Write your name here

Surname	Other names

Pearson Edexcel GCE

Centre Number

Candidate Number

A level Further Mathematics

Further Mechanics 1

Practice Paper 4

You must have:
Mathematical Formulae and Statistical Tables (Pink)

Total Marks

Instructions

- Use black ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- Fill in the boxes at the top of this page with your name, centre number and candidate number.
- Answer all the questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – there may be more space than you need.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are **7** questions in this question paper. The total mark for this paper is **75**.
- The marks for each question are shown in brackets – use this as a guide as to how much time to spend on each question.
- Calculators must not be used for questions marked with a * sign.

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

1. A ball of mass 0.5 kg is moving with velocity $12\mathbf{i}$ m s^{-1} when it is struck by a bat. The impulse received by the ball is $(-4\mathbf{i} + 7\mathbf{j})$ N s. By modelling the ball as a particle, find

(a) the speed of the ball immediately after the impact,

(4)

(b) the angle, in degrees, between the velocity of the ball immediately after the impact and the vector $\mathbf{i}$,

(2)

(c) the kinetic energy gained by the ball as a result of the impact.

(2)

(Total 8 marks)

[Mark scheme for Question 1]

[Examiner comment]

2.

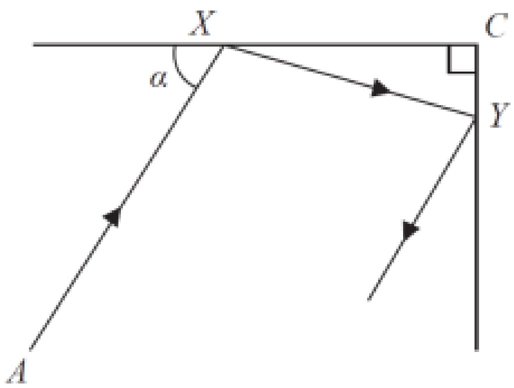


Figure 1

A small spherical ball P is at rest at the point A on a smooth horizontal floor. The ball is struck and travels along the floor until it hits a fixed smooth vertical wall at the point X. The angle between AX and this wall is α, where α is acute. A second fixed smooth vertical wall is perpendicular to the first wall and meets it in a vertical line through the point C on the floor. The ball rebounds from the first wall and hits the second wall at the point Y. After P rebounds from the second wall, P is travelling in a direction parallel to XA, as shown in Figure 1. The coefficient of restitution between the ball and the first wall is e. The coefficient of restitution between the ball and the second wall is ke.

Find the value of k.

(Total 9 marks)

[Mark scheme for Question 2]

[Examiner comment]

3.

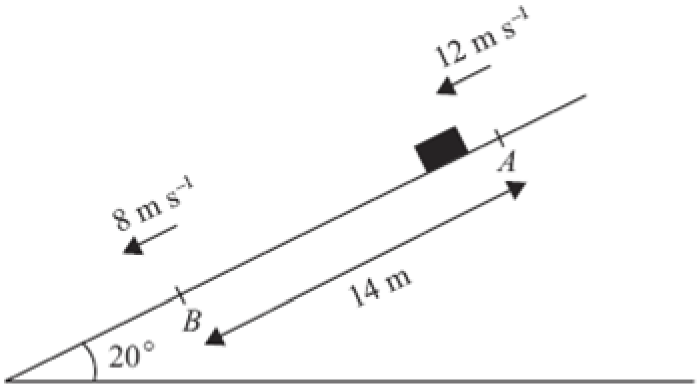


Figure 2

A package of mass 3.5 kg is sliding down a ramp. The package is modelled as a particle and the ramp as a rough plane inclined at an angle of 20° to the horizontal. The package slides down a line of greatest slope of the plane from a point A to a point B, where AB = 14 m. At A the package has speed 12 m s^{-1} and at B the package has speed 8 m s^{-1}, as shown in Figure 2.

Find

(a) the total energy lost by the package in travelling from A to B,

(5)

(b) the coefficient of friction between the package and the ramp.

(5)

(Total 10 marks)

Mark scheme for Question 3

Examiner comment

4.

