Pearson Edexcel

# Mark Scheme

# Mock Set 4

## Pearson Edexcel GCE In Mathematics (9MA0)
## Paper 1 Pure Mathematics

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**General Marking Guidance**

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

## EDEXCEL GCE MATHEMATICS
## General Instructions for Marking

1. The total number of marks for the paper is 100.

2. The Edexcel Mathematics mark schemes use the following types of marks:
   - **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
   - **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
   - **B** marks are unconditional accuracy marks (independent of M marks)
   - Marks should not be subdivided.

3. Abbreviations
   These are some of the traditional marking abbreviations that will appear in the mark schemes.
   - bod – benefit of doubt
   - ft – follow through
   - the symbol $\sqrt{}$ will be used for correct ft
   - cao – correct answer only
   - cso - correct solution only. There must be no errors in this part of the question to obtain this mark
   - isw – ignore subsequent working
   - awrt – answers which round to
   - SC: special case
   - oe – or equivalent (and appropriate)
   - dep – dependent
   - indep – independent
   - dp decimal places
   - sf significant figures
   - ∗ The answer is printed on the paper
   - ⌈ The second mark is dependent on gaining the first mark

4. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

5. Where a candidate has made multiple responses <u>and indicates which response they wish to submit</u>, examiners should mark this response.
   If there are several attempts at a question <u>which have not been crossed out</u>, examiners should mark the final answer which is the answer that is the <u>most complete</u>.
6. Ignore wrong working or incorrect statements following a correct answer.

7. Mark schemes will firstly show the solution judged to be the most common response expected from candidates. Where appropriate, alternatives answers are provided in the notes. If examiners are not sure if an answer is acceptable, they will check the mark scheme to see if an alternative answer is given for the method used.

## General Principles for Further Pure Mathematics Marking

(*But note that specific mark schemes may sometimes override these general principles*)

**<u>Method mark for solving 3 term quadratic:</u>**

**1. Factorisation**

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \ldots$

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \ldots$

**2. Formula**

Attempt to use the correct formula (with values for *a*, *b* and *c*)

**3. Completing the square**

Solving $x^2 + bx + c = 0$ : $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0, \quad q \neq 0$ , leading to $x = \ldots$

**<u>Method marks for differentiation and integration:</u>**

**1. Differentiation**

Power of at least one term decreased by 1. $(x^n \to x^{n-1})$

**2. Integration**

Power of at least one term increased by 1. $(x^n \to x^{n+1})$

**<u>Use of a formula</u>**

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

<u>Method mark</u> for quoting a correct formula and attempting to use it, even if there are small errors in the substitution of values.

Where the formula is <u>not</u> quoted, the method mark can be gained by implication from <u>correct</u> working with values but may be lost if there is any mistake in the working.

**<u>Exact answers</u>**

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **1(a)** | $m = \frac{5-(-2)}{7-(-3)}$ or $m = \frac{-2-5}{-3-7}$ | M1 | 1.1b |
| | $m = \frac{7}{10}$ | A1 | 1.1b |
| | | **(2)** | |
| **(b)** | $y - 5 = \frac{7}{10}(x-7)$ | M1 | 1.1b |
| | $7x - 10y + 1 = 0$ | A1 | 1.1b |
| | | **(2)** | |
| **(c)** | $m_N = -\frac{10}{7}$ | B1ft | 1.1b |
| | | **(1)** | |
| | | | **(5 marks)** |

**Notes**

(a)

M1: Correct method for the gradient

A1: Correct value $\left(\text{oe e.g. 0.7 but not } \frac{-7}{-10}\right)$

Correct answer implies both marks

(b)

M1: Correct straight line method using *A* or *B* and their *m*. For the $y = mx + c$ approach, must reach at least a value for *c* to score this mark.

