Write your name here

Surname	Other names

Pearson Edexcel GCE

Centre Number

Candidate Number

A level Further Mathematics
Further Mechanics 1
Practice Paper 2

You must have:
Mathematical Formulae and Statistical Tables (Pink)

Total Marks

Instructions

- Use black ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- Fill in the boxes at the top of this page with your name, centre number and candidate number.
- Answer all the questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – there may be more space than you need.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are **7** questions in this question paper. The total mark for this paper is **75**.
- The marks for each question are shown in brackets – use this as a guide as to how much time to spend on each question.
- Calculators must not be used for questions marked with a * sign.

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

1. A particle of mass 0.6 kg is moving with constant velocity $(c\mathbf{i} + 2c\mathbf{j})$ m s^{-1}, where c is a positive constant. The particle receives an impulse of magnitude $2\sqrt{10}$ N s.

Immediately after receiving the impulse the particle has velocity $(2c\mathbf{i} - c\mathbf{j})$ m s^{-1}.

Find the value of c.

(Total 6 marks)

Mark scheme for Question 1

Examiner comment

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2. A van of mass 600 kg is moving up a straight road inclined at an angle θ to the horizontal, where $\sin\theta = \dfrac{1}{16}$. The resistance to motion of the van from non-gravitational forces has constant magnitude R newtons. When the van is moving at a constant speed of 20 m s^{-1}, the van's engine is working at a constant rate of 25 kW.

(a) Find the value of R.

(4)

The power developed by the van's engine is now increased to 30 kW. The resistance to motion from non-gravitational forces is unchanged. At the instant when the van is moving up the road at 20 m s^{-1}, the acceleration of the van is a m s^{-2}.

(b) Find the value of a.

(4)

(Total 8 marks)

Mark scheme for Question 2

Examiner comment

3.

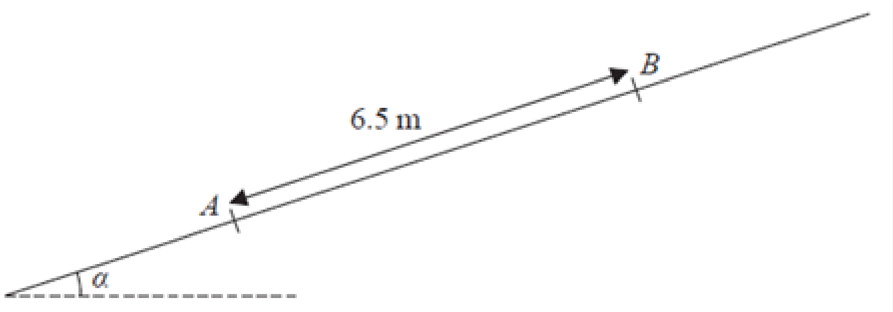


Figure 1

A particle P of mass 10 kg is projected from a point A up a line of greatest slope AB of a fixed rough plane. The plane is inclined at angle α to the horizontal, where $\tan \alpha = \frac{5}{12}$ and $AB = 6.5$ m, as shown in Figure 1. The coefficient of friction between P and the plane is μ. The work done against friction as P moves from A to B is 245 J.

(a) Find the value of μ.

(5)

The particle is projected from A with speed 11.5 m s^{-1}. By using the work-energy principle,

(b) find the speed of the particle as it passes through B.

(4)

(Total 9 marks)

Mark scheme for Question 3

Examiner comment

4. A small ball B, moving on a smooth horizontal plane, collides with a fixed smooth vertical wall. Immediately before the collision the angle between the direction of motion of B and the wall is α. The coefficient of restitution between B and the wall is $\frac{3}{4}$. The kinetic energy of B immediately after the collision is 60% of its kinetic energy immediately before the collision.

Find, in degrees, the size of angle α.

