# AQA Level 2 Further Mathematics Calculus

## Section 1: Introduction to differentiation

### Exercise

1. Differentiate the following with respect to $x$:
   (i) $y = 2x + 1$ (ii) $y = x^3 - 5x$ (iii) $y = x(x + 2)$.

2. Find the gradient function for each of the following graphs:
   (i) $y = 3x^2 - 4x + 1$ (ii) $y = (x + 2)(x - 1)$ (iii) $y = x^6(x - 1)$

3. For $y = 2x^5 - 3x^3 - x^2 + 3x$, find the rate of change of $y$ with respect to $x$ when $x = -1$.

4. For $y = (2x - 3)(x^2 + 1)$, find the rate of change of $y$ with respect to $x$ when $x = 2$.

5. Given that $y = 12x - x^3$,
   (i) Find the gradient of the curve at the origin.
   (ii) Find the coordinates of the two points where the gradient is zero.

6. Find the equation of the tangent to the curve $y = x^4 - x + 1$ at the point with $x$–coordinate 1.

7. Show that the equation of the normal to the curve $y = x^2 - x$ at the point (3, 6) is $x + 5y = 33$. Find the coordinates of the point where the normal meets the $x$–axis.

8. Given that $y = x^3 + 2x^2$, find $\frac{dy}{dx}$. Hence find the $x$–coordinates of the two points on the curve where the gradient is 4.

9. (i) Show that the point (1, 2) lies on both the curves $y = 2x^3$ and $y = 3x^2 - 1$.
   (ii) Show that the curves have the same gradient at this point.
   (iii) What do these results this tell you about the two curves?

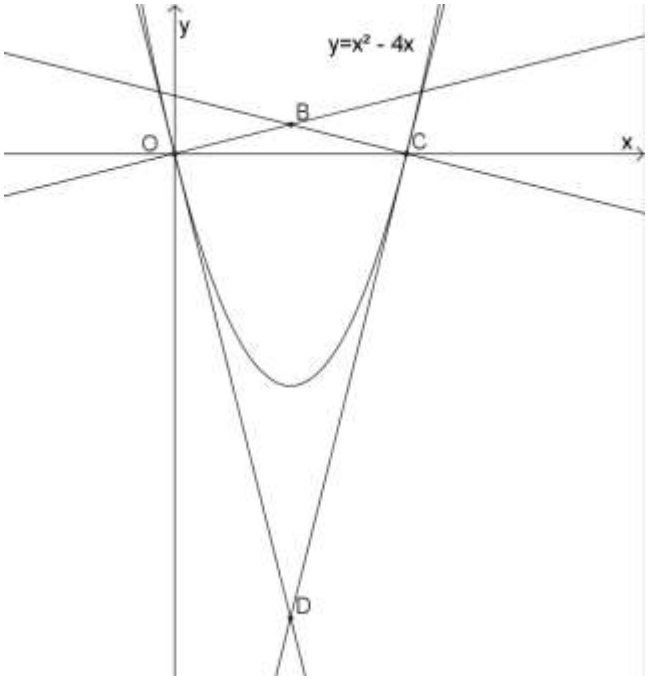
10. A curve has equation $y = ax^3 + bx$, where $a$ and $b$ are constants. At the point where $x = 1$, the $y$ –coordinate is 8 and the gradient is 12. Find $a$ and $b$.

11. Show that the tangent to the curve $y = x^3 + x + 2$ at the point P with $x$–coordinate 1 passes through the origin, and find the equation of the normal at this point. Given that the normal cuts the $x$ –axis at the point Q, find the area of triangle OPQ.

12. Show that the tangent to $y = x^2$ at the point (3, 9) crosses the $y$-axis at the point (0, -9).

# AQA FM Calculus 1 Exercise

13. O is the point (0, 0). C is the point (4, 0).
The curve $y = x^2 - 4x$ passes through points O and C.
Tangents to the curve at O and C cross at D.
Normals to the curve at O and C cross at B.




(i) Show that a circle can be drawn through points O, B, C and D.
(ii) Find the centre and radius of this circle.