)\right) + -3\left(\lim_{b\to 2^+}\left((2-5)^{\frac{2}{3}} - (2-b)^{\frac{2}{3}}\right)\right)$	M1	2.1
	$= -3\left(-2^{\frac{2}{3}} + (-3)^{\frac{2}{3}}\right) = -3\left(\sqrt[3]{9} - \sqrt[3]{4}\right)$	A1	2.2a
		(5)	
			(6 marks)

Notes

(a)
B1: A correct explanation why the integral is improper
(b)
M1: Integrates to obtain an expression of the form $k(2-x)^{\frac{2}{3}}$
A1: Correct integration
M1: Adopts the correct strategy of splitting the integral into two with limits 0→2 and 2→5
M1: Produces a rigorous argument that includes an upper limit for the first integral that approaches 2 from below and a lower limit for the second integral that starts from 2 from above
A1: Correct expression (allow exact equivalents)

Question	Scheme	Marks	AOs
9(a)	$m^2+2m+1=0 \Rightarrow m=...(-1)$	M1	1.1b
	$CF: y=(At+B)\mathrm{e}^{-t}$	M1	2.2a
	PI : Try $y=kt^2\mathrm{e}^{-t}+c$	B1	2.2a
	$\frac{\mathrm{d}y}{\mathrm{d}t}=2kt\mathrm{e}^{-t}-kt^2\mathrm{e}^{-t},\ \frac{\mathrm{d}^2y}{\mathrm{d}t^2}=2k\mathrm{e}^{-t}-2kt\mathrm{e}^{-t}+kt^2\mathrm{e}^{-t}-2kt\mathrm{e}^{-t}$ $2k\mathrm{e}^{-t}-4kt\mathrm{e}^{-t}+kt^2\mathrm{e}^{-t}+2\left(2kt\mathrm{e}^{-t}-kt^2\mathrm{e}^{-t}\right)+kt^2\mathrm{e}^{-t}+1=\mathrm{e}^{-t}+1 \Rightarrow k=...$	M1	1.1b
	$k=\frac{1}{2} \Rightarrow PI: y=\frac{1}{2}t^2\mathrm{e}^{-t}+1$	A1	1.1b
	$y=CF+PI=(At+B)\mathrm{e}^{-t}+\frac{1}{2}t^2\mathrm{e}^{-t}+1$	A1	1.1a
		(6)	
(b)	$t=0,\ y=1 \Rightarrow B=0$	M1	3.4
	$y=\left(At+0.5t^2\right)\mathrm{e}^{-t}+1 \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}t}=\left(A+t-At-0.5t^2\right)\mathrm{e}^{-t}$ $t=0,\ \frac{\mathrm{d}y}{\mathrm{d}t}=9 \Rightarrow A=9$	M1	3.4
	$y=\left(0.5t^2+9t\right)\mathrm{e}^{-t}+1$	A1	1.1b
		(3)	
(c)	$\frac{\mathrm{d}y}{\mathrm{d}t}=0 \Rightarrow 0.5t^2+8t-9=0 \Rightarrow t=...$	M1	3.1a
	$t>0 \Rightarrow t=1.0553... \Rightarrow y=...$	M1	3.4
	$y=4.50\mathrm{mg/l}$	A1	1.1b
		(3)	
(d)	$t=8 \Rightarrow y=\left(0.5\times8^2+9\times8\right)\mathrm{e}^{-8}+1=1.03488...$ • This is close to 1 so the model supports the suggestion that the concentration returns to its initial value after around 8 hours	M1 A1ft	3.4 3.2b
		(2)	
			(14 marks)

Notes

(a)
M1: Forms and solves the auxiliary equation
A1: Deduces the correct complementary function
B1: Deduces the correct form of the PI given the outcome for the CF
M1: Complete method to establish the value of *k*
A1: Correct PI
A1: Correct GS
(b)
M1: Uses the model and the initial conditions to find the value of *B*

M1: Uses the model by differentiating and using the other initial condition to find a value for A
A1: Correct PS
(c)
M1: Solves $\frac{\mathrm{d}y}{\mathrm{d}t} = 0$ to find t when the concentration is a maximum
M1: Uses their value of t and the model to find the maximum concentration
A1: Correct value
(d)
M1: Uses their model to finds the concentration when t = 8 in order to test the claim
A1ft: Follow through their solution but the comment must be consistent with their values.