Pearson Edexcel

Mark Scheme

Mock Set 4

Pearson Edexcel GCE In Mathematics (9MA0)
Paper 2 Pure Mathematics

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS
General Instructions for Marking

1. The total number of marks for the paper is 100.

2. The Edexcel Mathematics mark schemes use the following types of marks:
   - **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
   - **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
   - **B** marks are unconditional accuracy marks (independent of M marks)
   - Marks should not be subdivided.

3. Abbreviations
   These are some of the traditional marking abbreviations that will appear in the mark schemes.
   - bod – benefit of doubt
   - ft – follow through
   - the symbol $\sqrt{}$ will be used for correct ft
   - cao – correct answer only
   - cso - correct solution only. There must be no errors in this part of the question to obtain this mark
   - isw – ignore subsequent working
   - awrt – answers which round to
   - SC: special case
   - oe – or equivalent (and appropriate)
   - dep – dependent
   - indep – independent
   - dp decimal places
   - sf significant figures
   - ∗ The answer is printed on the paper
   - ⎾ The second mark is dependent on gaining the first mark

4. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

5. Where a candidate has made multiple responses <u>and indicates which response they wish to submit</u>, examiners should mark this response.
   If there are several attempts at a question <u>which have not been crossed out</u>, examiners should mark the final answer which is the answer that is the <u>most complete</u>.
6. Ignore wrong working or incorrect statements following a correct answer.

7. Mark schemes will firstly show the solution judged to be the most common response expected from candidates. Where appropriate, alternatives answers are provided in the notes. If examiners are not sure if an answer is acceptable, they will check the mark scheme to see if an alternative answer is given for the method used.

General Principles for Further Pure Mathematics Marking

(*But note that specific mark schemes may sometimes override these general principles*)

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \ldots$

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \ldots$

2. Formula

Attempt to use the correct formula (with values for *a*, *b* and *c*)

3. Completing the square

Solving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0, \quad q \neq 0$, leading to $x = \ldots$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. $(x^n \rightarrow x^{n-1})$

2. Integration

Power of at least one term increased by 1. $(x^n \rightarrow x^{n+1})$

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small errors in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Question	Scheme	Marks	AOs
1(a)(i)	$2x-10<3(5-x)\Rightarrow 2x-10<15-3x\Rightarrow 5x<25$	M1	1.1b
	$x<5$	A1	1.1b
		(2)	
(ii)	$x^2-11x+24=0\Rightarrow x=3,\ 8$	M1	1.1b
	$3\leqslant x\leqslant 8$	A1	1.1b
		(2)	
(b)	$3\leqslant x<5$	B1	2.2a
		(1)	
			(5 marks)

Notes

(a)(i)
M1: Expands the rhs and collects terms to obtain the form $ax<b$ or $ax>b$
A1: Correct answer
(ii)
M1: Attempts to solve the quadratic and get the critical values
A1: Correct range
(b)
B1: Deduces the correct range

Question	Scheme	Marks	AOs
2(a)	$f'(x) = 3x^2 - 10x - \frac{3}{x^2}$	M1 A1	1.1b 1.1b
		(2)	
(b)	Change of sign and f(x) is continuous so α lies between 0.5 and 0.6	B1	2.4
		(1)	
(c)	$x_0 = 0.5 \Rightarrow x_1 = 0.5 - \frac{f(0.5)}{f'(0.5)} = 0.5 - \frac{0.875}{3(0.5)^2 - 10(0.5) - 3(0.5)^{-2}}$	M1	1.1b
	$x_1 = 0.554$	A1	1.1b
		(2)	
			(5 marks)

Notes

(a)
M1: For $x^n \rightarrow x^{n-1}$
A1: Correct derivative
(b)
B1: Correct explanation
(c)
M1: Applies the N-R method correctly for their $f'(x)$
A1: For awrt 0.554

Question	Scheme	Marks	AOs
3(a)	$h = 0.3$	B1	1.1a
	$A \approx \frac{0.3}{2}\{1.811 + 2.944 + 2(2.342 + 2.718 + 2.941 + 3.011)\}$	M1	1.1b
	$= 4.02$	A1	1.1b
		(3)	
(b)	**Underestimate** and a relevant justification e.g. • {top of} trapezia lie below the curve • Area of trapezia < area under curve • An appropriate diagram which gives reference to the lost area • Curve is concave • The gradient of the curve is {continually} decreasing	B1	3.2b
		(1)	
(c)	$\int_{-0.6}^{0.9}(8 - 2\text{f}(x))\ \text{d}x = ... - 2\times"4.02"$	M1	1.1b
	$\int_{-0.6}^{0.9}(8 - 2\text{f}(x))\ \text{d}x = 8\times1.5 - ...$	M1	3.1a
	$\int_{-0.6}^{0.9}(8 - 2\text{f}(x))\ \text{d}x = 8\times1.5 - 2\times"4.02" = 3.96$	A1ft	2.2a
		(3)	
			(7 marks)

