Please check the examination details below before entering your candidate information

Candidate surname	Other names

Pearson Edexcel Level 3 GCE

Centre Number

Candidate Number

Mock Paper Set 3

Time: 2 hours

Paper Reference **9MA0/02**

Mathematics

Advanced

Paper 2: Pure Mathematics 2

You must have:
Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need*.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 15 questions in this question paper. The total mark for this paper is 100.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

Pearson

1. Find

$$\int (x^4 - 6x^2 + 7)\,dx$$

giving your answer in simplest form.

(3)

Question 1 continued

(Total for Question 1 is 3 marks)

2. The sequence $u_1, u_2, u_3, \ldots$ is defined by

$$u_1 = k$$
$$u_{n+1} = 3u_n - 2$$

where k is a constant.

(a) Find, in simplest form in terms of k,

(i) u_2

(ii) u_3

(2)

Given that $\sum_{r=1}^{4} u_r = 44$

(b) find the value of k.

(3)

Question 2 continued

(Total for Question 2 is 5 marks)

3. A student was asked to solve the simultaneous equations

$$x + y = 9$$
$$x^2 - 3xy + 2y^2 = 0$$

The student's solution is shown below:

line 1: $x + y = 9 \Rightarrow y = 9 - x$

line 2: $x^2 - 3xy + 2y^2 = 0 \Rightarrow x^2 - 3x(9 - x) + 2(9 - x)^2 = 0$

line 3: $x^2 - 27x - 3x^2 + 162 - 36x + 2x^2 = 0$

line 4: $63x = 162$

line 5: $x = \dfrac{162}{63} \Rightarrow y = 9 - \dfrac{162}{63} = \dfrac{45}{7}$

(a) Identify the error in line 3 of the solution.

(1)

(b) Using algebra and showing all your working, solve the simultaneous equations.

(4)

Question 3 continued

(Total for Question 3 is 5 marks)

4. Curve C has equation

$$y = (x + k)(2 - x)$$

where k is a constant and $k > 2$

(a) Sketch C, showing the coordinates of any points of intersection with the coordinate axes.

(3)

(b) Find, in simplest form in terms of k, the coordinates of the stationary point of C.

(3)

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Question 4 continued

(Total for Question 4 is 6 marks)

5. Relative to a fixed origin O,

- the point A has position vector $5\mathbf{i} + 3\mathbf{j} - 2\mathbf{k}$
- the point B has position vector $7\mathbf{i} + \mathbf{j} + 2\mathbf{k}$
- the point C has position vector $4\mathbf{i} + 8\mathbf{j} - 3\mathbf{k}$

(a) Find $|\overrightarrow{AB}|$ giving your answer as a simplified surd.

(2)

Given that $ABCD$ is a parallelogram,

(b) find the position vector of the point D.

(2)

The point E is positioned such that

- ACE is a straight line
- $AC : CE = 2 : 1$

(c) Find the coordinates of the point E.

(2)

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Question 5 continued

(Total for Question 5 is 6 marks)

6.

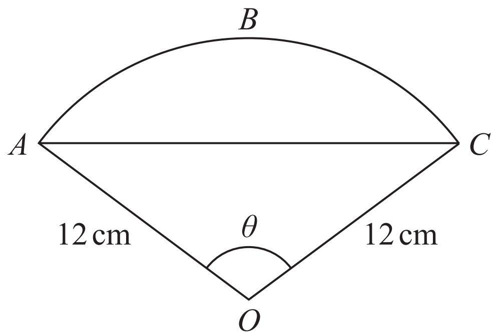

Diagram **NOT** accurately drawn

Figure 1

Figure 1 shows a sector $OABCO$ of a circle centre O.

Given that

- $OA = OC = 12\,\text{cm}$
- angle $AOC = \theta$ radians
- area triangle OAC : area segment $ABC = 3:1$

(a) show that

$$3\theta - 4\sin\theta = 0$$

(2)

(b) Taking 1.2 as a first approximation to θ, apply the Newton-Raphson method once to

$$f(\theta) = 3\theta - 4\sin\theta$$

to find a second approximation to θ

Give your answer to 3 decimal places.