(Total 8 marks)

Mark scheme for Question 4

Examiner comment

5. A particle P of mass $3m$ is moving in a straight line with speed $2u$ on a smooth horizontal table. It collides directly with another particle Q of mass $2m$ which is moving with speed u in the opposite direction to P. The coefficient of restitution between P and Q is e.

(a) Show that the speed of Q immediately after the collision is $\frac{1}{5}(9e + 4)u$.

(5)

The speed of P immediately after the collision is $\frac{1}{2}u$.

(b) Show that $e = \frac{1}{4}$.

(4)

The collision between P and Q takes place at the point A. After the collision Q hits a smooth fixed vertical wall which is at right-angles to the direction of motion of Q. The distance from A to the wall is d.

(c) Show that P is a distance $\frac{3}{5}d$ from the wall at the instant when Q hits the wall.

(4)

Particle Q rebounds from the wall and moves so as to collide directly with particle P at the point B. Given that the coefficient of restitution between Q and the wall is $\frac{1}{5}$,

(d) find, in terms of d, the distance of the point B from the wall.

(4)

(Total 17 marks)

Mark scheme for Question 5

Examiner comment

6.

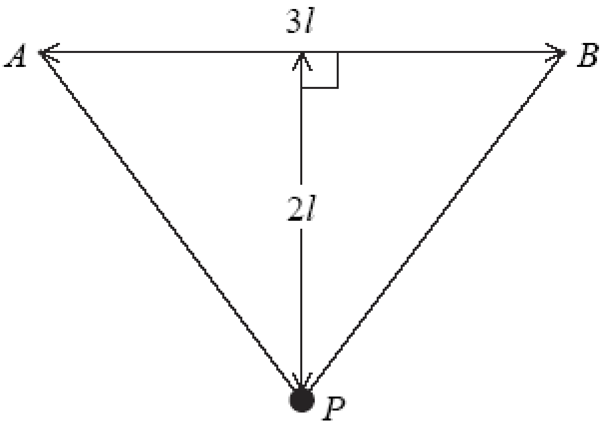


Figure 2

A light elastic string, of natural length $3l$ and modulus of elasticity λ, has its ends attached to two points A and B, where $AB = 3l$ and AB is horizontal. A particle P of mass m is attached to the mid-point of the string. Given that P rests in equilibrium at a distance $2l$ below AB, as shown in Figure 2,

(a) show that $\lambda = \dfrac{15mg}{16}$.

(9)

The particle is pulled vertically downwards from its equilibrium position until the total length of the elastic string is $7.8l$. The particle is released from rest.

(b) Show that P comes to instantaneous rest on the line AB.

(6)

(Total 15 marks)

[Mark scheme for Question 6]

[Examiner comment]

7.

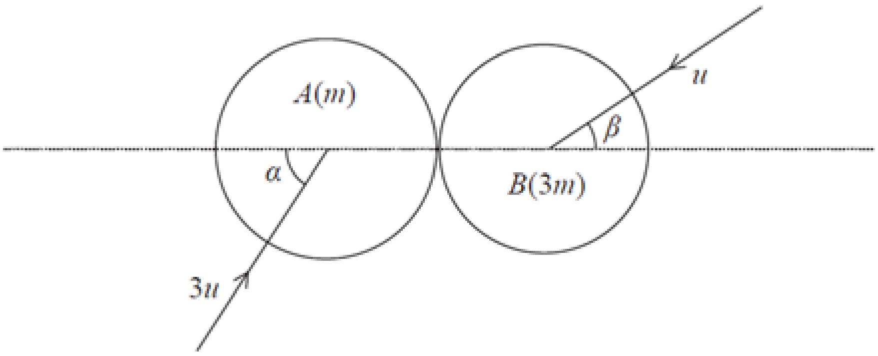


Figure 3

Two smooth uniform spheres A and B have equal radii. The mass of A is m and the mass of B is $3m$. The spheres are moving on a smooth horizontal plane when they collide obliquely. Immediately before the collision, A is moving with speed $3u$ at angle α to the line of centres and B is moving with speed u at angle β to the line of centres, as shown in Figure 3. The coefficient of restitution between the two spheres is $\frac{1}{5}$. It is given that $\cos \alpha = \frac{1}{3}$ and $\cos \beta = \frac{2}{3}$ and that α and β are both acute angles.