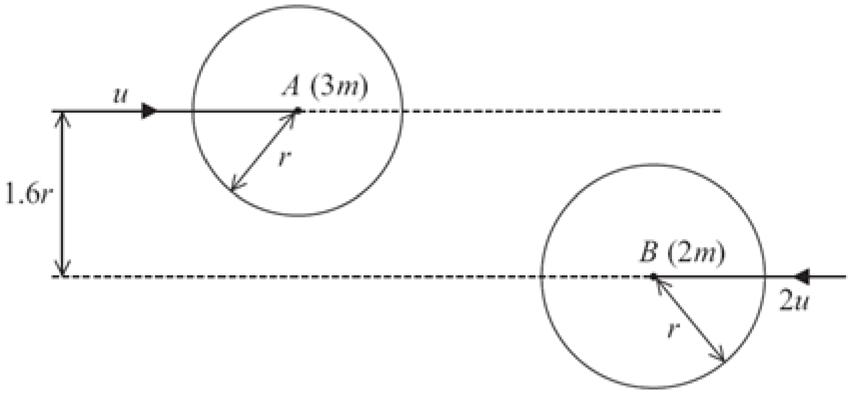


Figure 3

Two smooth uniform spheres A and B, of equal radius r, have masses $3m$ and $2m$ respectively. The spheres are moving on a smooth horizontal plane when they collide. Immediately before the collision they are moving with speeds u and $2u$ respectively. The centres of the spheres are moving towards each other along parallel paths at a distance $1.6r$ apart, as shown in Figure 3.

The coefficient of restitution between the two spheres is $\frac{1}{6}$.

Find, in terms of m and u, the magnitude of the impulse received by B in the collision.

(Total 10 marks)

Mark scheme for Question 4

Examiner comment

5. A truck of mass 900 kg is towing a trailer of mass 150 kg up an inclined straight road with constant speed 15 m s^{-1}. The trailer is attached to the truck by a light inextensible towbar which is parallel to the road. The road is inclined at an angle θ to the horizontal, where $\sin\theta = \frac{1}{9}$.

The resistance to motion of the truck from non-gravitational forces has constant magnitude 200 N and the resistance to motion of the trailer from non-gravitational forces has constant magnitude 50 N.

(a) Find the rate at which the engine of the truck is working.

(5)

When the truck and trailer are moving up the road at 15 m s^{-1} the towbar breaks, and the trailer is no longer attached to the truck. The rate at which the engine of the truck is working is unchanged. The resistance to motion of the truck from non-gravitational forces and the resistance to motion of the trailer from non-gravitational forces are still forces of constant magnitudes 200 N and 50 N respectively.

(b) Find the acceleration of the truck at the instant after the towbar breaks.

(3)

(c) Use the work-energy principle to find out how much further up the road the trailer travels before coming to instantaneous rest.

(4)

(Total 12 marks)

[Mark scheme for Question 5](#)

[Examiner comment](#)

6. Two particles A and B, of masses $3m$ and $4m$ respectively, lie at rest on a smooth horizontal surface. Particle B lies between A and a smooth vertical wall which is perpendicular to the line joining A and B. Particle B is projected with speed $5u$ in a direction perpendicular to the wall and collides with the wall. The coefficient of restitution between B and the wall is $\frac{3}{5}$.

(a) Find the magnitude of the impulse received by B in the collision with the wall.

(3)

After the collision with the wall, B rebounds from the wall and collides directly with A. The coefficient of restitution between A and B is e.

(b) Show that, immediately after they collide, A and B are both moving in the same direction.

(7)

The kinetic energy of B immediately after it collides with A is one quarter of the kinetic energy of B immediately before it collides with A.

(c) Find the value of e.

(4)

(Total 14 marks)

[Mark scheme for Question 6](#)

[Examiner comment](#)

7. A particle P of mass m is attached to one end of a light elastic string of natural length l and modulus of elasticity $3mg$. The other end of the string is attached to a fixed point O on a rough horizontal table. The particle lies at rest at the point A on the table, where $OA = \frac{7}{6}l$. The coefficient of friction between P and the table is μ.

(a) Show that $\mu \geq \frac{1}{2}$.

(4)

The particle is now moved along the table to the point B, where $OB = \frac{3}{2}l$, and released from rest. Given that $\mu = \frac{1}{2}$, find

(b) the speed of P at the instant when the string becomes slack,

(5)

(c) the total distance moved by P before it comes to rest again.

