Section 1: Basic algebra and simple linear equations

Exercise solutions

1. (i) $3\frac{3}{4} - 2\frac{2}{3} = 1 + \frac{3}{4} - \frac{2}{3}$

$= 1 + \frac{9-8}{12}$

$= 1\frac{1}{12}$

(ii) $1\frac{2}{5} \times 2\frac{1}{3} = \frac{7}{5} \times \frac{7}{3} = \frac{49}{15} = 3\frac{4}{15}$

(iii) $3\frac{3}{5} \div 2\frac{2}{3} = \frac{18}{5} \div \frac{8}{3} = \frac{18}{5} \times \frac{3}{8} = \frac{9}{5} \times \frac{3}{4} = \frac{27}{20} = 1\frac{7}{20}$

2. (i) $x : z = 2 : 5 = 6 : 15$

$y : z = 3 : 4 = 15 : 20$

$x : z = 6 : 20 = 3 : 10$

(ii) $2y : 5z = 6 : 20 = 3 : 10$

(iii) $x + 2y : y = 12 : 5$

3. $x : y = y : 4$

$\frac{x}{y} = \frac{y}{4}$

$4x = y^2$

(a) $x = y = 4$

(b) $x = 25, y = 10$ is one possible pair

(c) $x = 1, y = 2$

4. (i) Increasing by 20% is equivalent to multiplying by 1.2

$230 \times 1.2 = 276$

The price is £276.

(ii) $\frac{680}{800} = 0.85$, so the price has been multiplied by 0.85. So the new price is 85% of the old price, and so the price has been reduced by 15%.

5. (i) $2x + 3y - x + 5y + 4x = (2x - x + 4x) + (3y + 5y)$

$= 5x + 8y$

(ii) $5a-2b+3c-2a+5b=(5a-2a)+(-2b+5b)+3c$
$=3a+3b+3c$

(iii) $4p+q-6p-5q+5p+4q=(4p-6p+5p)+(q-5q+4q)$
$=3p$

6. (i) $3(2x+3y)=6x+9y$

(ii) $4(3a-2b)-3(a+2b)=12a-8b-3a-6b$
$=9a-14b$

(iii) $p(2p-q)+2q(p-3q)=2p^2-pq+2qp-6q^2$
$=2p^2+pq-6q^2$

7. (i) $(x+1)(x-3)=x^2-3x+x-3$
$=x^2-2x-3$

(ii) $(x+2)(2x+1)=2x^2+x+4x+2$
$=2x^2+5x+2$

(iii) $(x-3)(x-4)=x^2-4x-3x+12$
$=x^2-7x+12$

(iv) $(3x+2)(x-2)=3x^2-6x+2x-4$
$=3x^2-4x-4$

(v) $(2x+1)(4x-1)=8x^2-2x+4x-1$
$=8x^2+2x-1$

(vi) $(1-2x)(1+x)=1+x-2x-2x^2$
$=1-x-2x^2$

(vii) $(3+2x)(x-1)=3x-3+2x^2-2x$
$=2x^2+x-3$

(viii) $(5x-3)(2x+5)=10x^2+25x-6x-15$
$=10x^2+19x-15$

(ix) $(x+3)^3 = (x^2+6x+9)(x+3)$

$= x^3+6x^2+9x+3x^2+18x+27$

$= x^3+9x^2+27x+27$

8. (i) $(x-2)(2x^2-3x+1) = x(2x^2-3x+1)-2(2x^2-3x+1)$

$= 2x^3-3x^2+x-4x^2+6x-2$

$= 2x^3-7x^2+7x-2$

(ii) $(3x-2)(x^3-2x+4) = 3x(x^3-2x+4)-2(x^3-2x+4)$

$= 3x^4-6x^2+12x-2x^3+4x-8$

$= 3x^4-2x^3-6x^2+16x-8$

(iii) $(2x+1)(x^3+2x^2-3x-5)$

$= 2x(x^3+2x^2-3x-5)+(x^3+2x^2-3x-5)$

$= 2x^4+4x^3-6x^2-10x+x^3+2x^2-3x-5$

$= 2x^4+5x^3-4x^2-13x-5$

(iv) $(x+3)(2x-1)(x-4) = (x+3)(2x^2-8x-x+4)$

$= (x+3)(2x^2-9x+4)$

$= x(2x^2-9x+4)+3(2x^2-9x+4)$

$= 2x^3-9x^2+4x+6x^2-27x+12$

$= 2x^3-3x^2-23x+12$

(v) $(2x-1)^3 = (4x^2-4x+1)(2x-1)$

$= 8x^3-8x^2+2x-4x^2+4x-1$

$= 8x^3-12x^2+6x-1$

9. (i) $2x-3=8$

$2x=11$

$x=5.5$

(ii) $3y+2=y-5$

$2y+2=-5$

$2y=-7$

$y=-3.5$

(iii) $3-2a=3a-1$

$3=5a-1$

$4=5a$

$a=0.8$

(iv) $3(p-3)=2(2p+1)$

$3p-9=4p+2$

$-9=p+2$

$-11=p$

$p=-11$

(v) $2(1-z)+3(z+3)=4z+1$

$2-2z+3z+9=4z+1$

$11+z=4z+1$

$11=3z+1$

$10=3z$

$z=\frac{10}{3}$

(vi) $\frac{2b+1}{5}=\frac{3-b}{4}$

$4(2b+1)=5(3-b)$

$8b+4=15-5b$

$13b+4=15$

$13b=11$

$b=\frac{11}{13}$

10. Let the smallest angle be $x°$.
The largest angle is $3x°$.
The third angle is $(x+20)°$.
The three angles add up to 180°.

$x+3x+(x+20)=180$

$5x+20=180$

$5x=160$

$x=32$

The angles are 32°, 96° and 52°.

(Check: $32+96+52=180$).

11. Let the number of tables which seat 4 people be x.
The number of tables which seat 6 people is $24-x$.

Total number of seats $= 4x + 6(24 - x)$

$4x + 6(24 - x) = 114$

$4x + 144 - 6x = 114$

$30 = 2x$

$x = 15$

There are 15 tables which seat 4 people.

(Check: $15 \times 4 + 9 \times 6 = 60 + 54 = 114$)

12. Let x be the number of boys in the class
So number of girls is 30 – x.

Total of boys' heights $= 165x$
Total of girls' heights $= 159(30 - x)$
Total of heights for whole class $= 162.2 \times 30 = 4866$

$165x + 159(30 - x) = 4866$

$165x + 4770 - 159x = 4866$

$6x = 96$

$x = 16$

There are 16 boys and 14 girls in the class.