(a) Find the magnitude of the impulse on A due to the collision in terms of m and u.

(8)

(b) Express the kinetic energy lost by A in the collision as a fraction of its initial kinetic energy.

(4)

(Total 12 marks)

[Mark scheme for Question 7](#)

[Examiner comment](#)

TOTAL FOR PAPER: 75 MARKS

A level Further Mathematics – Further Mechanics 1 – Practice Paper 02 – Mark scheme –

Mark scheme for Question 1 (Examiner comment) (Return to Question 1)

Question	Scheme	Marks
1	Since this question is about the magnitude of the impulse, condone subtraction in the "wrong order" throughout.	
	$m\mathbf{v} - m\mathbf{u} = 0.6(2c\mathbf{i} - c\mathbf{j} - c\mathbf{i} - 2c\mathbf{j})$	M1
	$= 0.6(c\mathbf{i} - 3c\mathbf{j})$	A1
	Magnitude $= 0.6\sqrt{c^2 + 9c^2}$	DM1
	$= 0.6\sqrt{10}c \quad \left(= 0.6\sqrt{10c^2}\right)$	A1
	The next two marks are not available to a candidate who has equated a scalar to a vector.	
	$2\sqrt{10} = 0.6\sqrt{10}c$	DM1
	$c = \dfrac{10}{3}$	A1
		(6)
		(6 marks)

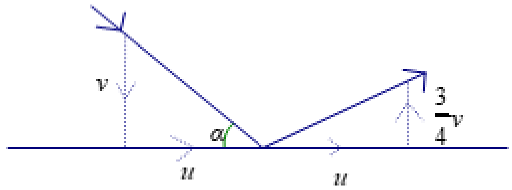
Mark scheme for Question 2 (Examiner comment) (Return to Question 2)

Question	Scheme	Marks
2(a)	Tractive force $= \dfrac{25000}{20} = 1250$ N	**B1**
	$1250 = R + 600g\sin\theta$	**M1**
	$R = 1250 - 600g \times \dfrac{1}{16}$ (=882.5)	**A1ft**
	$= 883$ or 880 N	**A1**
		(4)
(b)	T F $= \dfrac{30000}{20} = 1500$ N	
	$1500 - 600g \times \dfrac{1}{16} - R = 600a$	**M1A2**
	$a = \dfrac{1500 - 600 \times 9.8 \div 16 - 882.5}{600}$ (= 0.4166….)	
	$= 0.42$ or 0.417 m s^{-1}	**A1**
		(4)
		(8 marks)

Mark scheme for Question 3 (Examiner comment) (Return to Question 3)

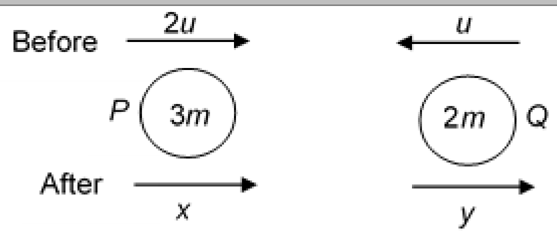
Question	Scheme	Marks
3(a)	Max friction $= \mu \times 10g\cos\alpha$	**B1**
	Work done against friction $= 6.5 \times 10g\mu\cos\alpha\,(= 245)$	**M1**
	Equation in μ: $6.5 \times 10g\mu \times \dfrac{12}{13} = 245$,	**dM1** **A1**
	$\mu = 0.417$ or 0.42	**A1**
		(5)
(b)	$\dfrac{1}{2} \times 10 \times 11.5^2 - 245 - 10g \times 6.5\sin\alpha = \dfrac{1}{2} \times 10 \times v^2$ or equivalent	**M1A2**
	$v = 5.85$ (5.9) (m s^{-1})	**A1**
		(4)
		(9 marks)