Notes

(a)
B1: States or uses $h = 0.3$
M1: Correct attempt at the trapezium rule. Must be an attempt at the correct structure e.g. $\frac{h}{2}\{y_{-0.6} + y_{0.3} + 2(y_0 + y_{0.3} + y_{0.6} + y_{0.9})\}$ with brackets as shown unless they are implied by subsequent work
A1: For awrt 4.02
(b)
B1: see main scheme
(c)
M1: For multiplying their answer to part (a) by ±2
M1: For a correct strategy for the "8" part of the integral. May see e.g. 8×1.5 or $8\times(0.9 + 0.6)$ or $\int_{-0.6}^{0.9} 8\ \text{d}x = [8x]_{-0.6}^{0.9} = 8\times0.9 - 8\times(-0.6)$
A1ft: For awrt 3.96 (or 3.97 if full accuracy used from (a)) or follow through $12 - 2\times$their answer to part (a)

Question	Scheme	Marks	AOs
4(a)	$\mathrm{f}\left(-\frac{1}{2}\right)=p\left(-\frac{1}{2}\right)^3-7\left(-\frac{1}{2}\right)^2-\left(-\frac{1}{2}\right)+q=0$	M1	1.1b
	$-\frac{p}{8}-\frac{7}{4}+\frac{1}{2}+q=0 \Rightarrow -p-10+8q=0$ $\Rightarrow 8q-p=10\,*$	A1*	2.1
		(2)	
(b)	$\mathrm{f}(1)=0 \Rightarrow p-7-1+q=0$ $8q-p=10,\ p+q=8 \Rightarrow p=...,\ \text{or}\ q=...$	M1	3.1a
	$p=6,\ q=2$	A1	1.1b
		(2)	
			(4 marks)

Notes

(a)

M1: Attempts $\mathrm{f}\left(-\frac{1}{2}\right)=0$ to obtain an equation in p and q

A1*: Proceeds with sufficient working and no errors to the printed answer

(b)

M1: Attempts $\mathrm{f}(1)=0$ to obtain another equation in p and q and then solves with the given equation from part (a) to obtain a value for p or q

A1: Correct values

Question	Scheme	Marks	AOs
5	$\dfrac{\cos 9\theta - 1}{\theta \sin\theta} = \dfrac{1 - \dfrac{(9\theta)^2}{2} - 1}{\ldots}$ **or** $\dfrac{\cos 9\theta - 1}{\theta \sin\theta} = \dfrac{\ldots}{\theta \times \theta}$	M1	1.1b
	$\dfrac{\cos 9\theta - 1}{\theta \sin\theta} = \dfrac{1 - \dfrac{(9\theta)^2}{2} - 1}{\theta \times \theta} = \ldots$	dM1	2.1
	$= -\dfrac{81}{2}$ oe	A1	1.1b
		(3)	
			(3 marks)

Notes

M1: Applies one correct approximation in either the numerator or the denominator
dM1: Applies both correct approximations in the numerator and the denominator and attempts to simplify
A1: Correct value

Question	Scheme	Marks	AOs
6(a)(i)	$\frac{dy}{dx} = 3ax^2 + 2bx + 12$	M1 A1	1.1b 1.1b
(ii)	$\frac{d^2y}{dx^2} = 6ax + 2b$	A1ft	1.1b
		(3)	
(b)	$\frac{13}{2} = a\left(\frac{3}{2}\right)^3 + b\left(\frac{3}{2}\right)^2 + 12\left(\frac{3}{2}\right) + 2, \quad 6a\left(\frac{3}{2}\right) + 2b = 0$ $\Rightarrow a = ..., b = ...$	M1	3.1a
	$a = 2, \ b = -9$	A1	1.1b
		(2)	
(c)	$\left(\frac{d^2y}{dx^2}\right)_{x=1} = 12(1) - 18, \quad \left(\frac{d^2y}{dx^2}\right)_{x=2} = 12(2) - 18$ **or** $\left(\frac{d^3y}{dx^3}\right)_{\left(x=\frac{3}{2}\right)} = 12$	M1	2.1
	$\left(\frac{d^2y}{dx^2}\right)_{x=1} < 0, \quad \left(\frac{d^2y}{dx^2}\right)_{x=2} > 0$ **or** $\left(\frac{d^3y}{dx^3}\right)_{\left(x=\frac{3}{2}\right)} \neq 0$ Hence point of inflection	A1	2.2a
		(2)	
			(7 marks)

