Section 1: Matrix arithmetic

Exercise

1. Work out:

(i) $\begin{pmatrix} 1 & 2 \\ -3 & 0 \end{pmatrix}\begin{pmatrix} 2 \\ 1 \end{pmatrix}$ (ii) $\begin{pmatrix} 4 & 2 \\ -1 & 3 \end{pmatrix}\begin{pmatrix} 1 & 5 \\ 0 & -2 \end{pmatrix}$

(iii) $\begin{pmatrix} 4 & 1 \\ 5 & -2 \end{pmatrix}\begin{pmatrix} 2 \\ -3 \end{pmatrix}$ (iv) $\begin{pmatrix} -2 & 5 \\ -1 & 4 \end{pmatrix}\begin{pmatrix} 3 & -1 \\ -2 & 1 \end{pmatrix}$

(v) $\begin{pmatrix} 6 & 5 \\ 4 & 3 \end{pmatrix}\begin{pmatrix} 1 \\ -2 \end{pmatrix}$ (vi) $\begin{pmatrix} 3 & 0 \\ 4 & -1 \end{pmatrix}\begin{pmatrix} 2 & 1 \\ -4 & 0 \end{pmatrix}$

(vii) $\begin{pmatrix} 8 & -6 \\ 3 & -2 \end{pmatrix}\begin{pmatrix} -1 \\ -4 \end{pmatrix}$ (viii) $\begin{pmatrix} 0 & 5 \\ 3 & 2 \end{pmatrix}\begin{pmatrix} -2 & 4 \\ 1 & -3 \end{pmatrix}$

2. If $\mathbf{A} = \begin{pmatrix} 1 & 0 \\ 2 & -1 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} -2 & 1 \\ 3 & 0 \end{pmatrix}$, find

(i) $5\mathbf{A}$
(ii) $-2\mathbf{B}$
(iii) $\mathbf{AB}$
(iv) $\mathbf{BA}$

3. If $\mathbf{A} = \begin{pmatrix} 3 & 1 \\ x & 2 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 6 & 2 \\ 4 & y \end{pmatrix}$ find the values of x and y given that $\mathbf{AB} = \mathbf{BA}$.

4. If $\mathbf{P} = \begin{pmatrix} 3 & a \\ b & 2 \end{pmatrix}$ and $\mathbf{Q} = \begin{pmatrix} 2 & c \\ -1 & d \end{pmatrix}$ find the values of a, b, c and d given that $\mathbf{PQ} = \mathbf{I}$.

5. $\mathbf{A} = \begin{pmatrix} 1+\sqrt{3} & 0 \\ 1 & \sqrt{3} \end{pmatrix}$ $\mathbf{B} = \begin{pmatrix} 1-\sqrt{3} & 1 \\ 1 & \sqrt{3} \end{pmatrix}$

(i) Work out $\mathbf{AB}$.
(ii) Find a matrix $\mathbf{C}$ such that $\mathbf{AC}$ consists of four non-zero whole numbers.

6. $\mathbf{M} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

(i) Work out $\mathbf{M}^2$.
$\mathbf{M}^3 = \mathbf{M}^2\mathbf{M}$, $\mathbf{M}^4 = \mathbf{M}^3\mathbf{M}$ etc
(ii) Work out $\mathbf{M}^3$.
(iii) Write down, with reasoning, $\mathbf{M}^{10}$.