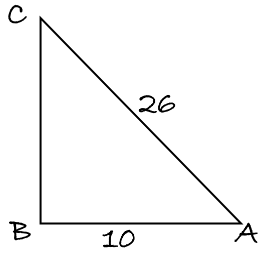
Section 1: Triangles, sine, cosine rule

Solutions to Exercise

1.



(i) $BC^2 = AC^2 - AB^2 = 26^2 - 10^2 = 576$

$BC = 24$ cm

(ii) $\sin A = \frac{24}{26} = \frac{12}{13}$

$\cos A = \frac{10}{26} = \frac{5}{13}$

$\tan A = \frac{24}{10} = \frac{12}{5}$

(iii) $\sin C = \frac{10}{26} = \frac{5}{13}$

$\cos C = \frac{24}{26} = \frac{12}{13}$

$\tan C = \frac{10}{24} = \frac{5}{12}$

(iv) $\sin A = \cos C$

$\cos A = \sin C$

$\tan A = \frac{1}{\tan C}$

(v) Since $C = 90° - A$, this can be generalised to

$\sin x = \cos(90° - x)$

$\cos x = \sin(90° - x)$

$\tan x = \frac{1}{\tan(90° - x)}$

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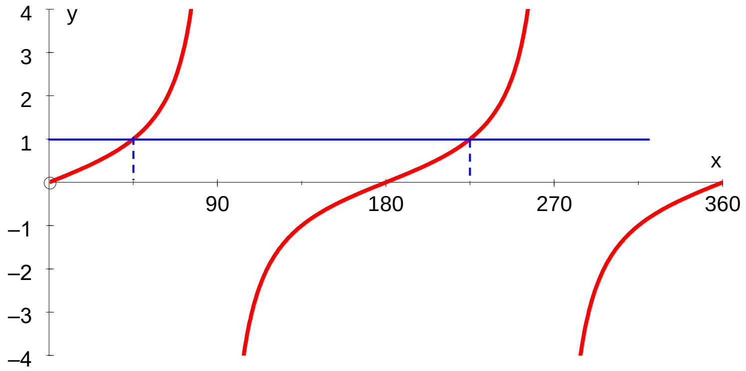
2. (i)



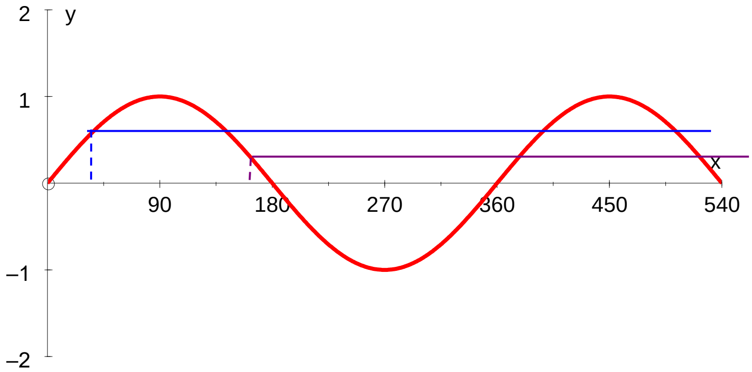
(ii) $\tan x = 1$

$x = 45^\circ$ or $180^\circ + 45^\circ$

$x = 45^\circ$ or 225°

(iii) By symmetry, angles are $180^\circ - 45^\circ = 135^\circ$ and $360^\circ - 45^\circ = 315^\circ$

3.



(i) $180^\circ - 40^\circ = 140^\circ$

$360^\circ + 40^\circ = 400^\circ$

$540^\circ - 40^\circ = 500^\circ$

(ii) $360^\circ + 20^\circ = 380^\circ$

$540^\circ - 20^\circ = 520^\circ$

4. (i) $x = 360^\circ - 25^\circ = 335^\circ$

(ii) $x = 180^\circ - 50^\circ = 130^\circ$

(iii) $x = 180^\circ + 120^\circ = 300^\circ$

(iv) $x = 180^\circ + 60^\circ = 240^\circ$ and $x = 360^\circ - 60^\circ = 300^\circ$

(v) $x = 180^\circ - 20^\circ = 160^\circ$ and $x = 180^\circ + 20^\circ = 200^\circ$

5. (i) $\sin 120° = \sin 60° = \frac{\sqrt{3}}{2}$

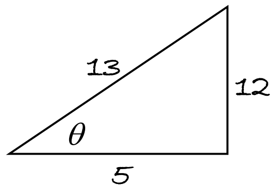
(ii) $\cos(-120°) = \cos 120° = -\cos 60° = -\frac{1}{2}$

(iii) $\tan 135° = -\tan 45° = -1$

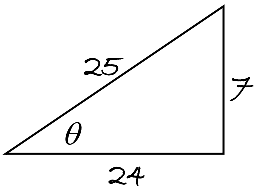
(iv) $\sin 300° = -\sin 60° = -\frac{\sqrt{3}}{2}$

(v) $\cos 270° = -\cos 90° = 0$

6. (i) $\cos\theta = \frac{5}{13}$

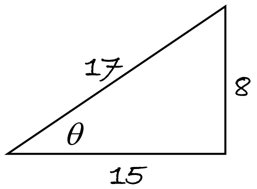


(ii) Since θ is in the second quadrant, cos θ and tan θ are both negative.