Notes

(a)(i)
M1: For $x^n \to x^{n-1}$
A1: Correct expression
(ii)
A1ft: Correct expression (follow through their first derivative)
(b)
M1: Substitutes the coordinates of *P* into the equation of the curve and substitutes $x = 1.5$ into their second derivative and sets = 0 and then solves 2 equations in *a* and *b*.
A1: Correct values
(c)
M1: Attempts to find the value of the second derivative either side of $x = 1.5$ or attempts the third derivative.
A1: Correct work with a suitable conclusion e.g. the second derivative changes sign either side of $x = 1.5$ or the third derivative is non-zero at $x = 1.5$

Question	Scheme	Marks	AOs
7	$\frac{(x-3)^2}{x^{\frac{3}{2}}} = \frac{x^2-6x+9}{x^{\frac{3}{2}}} = x^{\frac{1}{2}} - 6x^{-\frac{1}{2}} + 9x^{-\frac{3}{2}}$	M1	3.1a
	$\lim_{\delta x \to 0} \sum_{x=2}^{3} \frac{(x-3)^2}{x^{\frac{3}{2}}} \delta x = \int \left(x^{\frac{1}{2}} - 6x^{-\frac{1}{2}} + 9x^{-\frac{3}{2}} \right) dx = \frac{2}{3}x^{\frac{3}{2}} - 12x^{\frac{1}{2}} - 18x^{-\frac{1}{2}}$	M1 A1	2.1 1.1b
	$\left[\frac{2}{3}x^{\frac{3}{2}} - 12x^{\frac{1}{2}} - 18x^{-\frac{1}{2}} \right]_2^3 = 2\sqrt{3} - 12\sqrt{3} - \frac{18}{\sqrt{3}} - \left(\frac{4}{3}\sqrt{2} - 12\sqrt{2} - \frac{18}{\sqrt{2}} \right)$ $= ...\sqrt{2} + ...\sqrt{3}$	M1	2.1
	$= \frac{59}{3}\sqrt{2} - 16\sqrt{3}$	A1	1.1b
		(5)	
			(5 marks)

Notes

M1: Correct strategy to deal with the fraction. This requires expansion of the numerator followed by division by the denominator in order to reach an integrable form with at least 2 correct indices
M1: Interprets the demand correctly as an integral and applies $x^n \to x^{n+1}$ for at least one fractional power
A1: Fully correct integration (simplified or unsimplified)
M1: Applies the given limits the right way round to an expression of the form $ax^{\frac{3}{2}} + bx^{\frac{1}{2}} + cx^{-\frac{1}{2}}$ and combines 6 terms to reach an expression of the required form
A1: Correct expression

Question	Scheme	Marks	AOs
8(a)(i)	$150+29\times10=...$	M1	3.4
	$=440$	A1	1.1b
(ii)	$S_{30}=\frac{1}{2}(30)(2\times150+29\times10)=...$ or $S_{30}=\frac{1}{2}(30)(150+"440")=...$	M1	3.4
	$=8850$	A1	1.1b
		(4)	
(b)	$\frac{1}{2}(52)(2\times150+51d)=15000\Rightarrow d=...$	M1	3.1b
	$d=5.42...$ so the minimum value of d is 6	A1	3.2a
		(2)	
(c)	E.g. The increase in the number of candles sold each week is unlikely to be a constant	B1	3.5b
		(1)	
			(7 marks)

Notes

(a)(i)
M1: Uses the model and a correct term formula to find the required value
A1: For 440
(ii)
M1: Uses the model and a correct sum formula to find the required value
A1: For 8850
(b)
M1: Correct strategy to find the value of *d*. Requires the use of a correct sum formula with n = 52, a = 150, sets = 15 000 and solves for *d*. Allow use of e.g. "> 15 000" rather than "= 15 000"
A1: Interprets the value of *d* correctly and gives the answer 6 only
(c)
B1: See scheme