Mark scheme for Question 4 (Examiner comment) (Return to Question 4)

Question	Scheme	Marks
4		
	Velocity before & after: parallel to wall : u and u	**B1**
	Perpendicular to the wall : v and $\frac{3}{4}v$ Allow with ev	**B1**
	Kinetic energy: $\frac{1}{2}m\left(\frac{9}{16}v^2+u^2\right)=0.6\times\frac{1}{2}m\left(v^2+u^2\right)$	**M1A2**
	$\frac{90}{16}v^2+10u^2=6v^2+6u^2$	
	$4u^2=\frac{6}{16}v^2 \quad u^2=\frac{3}{32}v^2$	
	$\tan\alpha=\frac{v}{u}=\sqrt{\frac{32}{3}}$	**M1A1**
	$\alpha=73°$ (or better $72.976.....$)	**A1**
		(8)
	Alternative:	
		
	Velocity before & after: parallel to wall : $u\cos\alpha$ and $u\cos\alpha$	**B1**
	Perpendicular to the wall : $u\sin\alpha$ and $\frac{3}{4}u\sin\alpha$	**B1**
	Kinetic energy: $\frac{1}{2}m\left(\frac{9}{16}(u\sin\alpha)^2+(u\cos\alpha)^2\right)=0.6\times\frac{1}{2}m\left((u\sin\alpha)^2+(u\cos\alpha)^2\right)$	**M1A2**
	$\frac{9}{16}\sin^2\alpha+\cos^2\alpha=\frac{3}{5}=\frac{9}{16}+\frac{7}{16}\cos^2\alpha$	**M1**

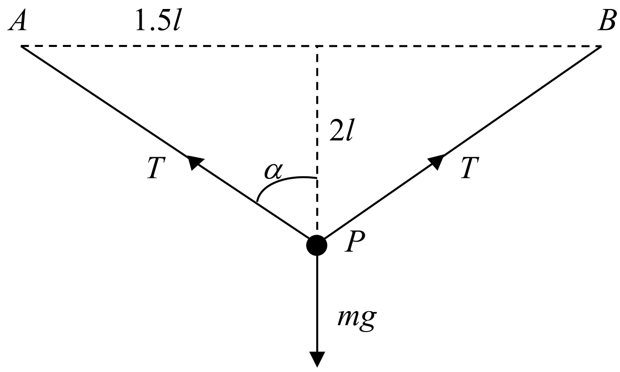
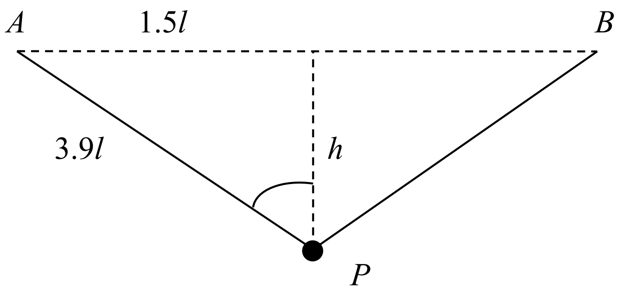
Question	Scheme	Marks
4 *continued*	$\cos^2\alpha = \frac{3}{35}$, $\alpha = \cos^{-1}\sqrt{\frac{3}{35}} = 73.0°$ (1.27 radians)	**A1A1**
		(8)
		(8 marks)

Mark scheme for Question 5 (Examiner comment) (Return to Question 5)

