Section 1: Matrix arithmetic

Solutions to Exercise

1. (i) $\begin{pmatrix} 1 & 2 \\ -3 & 0 \end{pmatrix}\begin{pmatrix} 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 4 \\ -6 \end{pmatrix}$

(ii) $\begin{pmatrix} 4 & 2 \\ -1 & 3 \end{pmatrix}\begin{pmatrix} 1 & 5 \\ 0 & -2 \end{pmatrix} = \begin{pmatrix} 4 & 16 \\ -1 & -11 \end{pmatrix}$

(iii) $\begin{pmatrix} 4 & 1 \\ 5 & -2 \end{pmatrix}\begin{pmatrix} 2 \\ -3 \end{pmatrix} = \begin{pmatrix} 5 \\ 16 \end{pmatrix}$

(iv) $\begin{pmatrix} -2 & 5 \\ -1 & 4 \end{pmatrix}\begin{pmatrix} 3 & -1 \\ -2 & 1 \end{pmatrix} = \begin{pmatrix} -16 & 7 \\ -11 & 5 \end{pmatrix}$

(v) $\begin{pmatrix} 6 & 5 \\ 4 & 3 \end{pmatrix}\begin{pmatrix} 1 \\ -2 \end{pmatrix} = \begin{pmatrix} -4 \\ -2 \end{pmatrix}$

(vi) $\begin{pmatrix} 3 & 0 \\ 4 & -1 \end{pmatrix}\begin{pmatrix} 2 & 1 \\ -4 & 0 \end{pmatrix} = \begin{pmatrix} 6 & 3 \\ 12 & 4 \end{pmatrix}$

(vii) $\begin{pmatrix} 8 & -6 \\ 3 & -2 \end{pmatrix}\begin{pmatrix} -1 \\ -4 \end{pmatrix} = \begin{pmatrix} 16 \\ 5 \end{pmatrix}$

(viii) $\begin{pmatrix} 0 & 5 \\ 3 & 2 \end{pmatrix}\begin{pmatrix} -2 & 4 \\ 1 & -3 \end{pmatrix} = \begin{pmatrix} 5 & -15 \\ -4 & 6 \end{pmatrix}$

2. (i) $5A = 5\begin{pmatrix} 1 & 0 \\ 2 & -1 \end{pmatrix}$

$= \begin{pmatrix} 5 & 0 \\ 10 & -5 \end{pmatrix}$

(ii) $-2BA = -2\begin{pmatrix} -2 & 1 \\ 3 & 0 \end{pmatrix}$

$= \begin{pmatrix} 4 & -2 \\ -6 & 0 \end{pmatrix}$

(iii) $AB = \begin{pmatrix} 1 & 0 \\ 2 & -1 \end{pmatrix}\begin{pmatrix} -2 & 1 \\ 3 & 0 \end{pmatrix}$

$= \begin{pmatrix} -2 & 1 \\ -7 & 2 \end{pmatrix}$

(iv) $\mathbf{BA} = \begin{pmatrix} -2 & 1 \\ 3 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 2 & -1 \end{pmatrix}$

$= \begin{pmatrix} 0 & -1 \\ 3 & 0 \end{pmatrix}$

3. $\mathbf{AB} = \begin{pmatrix} 3 & 1 \\ x & 2 \end{pmatrix}\begin{pmatrix} 6 & 2 \\ 4 & y \end{pmatrix} = \begin{pmatrix} 22 & 6+y \\ 6x+8 & 2x+2y \end{pmatrix}$

$\mathbf{BA} = \begin{pmatrix} 6 & 2 \\ 4 & y \end{pmatrix}\begin{pmatrix} 3 & 1 \\ x & 2 \end{pmatrix} = \begin{pmatrix} 18+2x & 10 \\ 12+xy & 4+2y \end{pmatrix}$

$\mathbf{AB} = \mathbf{BA} \Rightarrow \begin{pmatrix} 22 & 6+y \\ 6x+8 & 2x+2y \end{pmatrix} = \begin{pmatrix} 18+2x & 10 \\ 12+xy & 4+2y \end{pmatrix}$

$22 = 18 + 2x \quad \Rightarrow x = 2$

$6 + y = 10 \quad \Rightarrow y = 4$

Check: $6x + 8 = 12 + 8 = 20$ $\quad 12 + xy = 12 + 8 = 20$

$2x + 2y = 4 + 8 = 12$ $\quad 4 + 2y = 4 + 8 = 12$

4. $\begin{pmatrix} 3 & a \\ b & 2 \end{pmatrix}\begin{pmatrix} 2 & c \\ -1 & d \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$\begin{pmatrix} 6-a & 3c+ad \\ 2b-2 & bc+2d \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$6 - a = 1 \quad \Rightarrow a = 5$

$2b - 2 = 0 \quad \Rightarrow b = 1$

$3c + ad = 0 \quad \Rightarrow 3c + 5d = 0 \quad (1)$

$bc + 2d = 1 \quad \Rightarrow c + 2d = 1 \quad \Rightarrow c = 1 - 2d \quad (2)$

Substituting (2) into (1) gives $3(1 - 2d) + 5d = 0$

$3 - 6d + 5d = 0$

$d = 3$

$c = -5$

So $a = 5$, $b = 1$, $c = -5$, $d = 3$.

5. (i) $\mathbf{AB} = \begin{pmatrix} 1+\sqrt{3} & 0 \\ 1 & \sqrt{3} \end{pmatrix}\begin{pmatrix} 1-\sqrt{3} & 1 \\ 1 & \sqrt{3} \end{pmatrix} = \begin{pmatrix} -2 & 1+\sqrt{3} \\ 1 & 4 \end{pmatrix}$

(ii) One possible matrix is $\begin{pmatrix} 1-\sqrt{3} & 1-\sqrt{3} \\ 1 & 1 \end{pmatrix}$

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6. (i) $M^2 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I$

(ii) $M^3 = IM = M = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

(iii) $M^{10} = I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ because all even powers of M will be the identity.