Question	Scheme	Marks	AOs
9(a)	$x=\frac{7}{3}$	B1	2.2a
		(1)	
(b)	$x=2\Rightarrow\lvert 7-3x\rvert=\lvert 7-6\rvert=1,\ -2x^2+14x-19=1$ So the x coordinate of A is 2	B1	2.1
		(1)	
(c)	At B $3x-7=-2x^2+14x-19\Rightarrow 2x^2-11x+12=0\Rightarrow x=...$	M1	3.1a
	$2x^2-11x+12=0\Rightarrow x=4$	A1	1.1b
	$\Rightarrow y=5$	A1	1.1b
		(3)	
(d)	$\int\left(-2x^2+14x-19\right)\,\mathrm{d}x=-\frac{2x^3}{3}+7x^2-19x(+c)$	M1 A1	1.1b 1.1b
	Area of triangles: $\frac{1}{2}\left(\frac{7}{3}-2\right)\times1+\frac{1}{2}\left(4-\frac{7}{3}\right)\times5\quad\left(=\frac{13}{3}\right)$	M1	2.1
	Area of R is: $\left[-\frac{2x^3}{3}+7x^2-19x\right]_2^4-\frac{13}{3}=-\frac{128}{3}+112-76-\left(-\frac{16}{3}+28-38\right)-\frac{13}{3}$	ddM1	3.1a
	$=\frac{13}{3}$	A1	1.1b
		(5)	
	(d) Alternative:		
	$\int\left(-2x^2+14x-19-7+3x\right)\,\mathrm{d}x=-\frac{2x^3}{3}+7x^2-19x-7x+\frac{3x^2}{2}(+c)$ **or** $\int\left(-2x^2+14x-19+7-3x\right)\,\mathrm{d}x=-\frac{2x^3}{3}+7x^2-19x+7x-\frac{3x^2}{2}(+c)$	M1 A1	1.1b 1.1b
	$\left[-\frac{2x^3}{3}+7x^2-19x-7x+\frac{3x^2}{2}\right]_2^{\frac{7}{3}}$ **or** $\left[-\frac{2x^3}{3}+7x^2-19x+7x-\frac{3x^2}{2}\right]_{\frac{7}{3}}^4$	M1	2.1
	$\left[-\frac{2x^3}{3}+7x^2-19x-7x+\frac{3x^2}{2}\right]_2^{\frac{7}{3}}+\left[-\frac{2x^3}{3}+7x^2-19x+7x-\frac{3x^2}{2}\right]_{\frac{7}{3}}^4=...$	ddM1	3.1a
	$=\frac{13}{3}$	A1	1.1b
		(5)	
			(10 marks)

Notes
(a) M1: Deduces the correct value of x (b) B1: Substitutes $x = 2$ into both equations, obtains $y = 1$ both times and makes a conclusion. (c) M1: Correct strategy for B. E.g. attempts to solve $3x - 7 = -2x^2 + 14x - 19$ and solves the resulting 3TQ A1: For $x = 4$ A1: For $y = 5$ (d) M1: For $x^n \to x^{n+1}$ for the quadratic curve A1: Correct integration M1: For a complete attempt at the area of both triangles using their vertex coordinates and their A and B ddM1: For a correct overall strategy for the area of R. This requires an attempt at the area under the curve between A and B with the subtraction of the area of the 2 triangles. Depends on both previous M marks. A1: Correct value **(d) Alternative:** M1: For $x^n \to x^{n+1}$ for either subtraction A1: Correct integration for either part M1: For a complete attempt at the area of one of the required areas ddM1: For a correct overall strategy for the area of R. Depends on both previous method marks. A1: Correct value

Question	Scheme	Marks	AOs
10	$x = t^2,\ \ y = 2t \Rightarrow t^4 + 4t^2 = 10t^2 + k \Rightarrow t^4 - 6t^2 - k = 0$ or $y = 2t \Rightarrow x = \frac{y^2}{4} \Rightarrow \frac{y^4}{16} + y^2 = \frac{10y^2}{4} + k \Rightarrow y^4 - 24y^2 - 16k = 0$ or $x = t^2 \Rightarrow y = 2\sqrt{x} \Rightarrow x^2 + 4x = 10x + k \Rightarrow x^2 - 6x - k = 0$	M1 A1	3.1a 1.1b
	Roots must be real: $b^2 - 4ac > 0 \Rightarrow 6^2 + 4k > 0 \Rightarrow k > -9$ or e.g. $b^2 - 4ac > 0 \Rightarrow 24^2 + 64k > 0 \Rightarrow k > -9$	dM1 A1	3.1a 1.1b
	Both roots must be positive so e.g.: $6 - \sqrt{36 + 4k} > 0 \Rightarrow k < 0$	B1	2.2a
	$\{k : k < 0\} \cap \{k : k > -9\}$	A1	2.5
		(6)	
			(6 marks)

