**Set 9**

**MARK SCHEME**

**PURE MATHEMATICS**

**A level Practice Paper**

|  |  |
| --- | --- |
| Makes an attempt to substitute any of *n* = 1, 2, 3, 4, 5 or 6 into  **1** | **M1** |
| Successfully substitutes *n* = 1, 2, 3, 4, 5 **and** 6 into | **A1** |
| Draws the conclusion that as the statement is true for all numbers from 1 to 6 inclusive, it has been proved by exhaustion. | **B1** |
| **TOTAL: 3 marks** |  |

|  |  |
| --- | --- |
| Uses the double-angle formulae to write:  **2** | **M1** |
| Uses the fact thatandto write: | **M1** |
| Simplifies this expression to | **M1** |
| Correctly solves to find | **A1** |
| **TOTAL: 4 marks** |  |

|  |  |
| --- | --- |
| Makes an attempt to substitute *t* = 0 into  **3a**  For example, or  is seen. | **M1** |
| Concludes that the *TR* terms will always cancel at *t* = 0, therefore the room temperature  does not influence the initial coffee temperature. | **B1** |
|  | **(2 marks)** |
| Makes an attempt to substitute  and *t* = 10 into  **3b**  For example, is seen. | **M1** |
| Finds. Accept awrt 62.5°. | **A1** |
|  | **(2 marks)** |
| **TOTAL: 4 marks** |  |

|  |  |
| --- | --- |
| Makes an attempt to find. Raising the power by 1 would constitute an attempt.  **4** | **M1** |
| Correctly states | **A1** |
| States | **M1 ft** |
| Makes an attempt to solve this equation.  For example,oris seen. | **M1 ft** |
| Solves to find | **A1 ft** |
| **TOTAL: 5 marks** |  |

**NOTES:**

Student does not need to state ‘+C’ in an answer unless it is the final answer to an indefinite integral.

Award ft marks for a correct answer using an incorrect initial answer.

**5**

|  |  |
| --- | --- |
| Begins the proof by assuming the opposite is true.  ‘Assumption: there exist positive integer solutions to the statement’ | **B1** |
| Sets up the proof by factorising  and stating | **M1** |
| States that there is only one way to multiply to make 1:  and concludes this means that: *x* − *y* = 1 and *x* + *y* = 1 | **M1** |
| Solves this pair of simultaneous equations to find the values of *x* and *y*: *x* = 1 and *y* = 0 | **M1** |
| Makes a valid conclusion.  *x =* 1, *y =* 0 are not both positive integers, which is a contradiction to the opening statement. Therefore there do not exist positive integers *x* and *y* such that | **B1** |
| **TOTAL: 5 marks** |  |

|  |  |
| --- | --- |
| States that:  **6** | **M1** |
| Further states that: | **M1** |
| Equates the various terms. Equating the coefficients of *x*2:  Equating the coefficients of *x*:  Equating constant terms: | **M1** |
| Makes an attempt to manipulate the expressions in order to find *A*, *B* and *C*.  Obtaining two different equations in the same two variables would constitute an attempt. | **M1** |
| Finds the correct value of any one variable: either *A* = 4, *B* = −2 or *C* = 6 | **A1** |
| Finds the correct value of all three variables: *A* = 4, *B* = −2, *C* = 6 | **A1** |
| **TOTAL: 6 marks** |  |

Alternative method

Uses the substitution method, having first obtained this equation:

Substitutes *x* = 4 to obtain 13*B* = −26

Substitutes  to obtain 

Equates the coefficients of *x*2: 

Substitutes the found value of *C* to obtain 3*A* = 12

**7a**

|  |  |
| --- | --- |
| States or implies that | **M1** |
| States or implies that | **M1** |
| Makes an attempt to solve  For example,  or  is seen. | **M1** |
| States that . Must show all steps and a logical progression. | **A1** |
|  | **(4 marks)** |
| **7b** | **M1\*** |
| States that as there are no real solutions to the equation. | **B1\*** |
|  | **(2 marks)** |
| **TOTAL: 6 marks** |  |

NOTES: 7b Alternative Method

M1: Uses the method of completing the square to show that  or 

B1: Concludes that this equation will have no real solutions.

|  |  |
| --- | --- |
| Finds  8a | **M1** |
| Finds | **M1** |
| States thatfor alland concludes this implies *C* is concave  over the given interval. | **B1** |
|  | **(3 marks)** |
| States or implies that a point of inflection occurs when  8b | **M1** |
| Finds *x* = −2 | **A1** |
| Substitutes *x* = −2 into, obtaining *y* = 46 | **A1** |
|  | **(3 marks)** |
| **TOTAL: 6 marks** |  |