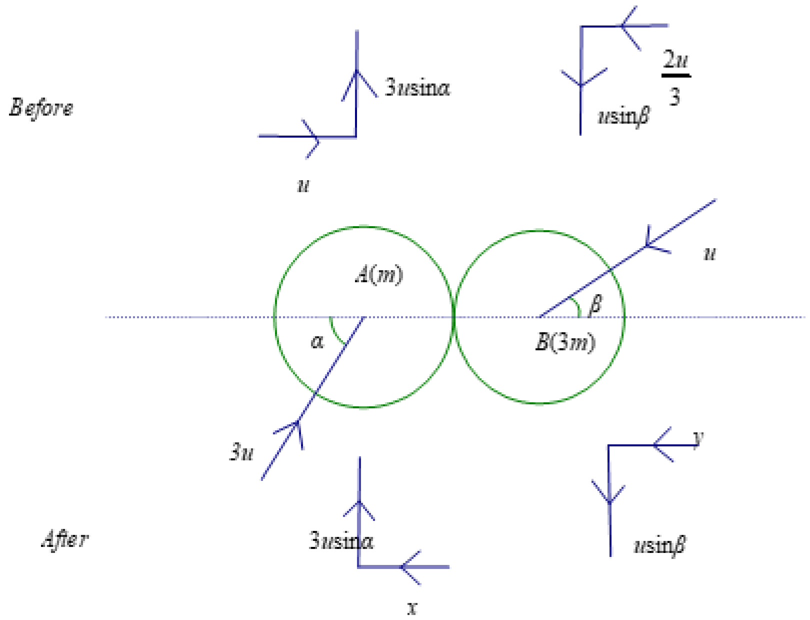
Question	Scheme	Marks
5(a)	 Correct use of NEL $y - x = e(2u + u)$ o.e.	M1* A1
	CLM $(\rightarrow)$: $3m(2u) + 2m(-u) = 3m(x) + 2m(y)$ ((□□□$u = 3x + 2y$)	B1
	Hence $x = y - 3eu$, $4u = 3(y - 3eu) + 2y$, ($u\,(9e + 4) = 5y$)	d*M1
	Hence, speed of Q $= \underline{\tfrac{1}{5}(9e + 4)u}$ **AG**	A1cso
		(5)
(b)	$x = y - 3eu = \dfrac{1}{5}(9e + 4)u - 3eu$	M1*
	Hence, speed P $= \dfrac{1}{5}(4 - 6e)u = \dfrac{2u}{5}(2 - 3e)$ o.e.	A1
	$x = \dfrac{1}{2}u = \dfrac{2u}{5}(2 - 3e) \Rightarrow 5u = 8u - 12eu, \Rightarrow 12e = 3$ & solve for e	d*M1
	gives, $e = \tfrac{3}{12} \Rightarrow \underline{e = \tfrac{1}{4}}$ **AG**	A1
		(4)
	Alternative	
	Using NEL correctly with given speeds of P and Q	M1#
	$3eu = \tfrac{1}{5}(9e + 4)u - \tfrac{1}{2}u$	A1
	$3eu = \tfrac{9}{5}eu + \tfrac{4}{5}u - \tfrac{1}{2}u$, $3e - \tfrac{9}{5}e = \tfrac{4}{5} - \tfrac{1}{2}$ & solve for e	d*M1
	$\tfrac{6}{5}e = \tfrac{3}{10} \Rightarrow e = \tfrac{15}{60} \Rightarrow e = \tfrac{1}{4}$.	A1
		(4)
(c)	Time taken by Q from A to the wall $= \underline{\dfrac{d}{y}} = \left\{\dfrac{4d}{5u}\right\}$	M1*
	Distance moved by P in this time $= \underline{\dfrac{u}{2} \times \dfrac{d}{y}}$ $\left(= \dfrac{u}{2}\left(\dfrac{4d}{5u}\right) = \dfrac{2}{5}d\right)$	A1
	Distance of P from wall $= d - x\left(\dfrac{d}{y}\right); = d - \tfrac{2}{5}d = \underline{\tfrac{3}{5}d}$ **AG**	d*M1 A1cso
		(4)