A1: Correct equation in the required form (allow any integer multiple)

(c)

B1ft: For $m_N = -\frac{10}{7}$ or the negative reciprocal of their answer to part (a)

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **2(a)** | $u_2 = 4p + q$ | B1 | 1.1b |
| | | **(1)** | |
| **(b)** | $4p + q = 10$ | B1ft | 1.1b |
| | $\sum_{r=1}^{3} u_r = 42 \Rightarrow 4 + 10 + 10p + q = 42 \Rightarrow p = ..., q = ...$ | M1 | 1.1b |
| | $p = 3,\ q = -2$ | A1 | 1.1b |
| | | **(3)** | |
| | | | **(4 marks)** |

**Notes**

(a)

B1: Correct expression

(b)

B1ft: Correct equation or correct follow through equation.
i.e. their part (a) in terms of $p$ and $q$ = 10

M1: Uses $\sum_{r=1}^{3} u_r = 42$ to establish another equation connecting $p$ and $q$ and solves simultaneously with their equation from part (a) to find values for $p$ and $q$

A1: Correct values

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **3(a)** | $y > 2$ | B1 | 2.5 |
| | | **(1)** | |
| **(b)** | $\mathrm{fg}(x) = \mathrm{e}^{3\ln x} + 2 = \mathrm{e}^{\ln x^3} + 2$ | M1 | 2.1 |
| | $= x^3 + 2$ | A1 | 1.1b |
| | | **(2)** | |
| **(c)** | $y = \mathrm{e}^{3x} + 2 \Rightarrow y - 2 = \mathrm{e}^{3x} \Rightarrow 3x = \ln(y-2)$ | M1 | 1.1b |
| | $\mathrm{f}^{-1}(x) = \frac{1}{3}\ln(x-2)$ | A1 | 1.1b |
| | $x > 2$ | B1ft | 2.2a |
| | | **(3)** | |
| | | | **(6 marks)** |

**Notes**

(a)

B1: Correct range. Allow f($x$) or f for $y$. Allow e.g. $\{y \in \mathbb{R}:\ y > 2\}$, $2 < y < \infty$, $(2, \infty)$

(b)

M1: Attempts the composite function the correct way round and applies $3\ln x = \ln x^3$

A1: Correct expression (ignore any domain given)

(c)

M1: Attempts the inverse function and reaches $\alpha x = \ln(y \pm 2)$ or $\alpha y = \ln(x \pm 2)$

A1: Correct inverse (the function must be in terms of $x$ but allow $\mathrm{f}^{-1}(x) = ...$ or e.g. $y = \ldots$ but not $x = \ldots$)

B1ft: Correct domain. Follow through their answer to part (a) but must be in terms of $x$.

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **4** | e.g. $\log_2(x+2)+\log_2(x+3)=\log_2(x+2)(x+3)$ <br> $2\log_2 x=\log_2 x^2$ | B1 | 1.2 |
| | $\log_2(x+2)(x+3)-\log_2 x^2=\log_2 2$ <br> $\Rightarrow\log_2\dfrac{(x+2)(x+3)}{x^2}=\log_2 2\Rightarrow(x+2)(x+3)=2x^2$ | M1 | 2.1 |
| | $x^2-5x-6=0\Rightarrow x=-1,\ 6$ | dM1 | 1.1b |
| | $x=6$ | A1 | 2.3 |
| | | **(4)** | |
| | | | **(4 marks)** |

**Notes**

B1: Recalls at least one rule of logs correctly (but not for just $1=\log_2 2$)

M1: Applies a correct strategy including the use of $1=\log_2 2$ to remove the logs to obtain a quadratic equation in $x$

dM1: Solves the resulting 3TQ (may only see the positive root)

A1: This is for $x = 6$ only and no other values offered or not clearly rejected

| Question | Scheme | Marks | AOs |
| --- | --- | --- | --- |
| **5(a)** | $(2+3x)^6 = 2^6 + \binom{6}{1}2^5(3x) + \binom{6}{2}2^4(3x)^2 + \binom{6}{3}2^3(3x)^3 + \dots$ | M1 | 1.1b |
| | $= 64 + \dots$ | B1 | 1.1b |
| | $= \dots + 576x + 2160x^2 + 4320x^3 + \dots$ | A1<br>A1 | 1.1b<br>1.1b |
| | | **(4)** | |
| **(a) ALT** | $(2+3x)^6 =$<br>$2^6\left(1+\frac{3x}{2}\right)^6 = 2^6\left(1+\binom{6}{1}\left(\frac{3x}{2}\right)+\binom{6}{2}\left(\frac{3x}{2}\right)^2+\binom{6}{3}\left(\frac{3x}{2}\right)^3+\dots\right)$ | M1 | 1.1b |
| | $= 64 + \dots$ | B1 | 1.1b |
| | $= \dots + 576x + 2160x^2 + 4320x^3 + \dots$ | A1<br>A1 | 1.1b<br>1.1b |
| | | **(4)** | |
| **(b)** | $3\times"64"$ or $\pm\frac{1}{8}\times"4320"$ | M1 | 1.1b |
| | Coefficient of $x$ is:<br>$3\times"64"-\frac{1}{8}\times"4320"$ | M1 | 3.1a |
| | $= -348$ | A1 | 1.1b |
| | | **(3)** | |
| | | | **(7 marks)** |

