GCE A level Further Mathematics (9FM0) – Shadow Paper (Set 1)

9FM0-02 Core Pure Mathematics 2

October 2020 Shadow Paper mark scheme

Please note that this mark scheme is not the one used by examiners for making scripts. It is intended more as a guide, indicating where marks are given for correct answers. As such, it may not show follow-through marks (marks that are awarded despite errors being made) or special cases.

It should also be noted that for many questions, there may be alternative methods of finding correct solutions that are not shown here – they will be covered in the formal mark scheme from the original paper.

This document is intended for guidance only and may differ significantly from the examiners' final mark scheme for the original paper which was published in December 2020.

Guidance on the use of codes within this document
M1 – method mark. This mark is generally given for an appropriate method in the context of the question. This mark is given for showing your working and may be awarded even if working is incorrect. A1 – accuracy mark. This mark is generally given for a correct answer following correct working. B1 – working mark. This mark is usually given when working and the answer cannot easily be separated. Some questions require all working to be shown; in such questions, no marks will be given for an answer with no working (even if it is a correct answer).

Question	Scheme	Marks	AOs
1	$\frac{dy}{dx} = -17\cosh x + 3\cosh 2x$	B1	1.1b
	$\frac{dy}{dx} = -17\cosh x + 3\left(2\cosh^2 x - 1\right)$	M1	3.1a
	$6\cosh^2 x - 17\cosh x - 3 = 0$	A1	1.1b
	$(6\cosh x + 1)(\cosh x - 3) = 0 \Rightarrow \cosh x = \ldots$	M1	1.1b
	$\cosh x = 3, \left(-\frac{1}{6}\right)$	A1	1.1b
	$\cosh x = \alpha \Rightarrow x = \pm\ln\left(\alpha + \sqrt{\alpha^2 - 1}\right)$ or $\ln\left(\alpha \pm \sqrt{\alpha^2 - 1}\right)$	M1	1.2
	$\pm\ln\left(3 + \sqrt{8}\right)$ or $\ln\left(3 \pm \sqrt{8}\right)$	A1	2.2a
		(7)	
		(7 marks)	

Question	Scheme	Marks
2(a)	Centre of circle C is (-1, –2)	B1
	$r=\frac{1}{2}\sqrt{(2--4)^2+(2--6)^2}=5$ or $r=\sqrt{(2--1)^2+(2--2)^2}=5$ or $r=\sqrt{(-1--4)^2+(-2--6)^2}=5$	M1
	$\lvert z+1+2i\rvert=5$ or $\lvert z-(-1-2i)\rvert=5$	A1
		(3)
(b)	$(x+1)^2+(y+2)^2=25,\ (x-3)^2+(y+4)^2=5$ $x^2+2x+1+y^2+4y+4=25$ $x^2-6x+9+y^2+8y+16=5$ $\Rightarrow 8x-4y=40$ $\Rightarrow x=\frac{y+10}{2}$ or $y=2x-10$	M1
	$\left(\frac{y+10}{2}\right)^2+2\left(\frac{y+10}{2}\right)+1+y^2+4y+4=25$ or $\left(\frac{y+10}{2}\right)^2-6\left(\frac{y+10}{2}\right)+9+y^2+8y+16=5$ or $x^2+2x+1+(2x-10)^2+4(2x-10)+4=25$ or $x^2-6x+9+(2x-10)^2+8(2x-10)+16=5$	M1
	$y^2+8y+12=0$ or $x^2-6x+8=0$	A1
	$y^2+8y+12=0\Rightarrow(y+2)(y+6)=0\Rightarrow y=-2,-6$ or $x^2-6x+8=0\Rightarrow(x-2)(x-4)=0\Rightarrow x=2,4$	M1
	$y=-2,-6\Rightarrow x=4,2$ or $x=2,4\Rightarrow y=-6,-2$	M1
	$2-6i, 4-2i$	A1
		(6)
		(9 marks)