(3)

(Total 12 marks)

[Mark scheme for Question 7](#)

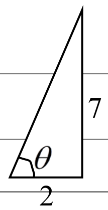
[Examiner comment](#)

TOTAL FOR PAPER: 75 MARKS

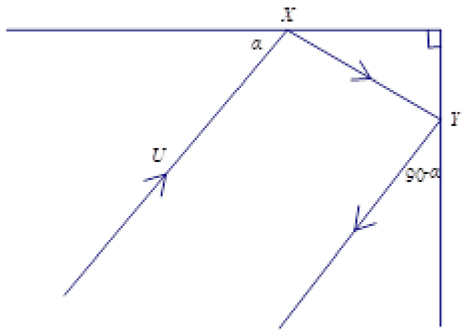
Further Mathematics – Further Mechanics 1– Practice Paper 04 –

Mark scheme –

Mark scheme for Question 1 (Examiner comment) (Return to Question 1)

Question	Scheme	Marks
1(a)	$\mathbf{I} = m\mathbf{v} - m\mathbf{u}$	**M1**
	$-4\mathbf{i} + 7\mathbf{j} = 0.5(\mathbf{v} - 12\mathbf{i})$	
	$4\mathbf{i} + 14\mathbf{j} = \mathbf{v}$	**A1**
	Speed $= \sqrt{16+196} = \sqrt{212}$ m s^{-1} (14.6 or better)	**M1A1**
		(4)
(b)	 $\tan\theta = \frac{7}{2}$	**M1**
	$\theta = 74.0....$	
	$\theta = 74°$	**A1ft**
		(2)
(c)	Gain in K.E. $= \frac{1}{2} \times 0.5(212 - 12^2)$, $= 17$ J	**M1A1**
		(2)
		(8 marks)

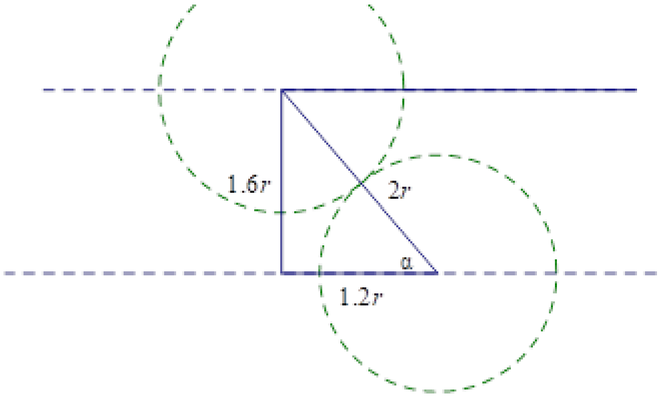
Mark scheme for Question 2 (Examiner comment) (Return to Question 2)

Question	Scheme	Marks
2		
	First impact:	
	Component parallel to wall: $=U\cos\alpha$	**B1**
	Perp to wall: NLR: $eU\sin\alpha$	**M1A1**
	Second impact: parallel to wall vel after $=eU\sin\alpha$	**B1**
	Perp to wall $ke\times U\cos\alpha$	**B1**
	Direction at $(90-\alpha)$ to the wall	**B1**
	$\Rightarrow \tan(90-\alpha)=\dfrac{keU\cos\alpha}{Ue\sin\alpha}$ or $\tan\alpha=\dfrac{eU\sin\alpha}{keU\cos\alpha}$	**M1**
	$\cot\alpha=k\cot\alpha$ or $\tan\alpha=\dfrac{1}{k}\tan\alpha$	**A1**
	$k=1$	**A1**
		(9)
		(9 marks)

Mark scheme for Question 3 (Examiner comment) (Return to Question 3)

Question	Scheme	Marks
3(a)	$\Delta \text{KE} = \frac{1}{2}\times 3.5\left(12^2 - 8^2\right)\ (=140)$ or KE at A, B correct separately	**B1**
	$\Delta \text{PE} = 3.5\times 9.8\times 14\sin 20°\ (\approx 164.238)$ or PE at A, B correct separately	**M1A1**
	$\Delta \text{E} = \Delta \text{KE} + \Delta \text{PE} \approx 304,\ \ 300$	**dM1** **A1**
		(5)
(b)	Using Work-Energy	
	$F_r = \mu \times 3.5g\cos 20°$	**M1** **A1**
	$304.238\ldots = F_r \times 14$ ft their (a), F_r	**M1** **A1 ft**
	$304.238\ldots = \mu 3.5g\cos 20° \times 14$	
	$\mu \approx 0.674\ ,\ 0.67$	**A1**
		(5)
		(10 marks)