(3)

Question 6 continued

(Total for Question 6 is 5 marks)

7. A ball is released from rest from a height of 5 m and bounces repeatedly on horizontal ground.

After hitting the ground for the first time, the ball rises to a maximum height of 3 m.

In a model for the motion of the ball

- the maximum height after each bounce is 60% of the previous maximum height
- the motion takes place in a vertical line

(a) Using the model

(i) show that the maximum height after the 3rd bounce is 1.08 m,

(ii) find the total distance the ball travels from release to when the ball hits the ground for the 5th time.

(3)

According to the model, after the ball is released, there is a limit, D metres, to the total distance the ball will travel.

(b) Find the value of D

(2)

With reference to the model,

(c) give a reason why, in reality, the ball will not travel D metres in total.

(1)

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Question 7 continued

(Total for Question 7 is 6 marks)

8. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

(a) Express $3\cos x + \sin x$ in the form $R\cos(x - \alpha)$ where

- R and α are constants
- $R > 0$
- $0 < \alpha < \dfrac{\pi}{2}$

Give the exact value of R and the value of α in radians to 3 decimal places.

(3)

The temperature, θ°C, inside a rabbit hole on a particular day is modelled by the equation

$$\theta = 6.5 + 3\cos\left(\frac{\pi t}{13} - 4\right) + \sin\left(\frac{\pi t}{13} - 4\right) \qquad 0 \leqslant t < 24$$

where t is the number of hours after midnight.

Using the equation of the model and your answer to part (a)

(b) (i) deduce the minimum value of θ during this day,

(ii) find the time of day when this minimum value occurs, giving your answer to the nearest minute.

(4)

(c) Find the rate of temperature increase in the rabbit hole at midday.

(2)

Question 8 continued

Question 8 continued

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Question 8 continued

(Total for Question 8 is 9 marks)

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Turn over

9. The function f is defined by

$$\mathrm{f}(x) = \frac{(x+5)(x+1)}{(x+4)} - \ln(x+4) \qquad x \in \mathbb{R} \qquad x > k$$

(a) State the smallest possible value of k.

(1)

(b) Show that

$$\mathrm{f}'(x) = \frac{ax^2 + bx + c}{(x+4)^2}$$

where a, b and c are integers to be found.

(4)

(c) Hence show that f is an increasing function.

(2)

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Question 9 continued

Question 9 continued

Question 9 continued

(Total for Question 9 is 7 marks)

10.

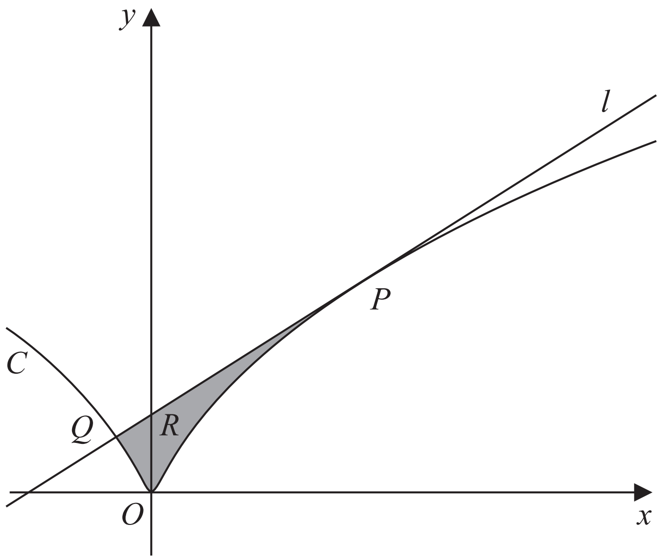

Figure 2

The curve C shown in Figure 2 has parametric equations

$$x = t^3 + 3t \qquad y = 3t^2 \qquad -2 < t < 4$$

The point P lies on C where $t = 3$

(a) Write down the coordinates of P

(1)

The line l is the tangent to C at P as shown in Figure 2.

