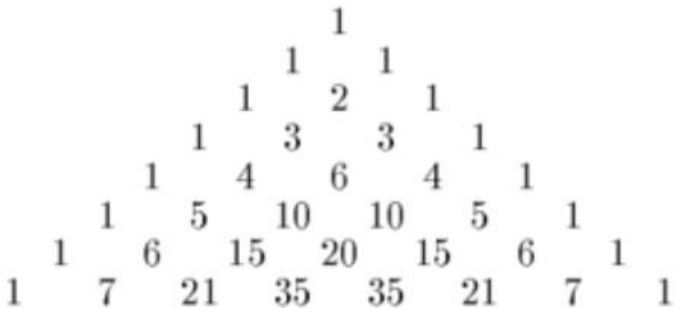
Section 2: The binomial expansion

Solutions to exercise

1.



2. (i) Coefficient of x^4 in $(x+1)^{10}$ is ${}^{10}C_4(1)^4(1)^6 = 210$

   (ii) Coefficient of x^4 in $(1-x)^7$ is ${}^{7}C_4(1)^3(-1)^4 = 35$

3. (i) coefficient of x^3 in $(3x+2)^5 = {}^{5}C_3(3)^3(2)^2 = 1080$

   (ii) coefficient of x^3 in $(3-2x)^6 = {}^{6}\mathbf{C_3}(\mathbf{3})^3(\mathbf{-2})^3 = \mathbf{-4320}$

4. Constant term in $(2x-\frac{5}{x})^6 = {}^{6}\mathbf{C_3}(2x)^3\left(\frac{5}{x}\right)^3 = 20000$

5. coefficient of a^2b^5 in $(3a-2b)^7 = {}^{7}\mathbf{C_2}(\mathbf{3}a)^2(\mathbf{-2}b)^5 = \mathbf{-6048}$

6. With this expression we can see that some of the terms will cancel out and some will be repeated. In fact the even powers will cancel out (as they will all be positive) and the odd powers will be repeated so we only need to look at the x and x^3 terms.
   x term of $(1+2x)^4 = {}^{4}\mathbf{C_1}(1)^3(\mathbf{2}x)^1 = 8x$
   x^3 term of $(1+2x)^4 = {}^{4}\mathbf{C_3}(1)^1(\mathbf{2}x)^3 = 32x^3$
   $(1+2x)^4 - (1-2x)^4 = 2(8x+32x^3) = 16x+64x^3$