Section 2: Matrix transformations

Solutions to Exercise

1. (i) Image of O = (0, 0)
   Image of A = (-0.5, 0)
   Image of B = (-0.5, -0.5)
   Image of C = (0, -0.5)

   The transformation is an enlargement, scale factor -0.5, centre the origin.

   (ii) Image of O = (0, 0)
   Image of A = (0, -1)
   Image of B = (-1, -1)
   Image of C = (-1, 0)

   The transformation is a reflection in the line y = -x.

2. A: $\begin{pmatrix} 1 & 0 \\ -2 & 1 \end{pmatrix}\begin{pmatrix} 3 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ -5 \end{pmatrix}$ so the image of A is (3, -5)

   B: $\begin{pmatrix} 1 & 0 \\ -2 & 1 \end{pmatrix}\begin{pmatrix} 3 \\ 3 \end{pmatrix} = \begin{pmatrix} 3 \\ -3 \end{pmatrix}$ so the image of B is (3, -3)

   C: $\begin{pmatrix} 1 & 0 \\ -2 & 1 \end{pmatrix}\begin{pmatrix} 6 \\ 3 \end{pmatrix} = \begin{pmatrix} 6 \\ -9 \end{pmatrix}$ so the image of C is (6, -9)

   D: $\begin{pmatrix} 1 & 0 \\ -2 & 1 \end{pmatrix}\begin{pmatrix} 6 \\ 1 \end{pmatrix} = \begin{pmatrix} 6 \\ -11 \end{pmatrix}$ so the image of D is (6, -11)

3. P: $\begin{pmatrix} 4 & 3 \\ -3 & -2 \end{pmatrix}\begin{pmatrix} 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ so the image of P is (0, 0)

   Q: $\begin{pmatrix} 4 & 3 \\ -3 & -2 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 7 \\ -5 \end{pmatrix}$ so the image of Q is (7, -5)

   R: $\begin{pmatrix} 4 & 3 \\ -3 & -2 \end{pmatrix}\begin{pmatrix} 0 \\ 2 \end{pmatrix} = \begin{pmatrix} 6 \\ -4 \end{pmatrix}$ so the image of R is (6, -4)

   S: $\begin{pmatrix} 4 & 3 \\ -3 & -2 \end{pmatrix}\begin{pmatrix} -1 \\ 1 \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$ so the image of S is (-1, 1)

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4. Under P, the point (1, 0) is mapped to the point (-1, 0) and the point (0, 1) is unchanged.

   So P is represented by $\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$.

   Under Q, the point (1, 0) is mapped to the point (0, -1) and the point (0, 1) is mapped to the point (1, 0).

   So Q is represented by $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$.

   The single matrix is $QP = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$.

   This transformation is a reflection in the line $y = x$.

5. Enlargements with either scale factor $\sqrt{2}$ or $\sqrt{2}$ would work.

   The relevant matrices are $\begin{pmatrix} \sqrt{2} & 0 \\ 0 & \sqrt{2} \end{pmatrix}$ and $\begin{pmatrix} -\sqrt{2} & 0 \\ 0 & -\sqrt{2} \end{pmatrix}$.

6. $\begin{pmatrix} 5 & 2 \\ 4 & a \end{pmatrix}\begin{pmatrix} 1 \\ -2 \end{pmatrix} = \begin{pmatrix} k \\ -2k \end{pmatrix}$

   $5 - 4 = k$

   $4 - 2a = -2k$

   $k = 1$, $a = 3$