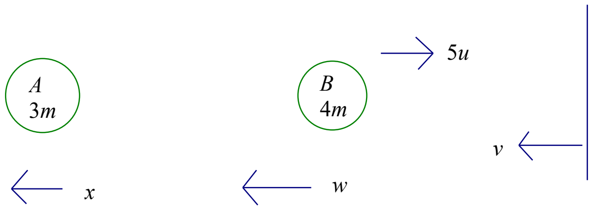
Mark scheme for Question 4 (Examiner comment) (Return to Question 4)

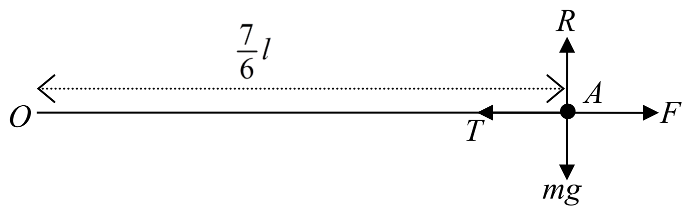
Question	Scheme	Marks
4		
	$0.6u$ or $u\cos\alpha$	B1
	$1.2u$ or $2u\cos\alpha$	B1
	$2m\times1.2u-3m\times0.6u=3ma+2mb$	M1
	$(3a+2b=0.6u)$	A1ft
	$e(1.2u+0.6u)=a-b$	M1
	$(a-b=0.3u)$	A1ft dM1
	$a=0.24u$ or $b=$ -$0.06u$	A1
	$(1.2u-(-0.06u))\times2m=2.52mu$	M1
	or $(0.24u-(-0.6u))\times3m=2.52mu$	A1
		(10)
		(10 marks)

Mark scheme for Question 5 (Examiner comment) (Return to Question 5)

Question	Scheme	Marks
5(a)	Constant speed $\Rightarrow$ no acceleration. Driving force $= 200 + 50 + 900g\sin\theta + 150g\sin\theta$	M1
	Or $D - T - 200 - 900g\sin\theta = 0$	A1
	And $T - 50 - 150g\sin\theta = 0$	A1
	$= 250 + 1050g \times \frac{1}{9} (= 1393.3333...)$	
	$P = \left(250 + 1050g \times \frac{1}{9}\right) \times 15$	M1
	$= 20900 \text{ W} (20.9 \text{ kW})$	A1
		(5)
(b)	$\left(their 1393\frac{1}{3}\right) - 200 - 900g \times \frac{1}{9} = 900a$	M1 A1ft
	$a = 0.237 \text{ m s}^{-2}$	A1
		(3)
(c)	$\frac{1}{2} \times 150 \times 15^2 = 50d + 150g\sin\theta d$	M1A1
	$\left(16875 = 50d + \frac{150}{9}gd\right)$	A1
	$d = 79 \text{ m} \ (79.1)$	A1
		(4)
		(12 marks)

Mark scheme for Question 6 (Examiner comment) (Return to Question 6)

Question	Scheme	Marks
6(a)		
	Impact with wall: $v=\frac{3}{5}\times 5u=3u$	**B1**
	Impulse $\pm 4m\left(3u-(-5u)\right)$	**M1**
	Magnitude 32*mu* (Ns)	**A1**
		(3)
(b)	CLM: $3mx+4mw=4m\times 3u$	**M1** **A1ft**
	Impact: $x-w=e\times 3u$	**M1** **A1ft**
	$3m(w+3eu)+4mw=7mw+9emu=12mu$	
	$7w=u(12-9e)$	**dM1**
	Use of $e\le 1$ in their w: $7w\ge 3u$	**M1**
	Hence $w>0$ and A and B are moving in the same direction	**A1**
		(7)
(c)	KE of B before collision $=\frac{1}{2}\times 4m\times(3u)^2\ (=18mu^2)$	**B1**
	$\Rightarrow \frac{1}{2}\times 4m\left(\frac{u}{7}(12-9e)\right)^2=\frac{1}{4}\left(\frac{1}{2}\times 4m\times 9u^2\right)$	**M1**
	$4(12-9e)^2=49\times 9,\ (4-3e)^2=\frac{49}{4}$	**A1**
	$e=\frac{1}{6}$	**A1**
		(4)
		(14 marks)