**Notes**

(a)
M1: For the correct structure of one of terms 2, 3 or 4. This requires a correct binomial coefficient combined with a correct power of 2 and a correct power of ($3x$) condoning missing brackets around the '$3x$'
B1: For 64
A1: For 2 of $+576x+2160x^2+4320x^3+\dots$ (Allow terms to be listed)
A1: All 3 of $+576x+2160x^2+4320x^3+\dots$ (Allow terms to be listed)
(a) **ALT**
M1: Takes out a factor of $2^6$ together with a correct structure for one of terms 2, 3 or 4 in the bracket. This requires a correct binomial coefficient combined a correct power of $\frac{3x}{2}$
B1: For 64
A1: For 2 of $+576x+2160x^2+4320x^3+\dots$ (Allow terms to be listed)
A1: All 3 of $+576x+2160x^2+4320x^3+\dots$ (Allow terms to be listed)
(b)
M1: For attempting one correct "term". Condone sign error on 2nd term.
M1: Fully correct strategy for the required coefficient using their expansion from part (a)
A1: For −348

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **6** | Assume there exists a positive real value of $x$ for which $x+\frac{4}{x}<4$ | B1 | 2.5 |
| | $x+\frac{4}{x}<4 \Rightarrow x^2+4<4x \Rightarrow x^2-4x+4<0$ | M1 | 3.1a |
| | $x^2-4x+4<0 \Rightarrow (x-2)^2<0$ | M1 | 2.1 |
| | Which is a contradiction as $(x-2)^2 \geqslant 0$<br>So $x+\frac{4}{x} \geqslant 4$ | A1 | 2.4 |
| | | **(4)** | |
| **ALT** | Assume there exists a positive real value of $x$ for which $x+\frac{4}{x}<4$ | B1 | 2.5 |
| | $x+\frac{4}{x}<4 \Rightarrow x^2+8+\frac{16}{x^2}<16 \Rightarrow x^2-8+\frac{16}{x^2}<0$ | M1 | 3.1a |
| | $\Rightarrow x^2-8+\frac{16}{x^2}<0 \Rightarrow \left(x-\frac{4}{x}\right)^2<0$ | M1 | 2.1 |
| | Which is a contradiction as $\left(x-\frac{4}{x}\right)^2 \geqslant 0$<br>So $x+\frac{4}{x} \geqslant 4$ | A1 | 2.4 |
| | | **(4)** | |
| | | | **(4 marks)** |

**Notes (this must be a proof by contradiction)**

B1: For using correct language and notation to set up the contradiction
M1: Recognises the requirement to multiply though by $x$ and collects terms to one side
M1: Uses appropriate algebra to progress to the contradiction
A1: Full and rigorous argument using fully correct algebra with a conclusion

**ALT**

B1: For using correct language and notation to set up the contradiction
M1: Recognises the requirement to squares both sides and collects terms to one side
M1: Uses appropriate algebra to progress to the contradiction
A1: Full and rigorous argument using fully correct algebra with a conclusion