Notes

M1: Makes the key step of using the Cartesian equation with the parametric equations to eliminate 2 of the variables
A1: Correct 3TQ in t^2, y^2 or x
dM1: Recognises the condition that $b^2 - 4ac > 0$ as roots must be real and uses this to find the minimum value for k
A1: For $k > -9$ seen as part of their solution
B1: Deduces that as both roots must be positive, $k < 0$
A1: Correct range using the correct notation. Allow equivalents e.g. $\{k : -9 < k < 0\}$, $k \in (-9, 0)$

Question	Scheme	Marks	AOs
11	$n = 3k+1$ **or** $n = 3k+2$ $n^2 - 1 = \ldots$	M1	3.1a
	$n = 3k+1 \Rightarrow n^2 - 1 = (3k+1)^2 - 1 = 9k^2 + 6k + 1 - 1 = 3(3k^2 + 2k)$ which is a multiple of 3 **or** $n = 3k+2 \Rightarrow n^2 - 1 = (3k+2)^2 - 1 = 9k^2 + 12k + 4 - 1 = 3(3k^2 + 4k + 1)$ which is a multiple of 3	A1	2.2a
	$n = 3k+1$ **and** $n = 3k+2$ $n^2 - 1 = \ldots$	dM1	2.1
	$n = 3k+1 \Rightarrow n^2 - 1 = (3k+1)^2 - 1 = 9k^2 + 6k + 1 - 1 = 3(3k^2 + 2k)$ which is a multiple of 3 **and** $n = 3k+2 \Rightarrow n^2 - 1 = (3k+2)^2 - 1 = 9k^2 + 12k + 4 - 1 = 3(3k^2 + 4k + 1)$ which is a multiple of 3 So if n is a positive integer that is not divisible by 3 then $n^2 - 1$ is divisible by 3	A1	2.4
		(4)	

(4 marks)

Notes

M1: Sets $n = 3k + 1$ or $n = 3k + 2$ (or e.g. $n = 3k - 1$) and attempts $n^2 - 1$ or $(n-1)(n+1)$

A1: Achieves e.g. $3(3k^2 + 2k)$ or $3(3k^2 + 4k + 1)$ oe and deduces that it is a multiple of 3

dM1: A full and rigorous attempt at the proof considering **both** $n = 3k + 1$ and $n = 3k + 2$ oe valid expressions e.g. $n = 3k + 1$ and $n = 3k - 1$

A1: Fully correct work with valid reasons and a final conclusion

Question	Scheme	Marks	AOs
12(a)	$\dfrac{dy}{dx}=\dfrac{3t^2-12}{2t+1}$	M1 A1	1.1b 1.1b
		(2)	
(b)	At P $3t^2-12=0 \Rightarrow t=\pm 2$ As P is in quadrant 1, $t=-2 \Rightarrow x=\ldots, y=\ldots$	M1	2.1
	$(6, 21)$	A1	2.3
		(2)	
(c)	At Q, $y=21$ $t^3-12t+5=21 \Rightarrow t^3-12t-16=0$	M1	3.1a
	$t^3-12t-16=(t+2)^2(t\pm\ldots)=0 \Rightarrow t=\ldots(4)$	dM1	2.1
	Q is $(24, 21)$	A1	2.2a
		(3)	
			(7 marks)

Notes

(a)

M1: Attempts $\dfrac{dx}{dt}$ and $\dfrac{dy}{dt}$ and then uses $\dfrac{dy}{dx}=\dfrac{dy}{dt}\div\dfrac{dx}{dt}$ (may be implied by their $\dfrac{dy}{dx}$)

A1: Correct expression

(b)

M1: Sets the numerator of their $\dfrac{dy}{dx}=0,$ solves a quadratic equation in t and uses it to find the coordinates of P

A1: Correct coordinates

(c)

M1: Recognises that at Q, the y coordinate is equal to the y coordinate of P and uses this to form a cubic equation in t

M1: Uses a correct strategy to find the value of t at Q. E.g. uses the repeated root at P to factorise the cubic equation.