Question	Scheme	Marks
5(c) *continued*	**Alternative**	
	Ratio speed P:speed Q = x:y = $\frac{1}{2}u : \frac{1}{5}(\frac{9}{4}+4)u = \frac{1}{2}u : \frac{5}{4}u$ = 2:5	**M1***
	So if Q moves a distance d, P will move a distance $\frac{2}{5}d$	**A1**
	Distance of P from wall $= d - \frac{2}{5}d\,; = \underline{\frac{3}{5}d}$ **AG** cso	**d*M1** **A1cso**
		(4)
		(17 marks)

Mark scheme for Question 6 (Examiner comment) (Return to Question 6)

Question	Scheme	Marks
6(a)		
	$AP = \sqrt{\left((1.5l)^2 + (2l)^2\right)} = 2.5l$	**M1A1**
	$\cos\alpha = \frac{4}{5}$	**B1**
	Hooke's Law $\quad T = \frac{\lambda(2.5l - 1.5l)}{1.5l} \left(= \frac{2\lambda}{3}\right)$	**M1A1**
	$\uparrow \quad 2T\cos\alpha = mg \quad \left(T = \frac{5mg}{8}\right)$	**M1A1**
	$2 \times \frac{2\lambda}{3} \times \frac{4}{5} = mg \quad \left(\frac{2\lambda}{3} = \frac{5mg}{8}\right)$	**M1**
	$\lambda = \frac{15mg}{16}$ * cso	**A1**
		(9)
(b)		
	$h = \sqrt{\left((3.9l)^2 - (1.5l)^2\right)} = 3.6l$	**M1A1**
	Energy $\quad \frac{1}{2}mv^2 + mg \times h = 2 \times \frac{15mg}{16} \times \frac{(2.4l)^2}{2 \times 1.5l}$ ft their h	**M1Aft A1**
	Leading to $\quad v = 0$ * cso	**A1**
		(6)
		(15 marks)

Mark scheme for Question 7 (Examiner comment) (Return to Question 7)

Question	Scheme	Marks
7(a)		
	CLM: $mx+3my=3m\times u\cos\beta-m\times 3u\cos\alpha=mu \quad (x+3y=u)$	**M1A1**
	NEL: $x-y=\frac{1}{5}(3u\cos\alpha+u\cos\beta)\left(=\frac{1}{5}\left(u+\frac{2}{3}u\right)=\frac{1}{3}u\right)$	**M1A1**
	$x=\frac{u}{2}$, or $y=\frac{u}{6}$	**DM1** **A1**
	Magnitude of the impulse on A = $mu-\left(m\times-\frac{u}{2}\right)=\frac{3mu}{2}$	**M1A1**
		(8)
(b)	Component of velocity perpendicular to the line of centres before = component after = $3u\sin\alpha=3u\times\frac{\sqrt{8}}{3}=\sqrt{8}u$	**B1**
	KE lost = $\frac{m}{2}\left(9u^2-\left(8u^2+\frac{1}{4}u^2\right)\right)\left[=\frac{3}{8}mu^2\right]$	**M1A1**
	Fraction lost = $\frac{3/8}{9/2}=\frac{3}{8}\times\frac{2}{9}=\frac{1}{12}$	**A1**
		(4)
		(12 marks)

A level Further Mathematics – Further Mechanics 1 – Practice Paper 02 – Examiner Report –

Examiner comment for Question 1 **(Mark scheme)** **(Return to Question 1)**

1. It was very common for candidates to start their response to this question by equating a scalar to a vector. Despite this serious error, most candidates did find the change in momentum and then found its magnitude in terms of c. When finding the impulse a small number of candidates ignored the vector nature of momentum and used the change in speed rather than the change in velocity. There were several basic computational errors: in simplifying the impulse vector a sign error was not uncommon and when applying Pythagoras' theorem, errors were made in squaring the terms or forgetting to square root or not squaring the given impulse of $2\sqrt{10}$.