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| 7(a) | $\overrightarrow{AB} = \left(\begin{pmatrix} t \\ 2t \\ 5t \end{pmatrix} - \begin{pmatrix} 8 \\ -3 \\ 2 \end{pmatrix}\right)$ | M1 | 1.1b |
| | $\overrightarrow{AB} = \begin{pmatrix} t-8 \\ 2t+3 \\ 5t-2 \end{pmatrix} \Rightarrow \left\|\overrightarrow{AB}\right\|^2 = (t-8)^2 + (2t+3)^2 + (5t-2)^2 = ...$ | M1 | 1.1b |
| | $= t^2 - 16t + 64 + 4t^2 + 12t + 9 + 25t^2 - 20t + 4$ <br> $= 30t^2 - 24t + 77\,*$ | A1* | 2.1 |
| | | **(3)** | |
| (b)(i) | $\left\|\overrightarrow{AB}\right\|^2 = 30t^2 - 24t + 77 \Rightarrow \frac{\mathrm{d}\left\|\overrightarrow{AB}\right\|^2}{\mathrm{d}t} = 60t - 24 = 0 \Rightarrow t = ...$ <br> or <br> $\left\|\overrightarrow{AB}\right\|^2 = 30t^2 - 24t + 77 = 30\left(t^2 - \frac{4}{5}t + \frac{77}{30}\right)$ <br> $= 30\left(\left(t - \frac{2}{5}\right)^2 + \frac{361}{150}\right) \Rightarrow t = ...$ | M1 | 3.1a |
| | $t = \frac{2}{5}$ | A1 | 2.2a |
| (ii) | $\left\|\overrightarrow{AB}\right\| = \sqrt{\left(\frac{2}{5} - 8\right)^2 + \left(2\left(\frac{2}{5}\right) + 3\right)^2 + \left(5\left(\frac{2}{5}\right) - 2\right)^2}$ | dM1 | 1.1b |
| | $\left\|\overrightarrow{AB}\right\| = \frac{19\sqrt{5}}{5}$ | A1 | 1.1b |
| | | **(4)** | |
| (c) | $\overrightarrow{OC} = \overrightarrow{OA} + \frac{5}{8}\overrightarrow{AB} = \begin{pmatrix} 8 \\ -3 \\ 2 \end{pmatrix} + \frac{5}{8}\begin{pmatrix} \frac{2}{5} - 8 \\ \frac{4}{5} + 3 \\ 0 \end{pmatrix}$ <br> or <br> $\overrightarrow{OC} = \overrightarrow{OB} + \frac{3}{8}\overrightarrow{BA} = \begin{pmatrix} \frac{2}{5} \\ \frac{4}{5} \\ 2 \end{pmatrix} + \frac{3}{8}\begin{pmatrix} 8 - \frac{2}{5} \\ -3 - \frac{4}{5} \\ 0 \end{pmatrix}$ | M1 | 3.1a |
| | $\left(\frac{13}{4}, -\frac{5}{8}, 2\right)$ | A1 | 1.1b |
| | | **(2)** | |
| | | | **(9 marks)** |

**Notes**

(a)
M1: Subtracts the 2 vectors either way round
M1: Applies Pythagoras to their vector and attempts to expand brackets
A1*: Obtains the printed answer with no errors.
(b)(i)
M1: Correct strategy for finding the value for *t* e.g. calculus or completing the square

| |
|---|
| A1: Correct value<br>(ii)<br>dM1: Substitutes their value for *t* and attempts the minimum distance<br>A1: Cao<br>(c)<br>M1: Fully correct strategy to find the coordinates of *C*<br>A1: Correct coordinates (condone if given as a vector) |

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **8(a)** | $\frac{\sin 2\theta + \sin\theta}{\cos 2\theta + \cos\theta + 1} \equiv \frac{2\sin\theta\cos\theta + \sin\theta}{\ldots}$ **or** $\frac{\ldots}{2\cos^2\theta - 1 + \cos\theta + 1}$ | M1 | 1.1b |
| | $\frac{\sin 2\theta + \sin\theta}{\cos 2\theta + \cos\theta + 1} \equiv \frac{2\sin\theta\cos\theta + \sin\theta}{2\cos^2\theta - 1 + \cos\theta + 1}$ | M1 | 2.1 |
| | $\equiv \frac{\sin\theta(2\cos\theta + 1)}{\cos\theta(2\cos\theta + 1)} = \tan\theta$* | A1* | 2.2a |
| | | **(3)** | |
| **(b)** | $\frac{\sin 2x + \sin x}{\cos 2x + \cos x + 1} = 2\cos x \Rightarrow \tan x = 2\cos x$<br>$\Rightarrow \sin x = 2\cos^2 x = 2(1 - \sin^2 x)$ | M1 | 3.1a |
| | $\Rightarrow 2\sin^2 x + \sin x - 2 = 0$ | A1 | 1.1b |
| | $\sin x = \frac{-1 \pm \sqrt{17}}{4}(0.780..., -1.280...) \Rightarrow x = ...$ | dM1 | 1.1b |
| | $x = 0.896,\ 2.246$ | A1<br>A1 | 1.1b<br>1.1b |
| | | **(5)** | |
| | | **(8 marks)** | |