(b) Use calculus to show that an equation for l is

$$3x - 5y + 27 = 0$$

(3)

The line l meets C again at the point Q

(c) Using algebra and showing all stages of your working, find the coordinates of Q

(3)

The finite region R, shown shaded in Figure 2, is bounded by the curve C and the line l

(d) Using algebraic integration, find the exact area of R

(5)

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Question 10 continued

Question 10 continued

Question 10 continued

(Total for Question 10 is 12 marks)

11. The number of bees in a colony is monitored over time.

There were 3 500 bees in the colony when monitoring began.
After 1 week there were only 2 000 bees in the colony.

In a simple model, the rate of decrease in the number of bees is assumed to be proportional to the square of the number of bees.

Given that there are x **thousand** bees in the colony t weeks after monitoring began,

(a) form and solve a differential equation to show that an equation of the model is

$$x = \frac{14}{3t + 4}$$

(6)

There are only 500 bees in the colony T weeks after monitoring began.

(b) Use the equation of the model to find T

(2)

Question 11 continued

Question 11 continued

Question 11 continued

(Total for Question 11 is 8 marks)

12. Given that

$$y = a^x$$

where a is a positive constant

(a) prove that

$$\frac{dy}{dx} = a^x \ln a$$

(3)

(b) Hence show that

$$\int_1^2 4^x \, dx = k(\ln 2)^n$$

where k and n are integers to be found.

(3)

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Question 12 continued

(Total for Question 12 is 6 marks)

13. The resting metabolic rate, R ml of oxygen consumed per hour, of a particular species of mammal is modelled by the formula,

$$R = aM^b$$

where

- M grams is the mass of the mammal
- a and b are constants

(a) Show that this relationship can be written in the form

$$\log_{10} R = b\log_{10} M + \log_{10} a$$

(2)

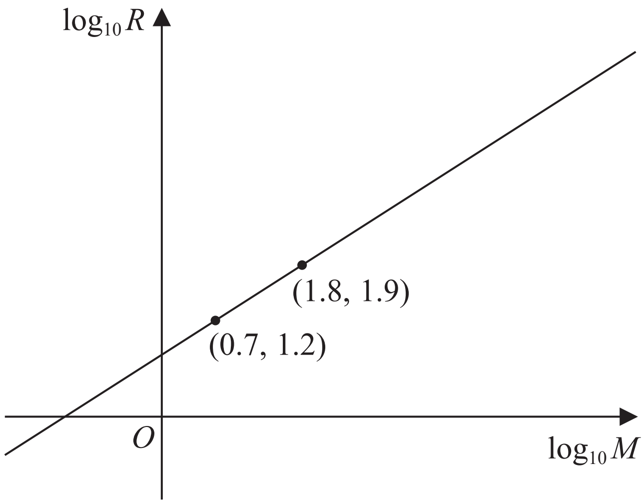

Figure 3

A student gathers data for R and M and plots a graph of $\log_{10} R$ against $\log_{10} M$

The graph is a straight line passing through points (0.7, 1.2) and (1.8, 1.9) as shown in Figure 3.

(b) Using this information, find a complete equation for the model.
Write your answer in the form

$$R = aM^b$$

giving the value of each of a and b to 3 significant figures.

(3)

(c) With reference to the model, interpret the value of the constant a

(1)

Question 13 continued

Question 13 continued

Question 13 continued

(Total for Question 13 is 6 marks)

14. (a) Use the substitution $u = 1 + \sin^2 x$ to show that

$$\int_0^{\frac{\pi}{6}} \frac{8\tan x}{1+\sin^2 x}\,dx = \int_p^q \frac{4}{u(2-u)}\,du$$

where p and q are constants to be found.

(5)

(b) Hence, using algebraic integration, show that

$$\int_0^{\frac{\pi}{6}} \frac{8\tan x}{1+\sin^2 x}\,dx = \ln A$$

where A is a rational number to be found.

(6)

Question 14 continued

Question 14 continued

Question 14 continued

(Total for Question 14 is 11 marks)

15. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

The first 3 terms of an arithmetic sequence are

$$\ln 3 \qquad \ln(3^k - 1) \qquad \ln(3^k + 5)$$

Find the exact value of the constant k.

(5)

Question 15 continued

Question 15 continued

(Total for Question 15 is 5 marks)

(TOTAL FOR PAPER IS 100 MARKS)