Please check the examination details below before entering your candidate information

Candidate surname	Other names

Pearson Edexcel Level 3 GCE

Centre Number	Candidate Number

Mock Paper Set 3

Time: 2 hours | Paper Reference **9MA0/01**

Mathematics

Advanced

Paper 1: Pure Mathematics 1

You must have:
Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 14 questions in this question paper. The total mark for this paper is 100.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

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Pearson

1. Curve C has equation

$$y = x^3 - 7x^2 + 5x + 4$$

(a) Find $\frac{dy}{dx}$

(2)

The point P (2, –6) lies on C

(b) Find the equation of the tangent to C at P
Give your answer in the form $y = mx + c$ where m and c are integers to be found.

(3)

Question 1 continued

(Total for Question 1 is 5 marks)

2. $f(x) = 3x^3 - 7x^2 + 7x - 10$

(a) Use the factor theorem to show that $(x - 2)$ is a factor of $f(x)$

(2)

(b) Find the values of the constants a, b and c such that

$$f(x) \equiv (x - 2)(ax^2 + bx + c)$$

(3)

(c) Using your answer to part (b) show that the equation $f(x) = 0$ has only one real root.

(2)

Question 2 continued

(Total for Question 2 is 7 marks)

3.

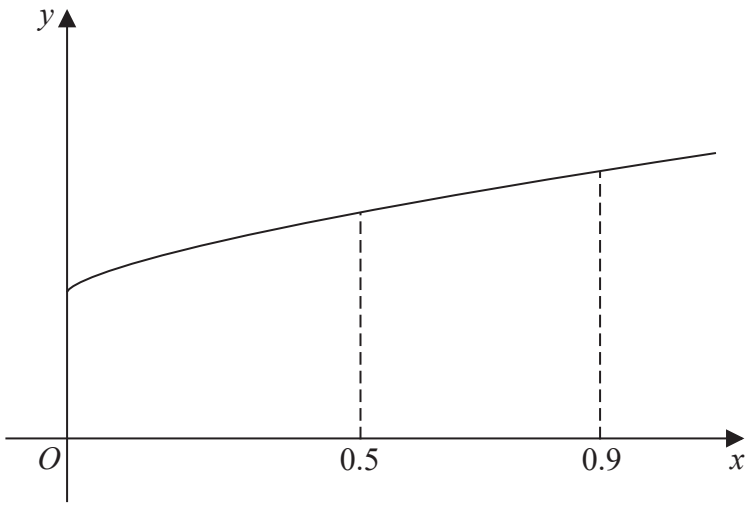

Figure 1

Figure 1 shows a sketch of part of the curve with equation $y = f(x)$

The table below shows corresponding values of x and y for this curve between $x = 0.5$ and $x = 0.9$

The values of y are given to 4 significant figures.

x	0.5	0.6	0.7	0.8	0.9
y	1.632	1.711	1.786	1.859	1.930

(a) Use the trapezium rule, with all the values of y in the table, to find an estimate for

$$\int_{0.5}^{0.9} f(x)\,dx$$

Give your answer to 3 significant figures.

(3)

(b) Using your answer to part (a), deduce an estimate for

$$\int_{0.5}^{0.9} (3f(x) + 2)\,dx$$

(3)

Question 3 continued

(Total for Question 3 is 6 marks)

4. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

(a) Express as an integral

$$\lim_{\delta x \to 0} \sum_{x=4}^{12} (1 + 2x)^{\frac{1}{2}} \delta x$$

(1)

(b) Using your answer to part (a) show that

$$\lim_{\delta x \to 0} \sum_{x=4}^{12} (1 + 2x)^{\frac{1}{2}} \delta x = \frac{98}{3}$$

(3)

Question 4 continued

(Total for Question 4 is 4 marks)

5. The functions f and g are defined by

$$f(x) = \frac{kx}{2x - 1} \qquad x \in \mathbb{R} \qquad x \neq \frac{1}{2}$$

$$g(x) = 2 + 3x - x^2 \qquad x \in \mathbb{R}$$

where k is a non-zero constant.

(a) Find in terms of k

(i) fg(4)

(ii) the range of f

(iii) f^{-1}

(6)

Given that

$$f^{-1}(2) = \frac{11}{3g(2)}$$

(b) find the exact value of k

(2)

Question 5 continued

(Total for Question 5 is 8 marks)

6. $$f(x) = \frac{10}{\sqrt{4 - 3x}}$$

(a) Show that the first 4 terms in the binomial expansion of $f(x)$, in ascending powers of x, are

$$A + Bx + Cx^2 + \frac{675}{1024}x^3$$

where A, B and C are constants to be found. Give each constant in simplest form.

(4)

Given that this expansion is valid for $|x| < k$

(b) state the largest value of k.

(1)

By substituting $x = \frac{1}{3}$ into $f(x)$ and into the answer for part (a),

(c) find an approximation for $\sqrt{3}$

Give your answer in the form $\frac{a}{b}$ where a and b are integers to be found.