Question	Scheme	Marks
7(a)		
	$T = \frac{3mg}{l}\left(\frac{1}{6}l\right) = \frac{1}{2}mg$	B1
	$\text{R}(\uparrow)\ \ R = mg \qquad \text{R}(\rightarrow)\ \ F = T = \frac{1}{2}mg$	M1
	$F \leqslant \mu R$	
	$\frac{1}{2}mg \leqslant \mu mg$	M1
	$\mu \geqslant \frac{1}{2}$ *	A1
		(4)
(b)	E.P.E. lost $= \frac{1}{2} \times \frac{3mg}{l}\left(\frac{1}{2}l\right)^2 = \frac{3mgl}{8}$	B1
	Work done by friction $= \frac{1}{2}mg\left(\frac{l}{2}\right)$	B1
	$\frac{3mgl}{8} = \frac{1}{2}mv^2 + \frac{1}{2}mg\left(\frac{l}{2}\right)$	M1 A1ft
	$v^2 = \frac{gl}{4}$	
	$v = \frac{1}{2}\sqrt{gl}$	A1
		(5)
(c)	$\frac{3mgl}{8} = \frac{1}{2}mgx$	M1 A1ft
	$x = \frac{3l}{4}$	A1
		(3)
		(12 marks)

Further Mathematics – Further Mechanics 1– Practice Paper 04 –

Examiner report –

Examiner comment for Question 1 (Mark scheme) (Return to Question 1)

1. Many candidates would have gained more marks in this question if they had checked that they had actually found the quantities asked for in the question. Despite some arithmetic errors in dividing by 0.5, many candidates successfully found the velocity after impact but only about half of those went on the find the speed asked for in the question.

In part (b), most candidates managed to find an appropriate angle, although there was some confusion over whether to use the velocity after impact or the impulse. Sometimes the fraction for $\tan\theta$ was the wrong way up, and occasionally sine or cosine were used, but often incorrectly. Those who had sign errors in their **v** often failed to realise that an obtuse angle would then be required.

Finding the change in kinetic energy in part (c)caused difficulties for those candidates who did not realise that energy is a scalar quantity and that the v^2 required was merely the square of the speed found in (a). Some candidates had accuracy errors due to the use of a rounded value for v^2. Some only found the final kinetic energy rather than the change in kinetic energy.

Examiner comment for Question 2 (Mark scheme) (Return to Question 2)

2. This standard result either produced good concise solutions or long, complicated and often inconclusive attempts at solution. The fact that it is a well-known result did not help those candidates who put more effort into trying to fix the result than into working through the problem in a systematic way.

The better solutions used clearly labeled diagrams with components of positive magnitude shown acting in realistic directions. Those candidates who expressed everything in terms of the initial speed and α were often more successful than those who introduced a new variable for the speed at each stage and for each angle. Some candidates incorrectly assumed that the stages on the path of P were perpendicular to each other, and some thought that for the final direction to be parallel to the initial direction the path needed to be at angle α to the wall after leaving Y.

The most elegant solution seen expressed the velocity as a column vector with components parallel to the walls and used $\mathbf{v} = \lambda\mathbf{u}$.

Examiner comment for Question 3 (Mark scheme) (Return to Question 3)

3. Many candidates would have gained more marks in this question if they had checked that they had actually found the quantities asked for in the question. Despite some arithmetic errors in dividing by 0.5, many candidates successfully found the velocity after impact but only about half of those went on the find the speed asked for in the question.

In part (b), most candidates managed to find an appropriate angle, although there was some confusion over whether to use the velocity after impact or the impulse. Sometimes the fraction for $\tan\theta$ was the wrong way up, and occasionally sine or cosine were used, but often incorrectly. Those who had sign errors in their **v** often failed to realise that an obtuse angle would then be required.

Finding the change in kinetic energy in part (c)caused difficulties for those candidates who did not realise that energy is a scalar quantity and that the v^2 required was merely the square of the speed found in (a). Some candidates had accuracy errors due to the use of a rounded value for v^2. Some only found the final kinetic energy rather than the change in kinetic energy.