**Notes**

(a)
M1: Applies sin2$\theta$ = 2sin$\theta$cos$\theta$ in the numerator **or** cos2$\theta$ = 2cos$^2\theta$ – 1 in the denominator
M1: Applies sin2$\theta$ = 2sin$\theta$cos$\theta$ in the numerator **and** cos2$\theta$ = 2cos$^2\theta$ – 1 in the denominator
A1*: Factorises to show cancelling and reaches tan $\theta$ with no errors
(b)
M1: Makes the connection with part (a), multiplies by cos $x$, applies $\cos^2 x = 1 - \sin^2 x$ and collects terms to obtain a 3TQ in sin $x$
A1: Correct 3TQ
dM1: Solves a 3TQ in sin $x$ and obtains at least one value for $x$
A1: Awrt one correct value
A1: Both correct, allowing awrt, and no other values in range

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| 9(a) | $t=0,\ \theta=80 \Rightarrow A=59$ | B1 | 2.2a |
| | $t=40,\ \theta=33 \Rightarrow 33=21+59\mathrm{e}^{-40k} \Rightarrow \mathrm{e}^{-40k}=\dfrac{12}{59}$ <br> $\Rightarrow -40k=\ln\left(\dfrac{12}{59}\right) \Rightarrow k=...$ | M1 | 3.1b |
| | $\Rightarrow k=-\dfrac{1}{40}\ln\left(\dfrac{12}{59}\right)\ \ (=0.0398)$ | A1 | 1.1b |
| | $\theta=21+59\mathrm{e}^{-0.0398t}$ | A1 | 3.3 |
| | | **(4)** | |
| (b) | $t=20 \Rightarrow \theta=21+59\mathrm{e}^{-20\times 0.0398}=...(47.6)$ | M1 | 3.4 |
| | This suggests that the model is appropriate because $47.6 \approx 48$ | A1ft | 3.5a |
| | | **(2)** | |
| (b) Alt | $\theta=48 \Rightarrow 48=21+59\mathrm{e}^{-20\times 0.0398} \Rightarrow \mathrm{e}^{-0.0398t}=\dfrac{27}{69} \Rightarrow t=...(19.6)$ | M1 | 3.4 |
| | This suggests that the model is appropriate because $19.6 \approx 20$ | A1ft | 3.5a |
| | | **(2)** | |
| (c) | $\theta=21+59\mathrm{e}^{-0.0398t} \Rightarrow \dfrac{\mathrm{d}\theta}{\mathrm{d}t}=-0.0398\times 59\mathrm{e}^{-0.0398t}=...$ | B1ft | 2.2a |
| | $\Rightarrow \dfrac{\mathrm{d}\theta}{\mathrm{d}t}=-0.0398\times 59\mathrm{e}^{-0.0398\times 20}=...$ | M1 | 1.1b |
| | Decreasing at a rate of $1.06\,°\mathrm{C}$ per minute | A1 | 3.2a |
| | | **(3)** | |
| | | | **(9 marks)** |

**Notes**

(a)
B1: Uses $t=0,\ \theta=80$ to deduce the correct value for $A$
M1: Uses the given equation for the model and $t=40,\ \theta=33$ and correct log work to establish a value for $k$
A1: Correct value for $k$ (exact or awrt 0.0398)
A1: Correct **equation** (allow exact $k$ or awrt 0.0398)
(b)
M1: Uses the model with their values to find the temperature after 20 minutes
A1ft: Compares their value with 48 and makes a correct interpretation
**Alt:**
M1: Uses the model with their values to find $t$ when $\theta = 48$
A1ft: Compares their value with 20 and makes a correct interpretation
(c)
B1ft: Deduces the correct derivative. Follow through their values.
M1: Substitutes $t = 20$ into an expression of the form $\beta\mathrm{e}^{-kt}$ to establish the required rate.
A1: Correct rate including units

| Question | Scheme | Marks | AOs |
| --- | --- | --- | --- |
| **10(a)** | $k = 20 \Rightarrow \frac{8-2k}{4+k} = \frac{-32}{24}$ which is $< -1$ so the series does not converge | B1 | 2.4 |
| | | **(1)** | |
| **(b)** | $\frac{8-2k}{4+k} < 1 \Rightarrow k > ...$ or $\frac{8-2k}{4+k} > -1 \Rightarrow k < ...$ | M1 | 1.1b |
| | $\frac{8-2k}{4+k} < 1 \Rightarrow k > ...$ and $\frac{8-2k}{4+k} > -1 \Rightarrow k < ...$ | M1 | 2.1 |
| | $k > \frac{4}{3}$ or $k < 12$ | A1 | 1.1b |
| | $\frac{4}{3} < k < 12$ | A1 | 2.2a |
| | | **(4)** | |
| | | | **(5 marks)** |