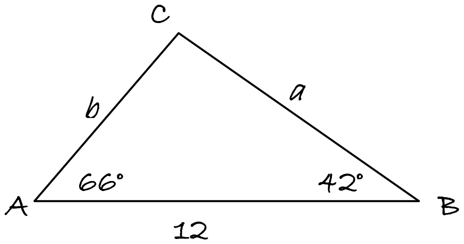
$\cos\theta = -\frac{24}{25}$
$\tan\theta = -\frac{7}{24}$

(iii) Since θ is in the second quadrant, sin θ is positive and cos θ is negative.



$\sin\theta = \frac{8}{17}$
$\cos\theta = -\frac{15}{17}$

7.



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Angle C $= 180° - 66° - 42° = 72°$

Using the sine rule: $\frac{a}{\sin A} = \frac{c}{\sin C}$

$\frac{a}{\sin 66°} = \frac{12}{\sin 72°}$

$a = \frac{12 \sin 66°}{\sin 72°} = 11.53$ cm

The length of BC is 11.53 cm

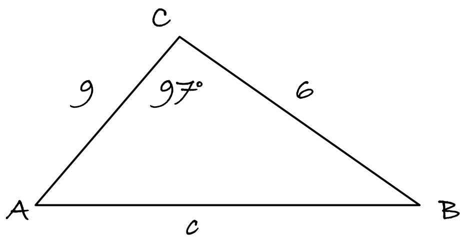
Using the sine rule: $\frac{b}{\sin B} = \frac{c}{\sin C}$

$\frac{b}{\sin 42°} = \frac{12}{\sin 72°}$

$b = \frac{12 \sin 42°}{\sin 72°} = 8.44$ cm

The length of AC is 8.44 cm.

8.



(i) Using the cosine rule: $c^2 = a^2 + b^2 - 2ab\cos C$

$= 9^2 + 6^2 - 2 \times 9 \times 6 \cos 97°$

$c = 11.4$ cm

(ii) Using the sine rule: $\frac{\sin A}{a} = \frac{\sin C}{c}$

$\frac{\sin A}{6} = \frac{\sin 97}{11.4}$

$\sin A = \frac{6 \sin 97}{11.4}$

$A = 31.5°$

$B = 180° - 97° - 31.5° = 51.5°$.

(iii) Area of triangle $= \frac{1}{2} ab \sin C$

$= \frac{1}{2} \times 6 \times 9 \sin 97°$

$= 26.8$ cm² (3 s.f.)

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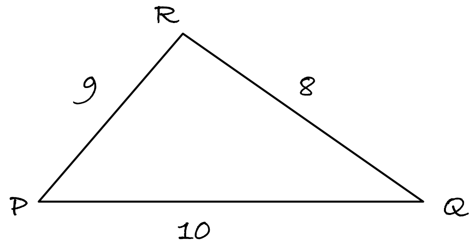
9.



Using the cosine rule: $\cos P = \dfrac{q^2 + r^2 - p^2}{2qr} = \dfrac{9^2 + 10^2 - 8^2}{2 \times 9 \times 10}$

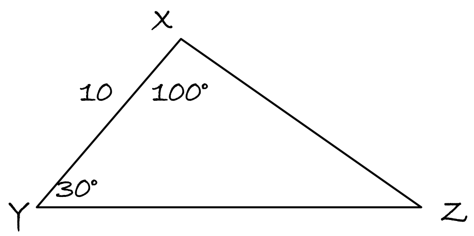
$P = 49.5°$

Using the cosine rule: $\cos Q = \dfrac{p^2 + r^2 - q^2}{2pr} = \dfrac{8^2 + 10^2 - 9^2}{2 \times 8 \times 10}$

$Q = 58.8°$

$R = 180° - 49.46° - 58.75° = 71.8°$

10.



Angle Z = 180° - 100° - 30° = 50°

Using the sine rule: $\dfrac{x}{\sin X} = \dfrac{z}{\sin z}$

$\dfrac{x}{\sin 100°} = \dfrac{10}{\sin 50°}$

$x = \dfrac{10 \sin 100°}{\sin 50°} = 12.86$

Area of triangle $= \frac{1}{2} xz \sin Y$

$= \frac{1}{2} \times 12.86 \times 10 \sin 30°$

$= 32.1 \text{ cm}^2$

11. Area of triangle ABC = ½ x 6 x 2 x sin B= 6 sin B
Area of triangle ADC = ½ x 3 x 4 x sin D= 6 sin D
ABCD is a cyclic quadrilateral so D = 180° - B
sin(180° - B) = sin B
So sin B = sin D and the two triangles have equal areas.

12. $7^2 = 8^2 + p^2 - 2 \times 8 \times p \times \cos 60$

$49 = 64 + p^2 - 8p$

$p^2 - 8p + 15 = 0$

$(p-3)(p-5) = 0$

$p = 3$ or 5

RQ could be either 3 cm or 5 cm long.