9a

|  |  |
| --- | --- |
| Forms a pair of simultaneous equations, using the given values | **M1** |
| Correctly solves to find *d* = −6 | **A1** |
| Finds *a* = 116 | **A1** |
| Uses  to find | **A1** |
|  | **(4 marks)** |
| Uses the sum of an arithmetic series to form the equation  9b | **M1 ft** |
| Successfully multiplies out the brackets and simplifies. Fully simplified quadratic  of  is seen or  is seen. | **M1 ft** |
| Correctly factorises: | **M1 ft** |
| States that *n* = 39 is the correct answer. | **A1** |
|  | **(4 marks)** |
| **TOTAL: 8 marks** |  |

**NOTES:**

**9a** Can use elimination or substitution to solve the simultaneous equations.

**9b** Award method marks for a correct attempt to solve the equation using their incorrect values from part **a**.

10a

|  |  |
| --- | --- |
| Interprets the stone hitting the ground as when | **M1** |
| Makes an attempt to use the quadratic formula to find *t*.  For example,is seen | **M1** |
| Finds | **M1** |
| Deducesm. Accept awrt 24.6 | **A1** |
|  | **(4 marks)** |
| Finds  10b | **M1** |
| Demonstrates an understanding that the greatest height will occur when  For example, | **M1** |
| Solves to find | **M1** |
| Makes an attempt to find the greatest height by substituting  into  For example, | **M1 ft** |
| Finds *y* ==13.265… m. Accept awrt 13.3 m | **A1 ft** |
|  | **(5 marks)** |
| **TOTAL: 9 marks** |  |

**NOTES:**

**10b:** can also be found using. This is an acceptable method.

**10b:** Award ft marks for correct sketch using incorrect values from earlier in part **b**.

|  |  |
| --- | --- |
| Findsvia *M*  11 | **M1** |
| Findsvia *N* | **M1** |
| Finds | **M1** |
| Finds | **M1** |
| Equates the two ways of moving from *O* to *P*. | **M1** |
| Equates coefficients of *a:* | **M1** |
| Equates coefficients of *b*. OR equates coefficients of *c*. | **M1** |
| Solves to find | **A1** |
| Concludes that at this value the lines intersect. | **B1** |
| Concludes that the lines must bisect one another as  and | **B1** |
| **TOTAL: 10 marks** |  |

|  |  |
| --- | --- |
| States that  12a | **M1** |
| Equates the various terms.  Equating *x*s  Equating numbers | **M1** |
| Multiplies or or both of the equations in an effort to equate one of the two variables. | **M1** |
| Finds *A* = 5 | **A1** |
| Find *B* = 6 | **A1** |
|  | **(5 marks)** |
| Writes as  12b | **M1 ft** |
| Makes an attempt to integrate the expression.  Attempt would constitute the use of logarithms. | **M1 ft** |
| Integrates the expression to find | **A1 ft** |
| Makes an attempt to substitute the limits | **M1 ft** |
| Simplifies to find  o.e. | **A1 ft** |
|  | **(5 marks)** |
| **TOTAL: 10 marks** |  |

**NOTES:**

Award ft marks for a correct answer to part **b** using incorrect values from part **a**.

|  |  |
| --- | --- |
| States that the local maximum occurs when  13a | **B1** |
| Makes an attempt to differentiate p(*t*) | **M1** |
| Correctly finds | **A1** |
| Finds  and | **M1** |
| Change of sign and continuous function in the interval  Therefore the gradient goes from positive to negative and so the function has reached a maximum. | **A1** |
|  | **(5 marks)** |
| States that the local minimum occurs when  13b | **B1** |
| Makes an attempt to differentiate | **M1** |
| Correctly finds | **A1** |
| Finds  and | **M1** |
| Attempts to find | **M1** |
| Finds | **A1** |
|  | **(6 marks)** |
| **TOTAL: 11 marks** |  |

**NOTES:**

**13a**

Minimum required is that answer states there is a sign change in the interval and that this implies a root in the given interval.

|  |  |
| --- | --- |
| Makes an attempt to set up a long division. For example,  is seen.  14a | **M1** |
| Long division completed so that a 2 is seen in the quotient and a remainder of –2*x* – 7 is also seen. | **M1** |
| States | **M1** |
| Either equates variables or makes a substitution in an effort to find *B* or *C*. | **M1** |
| Finds | **A1** |
| Finds | **A1** |
|  | **(6 marks)** |
| Correctly writes  or  as  14b | **M1 ft** |
| Simplifies to obtain | **A1 ft** |
| Correctly writes  as | **M1 ft** |
| Correctly writes  as | **M1 ft** |
| Simplifies to obtain | **A1 ft** |
| States the correct final answer: | **A1 ft** |
|  | **(6 marks)** |
| The expansion is only valid for  14c | **B1** |
|  | **(1 mark)** |
| **TOTAL: 13 marks** |  |

**NOTES:**

**14a**

Writes the RHS as a single fraction.



**14b**

Award all 6 marks for a correct answer using their incorrect values of *A*, *B* and/or *C* from part **a**.

**(TOTAL: 100 MARKS)**