**Notes**

(a)

B1: Correct explanation

(b)

M1: Attempts to solve either $\frac{8-2k}{4+k} < 1$ or $\frac{8-2k}{4+k} > -1$ to obtain one bound

M1: Attempts to solve both $\frac{8-2k}{4+k} < 1$ and $\frac{8-2k}{4+k} > -1$ to obtain both bounds

A1: One correct

A1: Deduces the correct range for $k$

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **11(a)(i)** | $74 = ab^{10},\ 198 = ab^{14} \Rightarrow \dfrac{198}{74} = b^4 \Rightarrow b = \sqrt[4]{\dfrac{198}{74}}$ | M1 | 3.1a |
| | $b = 1.279$ | A1 | 1.1b |
| **(a)(ii)** | $74 = ab^{10} \Rightarrow a = \dfrac{74}{b^{10}}$ or $198 = ab^{14} \Rightarrow a = \dfrac{198}{b^{14}}$ | dM1 | 3.4 |
| | $a = 6.3$ | A1 | 1.1b |
| | | **(4)** | |
| **(b)(i)** | *a* is the energy output in 1996 | B1 | 3.4 |
| **(b)(ii)** | *b* is the factor by which the energy output increases each year | B1 | 3.4 |
| | | **(2)** | |
| **(c)** | $E = 6.3 \times 1.279^{29} = \ldots$ | M1 | 3.4 |
| | $= 7917.46\ldots \quad (\text{GW})$ | A1 | 1.1b |
| | | **(2)** | |
| | | | **(8 marks)** |

**Notes**

(a)(i)
M1: Forms 2 equations in *a* and *b* and solves to obtain a value for *b*.
A1: $b = 1.279$
(a)(ii)
dM1: Uses either equation and their value for *b* to find a value for *a*
A1: $a = 6.3$
(b)(i)
B1: Correct interpretation for the constant *a*
(b)(ii)
B1: Correct interpretation for the constant *b*
(c)
M1: Uses their values of *a* and *b* with $t = 29$ in the equation for the model to obtain a value
A1: Correct value. Allow values between awrt 7920 to awrt 7935 following correct work.

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **12(a)** | $\int x^2 e^{x^3}\,dx = \frac{1}{3}e^{x^3}\ (+c)$ | M1<br>A1 | 1.1b<br>1.1b |
| | | **(2)** | |
| **(b)** | $u = x^3 \Rightarrow \frac{du}{dx} = 3x^2 \Rightarrow \int x^8 e^{x^3}\,dx = \int \frac{x^8 e^u}{3x^2}\,du = \frac{1}{3}\int u^2 e^u\,du$ | M1 | 3.1a |
| | $\frac{1}{3}\int u^2 e^u\,du = \frac{1}{3}u^2 e^u - \frac{2}{3}\int u e^u\,dx$ | M1<br>A1 | 2.1<br>1.1b |
| | $= \frac{1}{3}u^2 e^u - \frac{2}{3}u e^u + \frac{2}{3}\int e^u\,du$ | M1 | 1.1b |
| | $= \frac{1}{3}u^2 e^u - \frac{2}{3}u e^u + \frac{2}{3}e^u + c = \frac{1}{3}x^6 e^{x^3} - \frac{2}{3}x^3 e^{x^3} + \frac{2}{3}e^{x^3} + c$<br>$= \frac{1}{3}e^{x^3}\left(x^6 - 2x^3 + 2\right) + c$ | A1 | 2.1 |
| | | **(5)** | |
| | **Alternative:** | | |
| | $\int x^8 e^{x^3}\,dx = \int x^6 x^2 e^{x^3}\,dx$ | M1 | 3.1a |
| | $\int x^6 x^2 e^{x^3}\,dx = \frac{1}{3}x^6 e^{x^3} - 2\int x^5 e^{x^3}\,dx$ | M1<br>A1 | 2.1<br>1.1b |
| | $\int x^6 x^2 e^{x^3}\,dx = \frac{1}{3}x^6 e^{x^3} - 2\int x^3 x^2 e^{x^3}\,dx$<br>$= \frac{1}{3}x^6 e^{x^3} - 2\left[\frac{1}{3}x^3 e^{x^3} - \int x^2 e^{x^3}\,dx\right]$ | M1 | 1.1b |
| | $= \frac{1}{3}x^6 e^{x^3} - \frac{2}{3}x^3 e^{x^3} + \frac{2}{3}e^{x^3} + c = \frac{1}{3}e^{x^3}\left(x^6 - 2x^3 + 2\right) + c$ | A1 | 2.1 |
| | | | **(7 marks)** |

**Notes**

(a)

M1: For $\int x^2 e^{x^3}\,dx = ke^{x^3}\ (+c)$

A1: Correct integration (condone omission of $+ c$)

(b)

M1: Fully correct strategy for the substitution to reach an integral in terms of $u$ only.