A1: Deduces the correct coordinates

Question	Scheme	Marks	AOs
13(a)	$R=\frac{\sqrt{5}}{2}$	B1	1.1b
	$\tan\alpha=\frac{1}{2}\Rightarrow\alpha=\ldots$	M1	1.1b
	$\alpha=0.464$	A1	1.1b
	$\theta=\frac{\sqrt{5}}{2}\sin\left(x+0.464\right)$		
		(3)	
(b)	$\left(26-\frac{\sqrt{5}}{2}\right)$°C or awrt 24.9°C	B1ft	2.2a
		(1)	
(c)	$\left(\frac{\pi t}{3}-8\right)+0.464=\frac{7\pi}{2}$	M1	3.1b
	$t=17.686...$	A1	1.1b
	17:42 or 5:42 pm or 17 hours 42 minutes after midnight	A1	3.2a
		(3)	
(d)	e.g. The "26" should be increased	B1	3.5c
		(1)	
			(8 marks)

Notes

(a)

B1: $R=\frac{\sqrt{5}}{2}$ or the exact equivalent

M1: Proceeds to a value of α from $\tan\alpha=\pm\frac{1}{2}$ or $\tan\alpha=\pm2$ or $\cos\alpha=\pm\frac{1}{"R"}$ or $\sin\alpha=\pm\frac{\frac{1}{2}}{"R"}$

A1: $\alpha=$ awrt 0.464

(b)

B1ft: Deduces the correct temperature. Follow through their *R* and condone lack of units here.

(c)

M1: A complete method to find a value of *t* from $\left(\frac{\pi t}{3}-8\right)+0.464=\alpha$ where *α* is one of

$\frac{\pi}{2},\frac{3\pi}{2},\frac{7\pi}{2},\frac{11\pi}{2}$

A1: For awrt 17.7 following a suitable equation

A1: Deduces the correct time

(d)

B1: For any suitable refinement that increases the mean temperature but does not change the variation of temperature

Question	Scheme	Marks	AOs
14(a)	$\frac{dy}{dx}=\frac{(x^3+4)2e^{2x}-3x^2e^{2x}}{(x^3+4)^2}$	M1 A1 A1	1.1b 1.1b 1.1b
		(3)	
(b)	At P $\frac{dy}{dx}=\frac{(p^3+4)2e^{2p}-3p^2e^{2p}}{(p^3+4)^2}$ $\Rightarrow y-\frac{e^{2p}}{p^3+4}=\frac{(p^3+4)2e^{2p}-3p^2e^{2p}}{(p^3+4)^2}(x-p)$	M1	3.1a
	As l passes through (0, 0), $-\frac{e^{2p}}{p^3+4}=\frac{(p^3+4)2e^{2p}-3p^2e^{2p}}{(p^3+4)^2}(-p)$	M1	1.1b
	$\Rightarrow (p^3+4)e^{2p}=(p^3+4)2pe^{2p}-3p^3e^{2p}$ $\Rightarrow 2p^4-4p^3+8p-4=0$ $\Rightarrow p^4-2p^3+4p-2=0$ so p satisfies $x^4-2x^3+4x-2=0$*	A1*	2.1
		(3)	
(c)	$x_1=0.5\Rightarrow x_2=\frac{2(0.5)^3+2}{0.5^3+4}=...$	M1	1.1b
	$x_2=0.5455$	A1	1.1b
	$p=0.5646$	A1	2.2a
		(3)	
(d)	Gradient of l is $\frac{(p^3+4)2e^{2p}-3p^2e^{2p}}{(p^3+4)^2}=1.31$	B1	2.2a
		(1)	
			(10 marks)

Notes

(a)

M1: For $\frac{dy}{dx}=\frac{\alpha(x^3+4)e^{2x}-\beta x^2e^{2x}}{(x^3+4)^2}$

A1: For $\frac{dy}{dx}=\frac{2(x^3+4)e^{2x}-...}{(x^3+4)^2}$ or $\frac{dy}{dx}=\frac{...-3x^2e^{2x}}{(x^3+4)^2}$

A1: Fully correct derivative in any form

(b)

M1: Fully correct strategy for l. E.g. substitutes $x=p$ into their (a) result to find the gradient of l and uses this with $\left(p, \frac{e^{2p}}{p^3+4}\right)$ to form an equation for l which may be implied