(2)

Question 6 continued

(Total for Question 6 is 7 marks)

7. Curve C has equation

$$y = (x^2 - 5x + 8)e^{x^2} \qquad x \in \mathbb{R}$$

(a) Show that

$$\frac{dy}{dx} = (2x^3 - 10x^2 + 18x - 5)e^{x^2}$$

(3)

Given that

- C has only one stationary point
- the stationary point has x coordinate α
- $\frac{dy}{dx} \approx -0.5$ at $x = 0.3$
- $\frac{dy}{dx} \approx 0.9$ at $x = 0.4$

(b) explain why $0.3 < \alpha < 0.4$

(1)

(c) Show that α is a solution of the equation

$$x = \frac{5(2x^2 + 1)}{2(x^2 + 9)}$$

(3)

(d) Using the iteration formula

$$x_{n+1} = \frac{5(2x_n^2 + 1)}{2(x_n^2 + 9)} \qquad \text{with } x_1 = 0.3$$

find

(i) the value of x_3 to 4 decimal places,

(ii) the value of α to 4 decimal places.

(3)

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Question 7 continued

Question 7 continued

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Question 7 continued

(Total for Question 7 is 10 marks)

8. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

(a) Show that

$$\frac{1-\cos 2\theta}{\sin^2 2\theta} \equiv k\sec^2\theta \qquad \theta \neq \frac{n\pi}{2} \qquad n \in \mathbb{Z}$$

where k is a constant to be found.

(3)

(b) Hence solve, for $-\frac{\pi}{2} < x < \frac{\pi}{2}$

$$\frac{1-\cos 2x}{\sin^2 2x} = (1+2\tan x)^2$$

Give your answers to 3 significant figures where appropriate.

(4)

Question 8 continued

Question 8 continued

Question 8 continued

(Total for Question 8 is 7 marks)

9. The vertical speed, $v\,\text{m s}^{-1}$, of a skydiver, t seconds after their parachute opened, is modelled by the equation

$$v = A + Be^{-0.5t}$$

where A and B are constants.

Given that the vertical speed of the skydiver was

- $56\,\text{m s}^{-1}$ at the instant the parachute opened
- $10\,\text{m s}^{-1}$ exactly 5 seconds after the parachute opened

(a) find a complete equation for the model.
Give the values of A and B to 3 significant figures.

(4)

Given also that the skydiver eventually descended safely to the ground at a constant vertical speed of $6\,\text{m s}^{-1}$

(b) evaluate the model.

(2)

Question 9 continued

Question 9 continued

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Question 9 continued

(Total for Question 9 is 6 marks)

10. (a) Sketch the graph with equation

$$y = |3x - 2a|$$

where a is a positive constant.
State the coordinates of each point where the graph cuts or meets the coordinate axes.

(2)

(b) Solve, in terms of a, the inequality

$$|3x - 2a| \leqslant x + a$$

(4)

Given that $|3x - 2a| \leqslant x + a$

(c) find, in terms of a, the range of possible values of $g(x)$, where

$$g(x) = 5a - \left|\frac{1}{2}a - x\right|$$

(3)

Question 10 continued

Question 10 continued

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Question 10 continued

(Total for Question 10 is 9 marks)

11. The mean yearly concentration, C parts per million (ppm), of carbon dioxide in the Earth's atmosphere was first measured in 1960.

The equation

$$C = ab^t \qquad \text{where } a \text{ and } b \text{ are constants}$$

models the mean yearly concentration of carbon dioxide t years after 1960.

Given that the mean yearly concentration of carbon dioxide was

- 339 ppm in 1980
- 414 ppm in 2020

(a) (i) find the value of b to 3 decimal places,

(ii) find the value of a to the nearest integer.

(4)

(b) With reference to the model,

(i) interpret the value of a,

(ii) interpret the value of b.

(2)

Using the model,

(c) find the year when the mean yearly concentration of carbon dioxide is predicted to reach 450 ppm.

(*Solutions based entirely on calculator technology are not acceptable*.)

(4)

Question 11 continued

Question 11 continued

Question 11 continued

(Total for Question 11 is 10 marks)

12. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

(a) Show that

$$\int x \sin kx \, dx = \frac{1}{k^2} \sin kx - \frac{1}{k} x \cos kx + c$$

where k is a constant and where c is an arbitrary constant.

(3)

A theme park ride lasts for 70 seconds.
The height above ground, H metres, of a passenger on the theme park ride is modelled by the differential equation

$$\frac{dH}{dt} = \frac{t \sin\left(\frac{\pi t}{5}\right)}{10H} \qquad 0 \leqslant t \leqslant 70$$

where t seconds is the time from the start of the ride.

Given that the passenger is 5 m above ground at the start of the ride

(b) find the height above ground of the passenger 52 seconds after the start of the ride.

(6)

Question 12 continued

Question 12 continued

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Question 12 continued

(Total for Question 12 is 9 marks)

13. Given that

$$y^2 - x^2 = 8$$

show that

$$\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} = \frac{8}{y^3}$$

(5)

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Question 13 continued

Question 13 continued

Question 13 continued

(Total for Question 13 is 5 marks)

14. (i) Prove that the sum of the squares of 2 consecutive odd integers is always 2 more than a multiple of 8

(3)

(ii) Use proof by contradiction to show that $\log_2 5$ is irrational.

(4)

Question 14 continued

Question 14 continued

(Total for Question 14 is 7 marks)

(TOTAL FOR PAPER IS 100 MARKS)

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