Question	Scheme	Marks
3(a)	$2m^2+7m+3=0$ $(2m+1)(m+3)=0$ $m=-\frac{1}{2},-3$	M1
	C.F. is $y=Ae^{-\frac{1}{2}t}+Be^{-3t}$	A1
	P.I $y=at^2+bt+c$	B1
	$y'=2at+b,\ y''=2a$	
	$2(2a)+7(2at+b)+3(at^2+bt+c)\equiv 3t^2+11t$	M1
	$3a=3,\ a=1$ $14+3b=11,\ b=-1$	A1
	$4-7+3c=0,\ c=1$	M1 A1
	General solution: $y=Ae^{-\frac{1}{2}t}+Be^{-3t}+(t^2-t+1)$	A1 (ft)
		(8)
(b)	$y'=-\frac{1}{2}Ae^{-\frac{1}{2}t}-3Be^{-3t}+(2t-1)$	M1
	$t=0, y'=1:\ \ 1=-1-\frac{1}{2}A-3B$ $t=0, y=1:\ \ 1=1+A+B$ one of these	M1 A1
	Solve: $A+B=0,\ A+6B=-4$	
	$A=\frac{4}{5}, B=-\frac{4}{5}$	M1
	$y=(t^2-t+1)+\frac{4}{5}\left(e^{-\frac{1}{2}t}-e^{-3t}\right)$	A1
		(5)
(c)	$t=1:\ \ y=\frac{4}{5}\left(e^{-\frac{1}{2}}-e^{-3}\right)+1$ (= 1.445…)	A1
		(1)
		(14 marks)

Question	Scheme	Marks
4(a)	$(\cos\theta + \mathrm{i}\sin\theta)^6 = \cos^6\theta + \binom{6}{1}\cos^5\theta\ \mathrm{i}\sin\theta + \binom{6}{2}\cos^4\theta\ (\mathrm{i}\sin\theta)^2 + \ldots$	M1
	$\mathrm{i}\sin 6\theta = {}^6C_1c^5\ \mathrm{i}s\ + {}^6C_3c^3\mathrm{i}^3s^3 + {}^6C_5c^1\mathrm{i}^5s^5$	M1
	$\cos 6\theta = c^6 - 15c^4s^2 + 15c^2s^4 - s^6$	A1
	$\sin^4\theta = \left(\sin^2\theta\right)^2$ $c^6 - 15c^4(1-c^2) + 15c^2(1-c^2)^2 - (1-c^2)^3$ $= c^6 - 15c^4 + 15c^6 + 15c^2 - 30c^4 + 15c^6 - 1 + 3c^2 - 3c^4 + c^6$	M1
	$\cos 6\theta = 32\cos^6\theta - 48\cos^4\theta + 18\cos^2\theta - 1$	A1
		(5)
(b)	$\cos 6\theta - 1 = 0 \Rightarrow \cos 6\theta = 1$	M1
	$6\theta = -720°, -360°, 0°, 360°, \ldots$ $6\theta = -4\pi, -2\pi, 0, 2\pi, \ldots$	A1
	$\theta = -\frac{720°}{6}, -\frac{360°}{6}, 0, \frac{360°}{6}, \ldots \Rightarrow \cos\theta = \ldots$ or $\theta = -\frac{4\pi}{6}, -\frac{2\pi}{6}, 0, \frac{2\pi}{6}, \ldots \Rightarrow \cos\theta = \ldots$	M1
	$x = \pm\frac{1}{2}, \pm 1$	A1 A1
		(5)
		(10 marks)

Question	Scheme	Marks
5(a)	$y=\tan^{-1}x \Rightarrow \tan y = x \Rightarrow \dfrac{dx}{dy}=\sec^2 y$	M1
	$\dfrac{dx}{dy}=1+\tan^2 y$	M1
	$\dfrac{dy}{dx}=\dfrac{1}{1+\tan^2 y}=\dfrac{1}{1+x^2}$	A1
		(3)
(b)	$u=\tan^{-1}x, \dfrac{du}{dx}=\dfrac{1}{1+x^2}$	B1
	$\dfrac{1}{4}x^4\tan^{-1}x-\dfrac{1}{4}\int\dfrac{x^4}{1+x^2}dx$	M1
	$\dfrac{x^4}{1+x^2}=x^2-1+\dfrac{1}{x^2+1}$	M1
	$\int\dfrac{x^4}{1+x^2}dx=\dfrac{1}{3}x^3-x+\tan^{-1}x+k$	M1
	$\dfrac{1}{4}\left(x^4\tan^{-1}x-\dfrac{1}{3}x^3+x-\tan^{-1}x\right)+k$	A1
		(5)
(c)	Mean value = $\left(\dfrac{\sqrt{3}}{4}\right)\left[x^4\tan^{-1}x-\dfrac{1}{3}x^3+x-\tan^{-1}x\right]_0^{\frac{1}{\sqrt{3}}}$ $=\dfrac{\sqrt{3}}{4}\left[\left(\dfrac{1}{9}\cdot\dfrac{\pi}{6}-\dfrac{1}{3}\dfrac{\sqrt{3}}{9}+\dfrac{1}{\sqrt{3}}-\dfrac{\pi}{6}\right)-0\right]$	M1
	$=\dfrac{-\sqrt{3}\pi+6}{27}$ oe	A1
		(2)
		(10 marks)