Examiner comment for Question 4 (Mark scheme) (Return to Question 4)

4. A clear diagram showing the components of the velocities of A and B parallel to the line of centres immediately before the collision was a vital start for most candidates. Choosing simple names for the components parallel to the line of centres after collision rather than introduce two unknown angles led to simpler equations to work with, and usually less confused working. The majority of candidates did try to write down an equation for conservation of linear momentum and to use the impact law parallel to the line of centres, but signs were not always used consistently and there was some confusion with angles. Errors in solving the resulting simultaneous equations were common, often because the candidate's equations were not expressed in the simplest possible form. Most candidates knew the appropriate formula for impulse, but they did not always take account of the change in direction of motion and some candidates used a mixture of speeds and components of velocity.

Examiner comment for Question 5 (Mark scheme) (Return to Question 5)

5. (a) Most students found the driving force correctly, and hence the required rate of working. The few errors seen were through missing out a required term or trig. or sign errors. Students who considered the truck and trailer separately tended to make more errors in setting up their equations.

(b) Some students did not understand that the only forces they needed to consider here were the forces acting on the truck. Most errors were due to including the trailer in the equation. Some students formed a dimensionally incorrect equation by using power rather than driving force in their equation of motion. A few students, especially where there had not been a clear diagram, could not identify the remaining forces acting on the truck after separation.

(c) Students who used suvat equations rather than the work-energy principle (required by the question) scored no marks here. Many students did attempt to use work and energy, and almost half of all students reached the correct solution. Here again there was some confusion about what forces were involved, with several students including the truck in their working. Other common errors were to leave out the work done against the resistance, or to double count the change in GPE (which many students regard as being distinct from the work done against the weight).

Examiner comment for Question 6 (Mark scheme) (Return to Question 6)

6. (a) The relationship between impulse and momentum was well understood, but a significant number of students failed to appreciate the effect of the change of direction on the impulse whilst other solutions lost at least one of the factors m and u.

(b) Many students did draw some form of diagram, but that did not prevent a large number of solutions from assuming that A and B would both be moving towards the wall after their collision -an assumption that suggests that students struggled with their intuitive understanding of the problem. Nearly all students were able to use the conservation of linear momentum and Newton's Experimental Law correctly as well as work through the algebra needed to find an expression for the velocity of B after the impact. The necessity to show that the particles were moving in the same direction after the impact, required explicit reference to $e \leqslant 1$, something that was often implied but not stated. Several students analysed the direction of motion of A after the motion, but this was not expected as the nature of the impact leads to only one possible outcome.

(c) Many students offered correct solutions to this problem. The working to find the value of e was often made more complicated than necessary by students multiplying out the brackets in their equation for kinetic energy rather than simply spotting that they were equating two perfect squares and using what they knew about the direction of motion. Some errors were caused by students placing the $\frac{1}{4}$ on the wrong side of their equation, and several students included the kinetic energy of A in their equation.

Examiner comment for Question 7 (Mark scheme) (Return to Question 7)

7. Part (a) highlighted the difficulty many candidates have in setting out a plausible proof, especially when an inequality is involved. All too often the tension in the string was correctly equated to the frictional force, which was then incorrectly stated to be equal to μR. Given that $F \leq \mu R$ is also regularly examined in M2, this showed a poor understanding of friction forces. Once $\mu = \frac{1}{2}$ had been obtained it was common to see the required inequality appear. Sometimes an attempt to explain the inequality was given, but only in rare cases would this be satisfactory.

Most attempted parts (b) and (c) using the change in energy, successfully equating elastic potential energy, kinetic energy and work done against friction although there were some sign errors in the equations. However, many omitted the work done against friction from their calculation, in spite of the attention drawn to friction by part (a). Those who attempted to use the equation of motion had more success in part (c) when they could consider the motion after the string had become slack hence constant deceleration applied. Too often the attempts in part (b) had T as a fixed value, rather than involving x, so no solution of a differential equation was attempted. Some candidates attempted an SHM solution. The majority of these were aware that the centre of the motion was where the extension was $\frac{1}{6}l$ and set up their equation accordingly although again some assumed that T was constant. Candidates adopting this approach must be careful that they prove that the motion is simple harmonic while the string is stretched.

It was surprising how many candidates who had omitted friction from their attempt at part (b) then included it in part (c). This did, however, allow them to gain some of the marks for (c). The most common error in (c) was to obtain the distance moved after the string became slack and forget to add $\frac{1}{2}l$.