M1: Sets $x = 0$ and $y = 0$ to establish an equation in p

A1*: Completes to the given answer with no errors

(c)

M1: Attempts to use the given recurrence relation with $x = 0.5$

A1: Awrt 0.5455

A1: For deducing that $p = 0.5646$ only
(d)
B1: Uses the value of p to deduce the gradient of l. Allow awrt 1.31

Question	Scheme	Marks	AOs
15(a)	$A=1$	B1	1.1b
	$\frac{u^2}{u^2-1}\equiv A+\frac{B}{u+1}+\frac{C}{u-1}$ $\Rightarrow u^2\equiv A\left(u^2-1\right)+B\left(u-1\right)+C\left(u+1\right)\Rightarrow B=\ldots,\ C=\ldots$	M1	1.1b
	$B=-\frac{1}{2},\quad C=\frac{1}{2}$	A1	1.1b
		(3)	
(b)	$u=\sqrt{1+e^{3x}}\Rightarrow u^2=1+e^{3x}\Rightarrow 2u\frac{du}{dx}=3e^{3x}$ **or** $u=\sqrt{1+e^{3x}}\Rightarrow \frac{du}{dx}=\frac{3}{2}e^{3x}\left(1+e^{3x}\right)^{-\frac{1}{2}}$	B1	1.1b
	$x=\frac{1}{3}\ln 3\Rightarrow u=\sqrt{1+e^{\ln 3}}=\ldots$ $x=\frac{1}{3}\ln 8\Rightarrow u=\sqrt{1+e^{\ln 8}}=\ldots$	M1	1.1b
	$\int\sqrt{1+e^{3x}}\,dx=\int u\times\frac{2u}{3e^{3x}}\,du=\int u\times\frac{2u}{3\left(u^2-1\right)}\,du$ **or** $\int\sqrt{1+e^{3x}}\,dx=\int\sqrt{1+e^{3x}}\times\frac{2}{3}\frac{\sqrt{1+e^{3x}}}{e^{3x}}\,dx=\int\frac{2u^2}{3\left(u^2-1\right)}\,du$	M1 A1	3.1a 1.1b
	$\int_{\frac{1}{3}\ln 3}^{\frac{1}{3}\ln 8}\sqrt{1+e^{3x}}\,dx=\frac{2}{3}\int_2^3\frac{u^2}{u^2-1}\,du$	A1	2.1
		(5)	
(c)	$\int\frac{u^2}{u^2-1}\,du=\int\left(1-\frac{1}{2\left(u+1\right)}+\frac{1}{2\left(u-1\right)}\right)du$ $=u-\frac{1}{2}\ln\left(u+1\right)+\frac{1}{2}\ln\left(u-1\right)\left(+c\right)$	M1 A1	2.1 1.1b
	$\frac{2}{3}\int_2^3\frac{u^2}{u^2-1}\,du=\frac{2}{3}\left[u-\frac{1}{2}\ln\left(u+1\right)+\frac{1}{2}\ln\left(u-1\right)\right]_2^3$ $\frac{2}{3}\left(3-\frac{1}{2}\ln\left(4\right)+\frac{1}{2}\ln\left(2\right)-\left(2-\frac{1}{2}\ln\left(3\right)+\frac{1}{2}\ln\left(1\right)\right)\right)=\ldots$	M1	2.1
	$=\frac{2}{3}+\frac{1}{3}\ln\frac{3}{2}$	A1	1.1b
		(4)	
			(12 marks)

Notes
(a) B1: Correct value for A M1: Complete method to find B and C e.g. substitutes values or compares coefficients. A1: Correct values (or fractions) (b) B1: Any correct equation connecting du with dx M1: Attempts to find limits in terms of u using the given substitution M1: A completely correct strategy to achieve an integral in terms of u only A1: Obtains $\frac{2}{3}\int \frac{u^2}{u^2-1}\,\mathrm{d}u$ with no errors ignoring limits A1: All correct with no errors and with correct limits (c) M1: Recognises the form for the integration for at least one of the fractions e.g. $\int \frac{A}{u+1}\,\mathrm{d}u \to k\ln(u+1)$ or $\int \frac{B}{u-1}\,\mathrm{d}u \to k\ln(u-1)$ A1: Fully correct integration M1: Substitutes both u limits and subtracts and reaches an expression of the required form A1: Correct answer

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