M1: Applies integration by parts in the correct direction on $u^2 e^u$

A1: Correct integral for the first application of parts

M1: Applies parts again on $u e^u$

A1: Completes the process and obtains the correct answer in the form required

**Alt:**

M1: Makes the key step of writing $x^8$ as $x^6 \times x^2$

M1: Applies integration by parts in the correct direction

A1: Correct integral for the first application of parts

M1: Applies integration by parts again, in the correct direction, after writing $x^5$ as $x^3 \times x^2$

A1: Completes the process and obtains the correct answer in the form required

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| 13(a) | $\dfrac{1}{x(100-x)} \equiv \dfrac{P}{x} + \dfrac{Q}{100-x} \Rightarrow P = \ldots,\ Q = \ldots$ | M1 | 1.1b |
| | $\dfrac{1}{x(100-x)} \equiv \dfrac{1}{100x} + \dfrac{1}{100(100-x)}$ | A1 | 1.1b |
| | | (2) | |
| (b) | $500\dfrac{dx}{dt} = x(100-x) \Rightarrow \int \dfrac{500}{x(100-x)}dx = \int dt$ or $\int \dfrac{1}{x(100-x)}dx = \int \dfrac{1}{500}dt$ <br> $\Rightarrow 5\int\left(\dfrac{1}{x}+\dfrac{1}{100-x}\right)dx = \int dt$ or e.g. $\int\left(\dfrac{1}{x}+\dfrac{1}{100-x}\right)dx = \int \dfrac{1}{5}dt$ | M1 | 2.1 |
| | $5\ln x - 5\ln(100-x) = t + c$ | M1<br>A1ft | 3.1a<br>1.1b |
| | $x = 5,\ t = 0 \Rightarrow 5\ln\dfrac{1}{19} = c$ | M1 | 3.4 |
| | $5\ln x - 5\ln(100-x) = t + 5\ln\dfrac{1}{19} \Rightarrow 5\ln\dfrac{x}{100-x} = t + 5\ln\dfrac{1}{19}$ <br> $\Rightarrow \ln\dfrac{x}{100-x} = \dfrac{t}{5} + \ln\dfrac{1}{19} \Rightarrow \dfrac{x}{100-x} = \dfrac{1}{19}e^{\frac{t}{5}} \Rightarrow x = \ldots$ | M1 | 2.1 |
| | $x = \dfrac{100}{1+19e^{-\frac{1}{5}t}}$ | A1 | 1.1b |
| | | (6) | |
| (c) | $x = \dfrac{100}{1+19e^{-\frac{1}{5}\times 10}} = \ldots$ | M1 | 3.4 |
| | $x = 28\ (\text{m}^2)$ | A1 | 1.1b |
| | | (2) | |
| | | | **(10 marks)** |

**Notes**

(a)

M1: Correct method of partial fractions to find values for *P* and *Q*. May be implied by correct values or correct fractions.

A1: Correct partial fractions

(b)

M1: Separates the variables and uses the result from part (a)

M1: Correct attempt at the integration.

Look for $\alpha\ln x + \beta\ln(100-x) = t$ or equivalent

A1ft: Correct integration for their PFs of the form $\dfrac{A}{x} + \dfrac{B}{100-x}$ (condone omission of + *c*)

M1: Uses the conditions in the model of $x = 5$, $t = 0$ to find their constant of integration

M1: Uses correct processing to make *x* the subject to reach an expression of the required form

A1: Correct expression

(c)