Examiner comment for Question 2 **(Mark scheme)** **(Return to Question 2)**

2. Most candidates were well prepared for a question on power, and gave confident answers. The most common error was an over-specified answer at the end of part (a). Many candidates left their answer as 882.5, which is inappropriate after the use of 9.8 as an approximate value for g.

Examiner comment for Question 3 **(Mark scheme)** **(Return to Question 3)**

3. (a) There were many correct answers to this part of the question. The method for finding the work done against the friction was well understood, but some candidates also included the gain in gravitational potential energy in their answer. In the course of the working an approximate value for g is used, so an "exact" answer of $\frac{5}{12}$ is inadmissible.

   (b) The majority of candidates followed the request to answer this using the work-energy principle. There were many correct answers. Common errors involved sign errors in the work-energy equation, or omitting a term (often the work done against friction) from the equation. As the question specifies the method to be used, attempts to answer the question using *suvat* equations scored no marks.

Examiner comment for Question 4 **(Mark scheme)** **(Return to Question 4)**

4. Many students obtained the correct final answer. The most concise solutions started by creating variables for the components of the initial velocity perpendicular and parallel to the wall, and worked with these throughout the solution. Those students who worked with trigonometric ratios of the angles tended to have a more difficult task eliminating the unwanted variables. The most common error in setting up the initial equations was to place the 0.6 on the wrong side of the energy equation.

Examiner comment for Question 5 **(Mark scheme)** **(Return to Question 5)**

5. Candidates made a confident start to this question, but in parts (a) and (b), poor algebraic skills and the lack of a clear diagram with the directions marked on it hampered weaker candidates' attempts to set up correct and consistent (or even physically possible) equations. The direction of *P* after impact was not given and those candidates who took its direction as reversed ran into problems when finding the value of *e*. Many realised that they had chosen the wrong direction and went on to answer part (b) correctly but some did not give an adequate explanation for a change of sign for their velocity of *P*. Algebraic and sign errors were common, and not helped by candidates' determination to reach the given answers.

   Parts (c) and (d) caused the most problems. They could be answered using a wide variety of methods, some more formal than others. Many good solutions were seen but unclear reasoning and methods marred several attempts. Too many solutions were sloppy, with *u* or *d* appearing and disappearing through the working. A few words describing what was being calculated or expressed at each stage would have helped the clarity of solutions greatly. Students need to be reminded yet again that all necessary steps need to be shown when reaching a given answer. Too many simply stated the answer $\frac{3d}{5}$ without the explanation to support it.

Examiner comment for Question 6 **(Mark scheme)** **(Return to Question 6)**

6. Part (a) was well done. A correct use of Hooke's Law, combined with vertical resolution and the geometry of the situation, was the standard approach. A few made the error of not realising that there were two tensions for the resolution but only one for Hooke's Law. Many good arguments were seen in part (b). There were two main methods; using conservation of energy with three terms and showing v = 0 , and finding the elastic potential energy loss and showing that it was equal to the gain in gravitational potential energy at the level of AB. There was a tendency for candidates to drop the l in their lengths. As energy is involved, this gave dimensionally incorrect solutions but candidates who did this could usually gain 4 of the 6 marks if their solution was otherwise correct.

Examiner comment for Question 7 **(Mark scheme)** **(Return to Question 7)**

7. In part (a) although the majority of students produced appropriate momentum and restitution equations, the directions of their velocities were often not well defined nor clearly shown on a diagram. This led to frequent sign errors. Mistakes in solving the resulting simultaneous equations were quite common. The definition of impulse was generally known but occasionally speeds rather components of velocity (or a mixture of the two) were used.

   In part (b) there was confusion between speeds and components of velocity in the calculation of kinetic energy. Although it was not necessary to find the perpendicular components in finding the loss in energy, it was necessary to use the whole speed in the calculation of initial energy. Some found the final energy (rather than loss) as a fraction of the initial energy, and occasionally the energies for both spheres were combined. Some students used $\frac{1}{2}mu^2$ in place of $\frac{1}{2}m(3u)^2$.