Question	Scheme	Marks	AOs
6(a)	$\|\mathbf{M}\| = 6(3) - k(1+k) - 1(2-k)$	M1	1.1b
	Minors: $\begin{pmatrix} 3 & 1+k & 2-k \\ k+2 & 6+k & 12-k^2 \\ -k+1 & -5 & 6-k \end{pmatrix}$ Cofactors: $\begin{pmatrix} 3 & -1-k & 2-k \\ -2-k & 6+k & k^2-12 \\ -k+1 & 5 & 6-k \end{pmatrix}$	B1	1.1b
	$\mathbf{M}^{-1} = \dfrac{1}{16-k^2}\begin{pmatrix} 3 & -2-k & -k+1 \\ -1-k & 6+k & 5 \\ 2-k & k^2-12 & 6-k \end{pmatrix}$	M1 A1	
		(4)	
(b)	$\mathbf{M}^{-1} = \dfrac{1}{12}\begin{pmatrix} 3 & -4 & -1 \\ -3 & 8 & 5 \\ 2 & -8 & 4 \end{pmatrix}\begin{pmatrix} 5 \\ p \\ 9 \end{pmatrix} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$	M1	
	$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \dfrac{1}{12}\begin{pmatrix} 6-4p \\ 30+8p \\ 36-8p \end{pmatrix}$	A1	
	$\left(\dfrac{6-4p}{12}, \dfrac{30+8p}{12}, \dfrac{36-8p}{12}\right)$ oe	A1	
		(3)	
(c)(i)			
	For consistency: e.g. $10x+6y=14$ and $5x+3y=5-q$	M1	
	$18+2q=14$	M1	
	$q=-2$	A1	
(ii)	$5x+3y=7$	M1	
	$x=\dfrac{7-3y}{5}$	A1	
	$r = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} + \mu\begin{pmatrix} -3 \\ 5 \\ 2 \end{pmatrix}$ Alternatives acceptable	M1 A1	
		(7)	
			(14 marks)

Question	Scheme	Marks	AOs
7(a)	$2=\frac{a}{1+b}, 1=\frac{a}{3+b} \Rightarrow a=..., b=...$	M1	3.3
	$a = 4$, $b = 1$	A1	1.1b
		(2)	
(b)	$V_1=\pi\int x^2 dy=\pi\int\left(\frac{"4"}{y+"1"}\right)^2 dy$	B1ft	3.4
	$=\pi\int_1^3\left(\frac{"4"}{y+"1"}\right)^2 dy$	M1	1.1a
	$16\pi\left[-(y+1)^{-1}\right]_1^3 (=4\pi)$	M1	1.1b
	$x^2+(y-4)^2=2$	B1	2.2a
	$V_2=\pi\int x^2 dy=\pi\int\left(2-(y-4)^2\right)dy$	M1	1.1b
	$=\pi\int_3^{4+\sqrt{2}}\left(2-(y-4)^2\right)dy$	M1	3.3
	$=\pi\left[2y-\frac{1}{3}(y-4)^3\right]_3^{4+\sqrt{2}} \quad \left(=\frac{\pi}{3}\left(5+4\sqrt{2}\right)\right)$	A1	1.1b
	$V_1+V_2+\text{cylinder}=4\pi+\frac{\pi}{3}\left(5+4\sqrt{2}\right)+4\pi$	dM1	3.4
	$\frac{\pi}{3}\left(29+4\sqrt{2}\right)\approx 36.3 \text{ (cm}^3\text{)}$	A1	2.2b
		(9)	
		(11 marks)	