M1: Uses their equation with $t = 10$ to find a value for *x*

A1: $x = 28.00045\ldots$ awrt 28

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| 14(a) | $x=\frac{\pi\sqrt{3}}{12}$ **or** $y=\frac{2}{3}$ | B1 | 1.1b |
| | $x=\frac{\pi\sqrt{3}}{12}$ **and** $y=\frac{2}{3}$ | B1 | 1.1b |
| | | **(2)** | |
| (b) | $\frac{dx}{d\theta}=\frac{1}{2}\sin\theta+\frac{1}{2}\theta\cos\theta$ | M1 | 1.1b |
| | Area under $C=\int y\frac{dx}{d\theta}d\theta=\int\frac{1}{3}\sec\theta\left(\frac{1}{2}\sin\theta+\frac{1}{2}\theta\cos\theta\right)d\theta$ | M1<br>A1 | 2.1<br>1.1b |
| | Area required is $\frac{\pi\sqrt{3}}{12}\times\frac{2}{3}-\frac{1}{6}\int(\tan\theta+\theta)d\theta$ | ddM1 | 3.1a |
| | $=\frac{1}{18}\pi\sqrt{3}-\frac{1}{6}\int_0^{\frac{\pi}{3}}(\tan\theta+\theta)d\theta$ | A1 | 2.1 |
| | | **(5)** | |
| (c) | $\int(\tan\theta+\theta)d\theta=\ln(\sec\theta)+\frac{\theta^2}{2}(+c)$ | B1 | 2.2a |
| | Area $=\frac{1}{18}\pi\sqrt{3}-\frac{1}{6}\left[\ln(\sec\theta)+\frac{\theta^2}{2}\right]_0^{\frac{\pi}{3}}$<br>$=\frac{1}{18}\pi\sqrt{3}-\frac{1}{6}\left(\ln\left(\sec\frac{\pi}{3}\right)+\frac{\pi^2}{18}-(0)\right)$ | M1 | 2.1 |
| | $=\frac{1}{18}\pi\sqrt{3}-\frac{1}{6}\ln 2-\frac{\pi^2}{108}$ | A1 | 1.1b |
| | | **(3)** | |
| | | **(10 marks)** | |

**Notes**

(a)
B1: One correct coordinate
B1: Both correct coordinates
(b)
M1: Applies the product rule to the *x* parameter to obtain $\frac{dx}{d\theta}=\alpha\sin\theta+\beta\theta\cos\theta$
M1: Applies $\int y\frac{dx}{d\theta}d\theta$ for the area under the curve
A1: Correct integral
ddM1: Fully correct strategy for the area of *R*.
A1: Fully correct expression in the form required
(c)
B1: Deduces the correct expression for $\int(\tan\theta+\theta)d\theta$
M1: Completes the problem by applying the correct limits
A1: Correct expression

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **15(a)** | $y=\arccos\frac{1}{2}x \Rightarrow \cos y=\frac{1}{2}x \Rightarrow x=2\cos y$<br>$y=\arcsin x \Rightarrow \sin y = x$ | M1 | 1.1b |
| | $\sin y = 2\cos y \Rightarrow \tan y = 2*$<br>or<br>$\sin y = x,\ 2\cos y = x \Rightarrow \tan y = \frac{x}{x/2} = 2*$ | A1* | 2.1 |
| | | | **(2)** |
| **(b)** | $\tan y = 2 \Rightarrow x = \sin y = \frac{2}{\sqrt{5}}$<br>or<br>$\tan y = 2 \Rightarrow x = 2\cos y = \frac{2}{\sqrt{5}}$<br>or<br>$\tan y = 2 \Rightarrow \tan^2 y = 4 \Rightarrow \sec^2 y = 5 \Rightarrow \cos y = \frac{1}{\sqrt{5}} \Rightarrow x = \frac{2}{\sqrt{5}}$<br>or<br>$\tan y = 2 \Rightarrow \cot^2 y = \frac{1}{4} \Rightarrow \text{cosec}^2 y = \frac{5}{4} \Rightarrow \sin y = \frac{2}{\sqrt{5}} = x$ | M1<br>A1 | 3.1a<br>2.2a |
| | | | **(2)** |
| | | | **(4 marks)** |

**Notes**

(a)

M1: Uses inverse trigonometric functions to express $x$ in terms of $\cos y$ and $x$ in terms of $\sin y$

A1*: Sets the $x$'s equal and rearranges to the given answer or uses $\sin y$ and $\cos y$ in terms of $x$ and applies $\tan y = \frac{\sin y}{\cos y}$ to obtain the given answer.

(b)

M1: Uses Pythagoras directly or trigonometric identities to establish the value of $x$

A1: Correct exact value (accept any exact equivalent e.g. $\frac{2\sqrt{5}}{5}$)

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