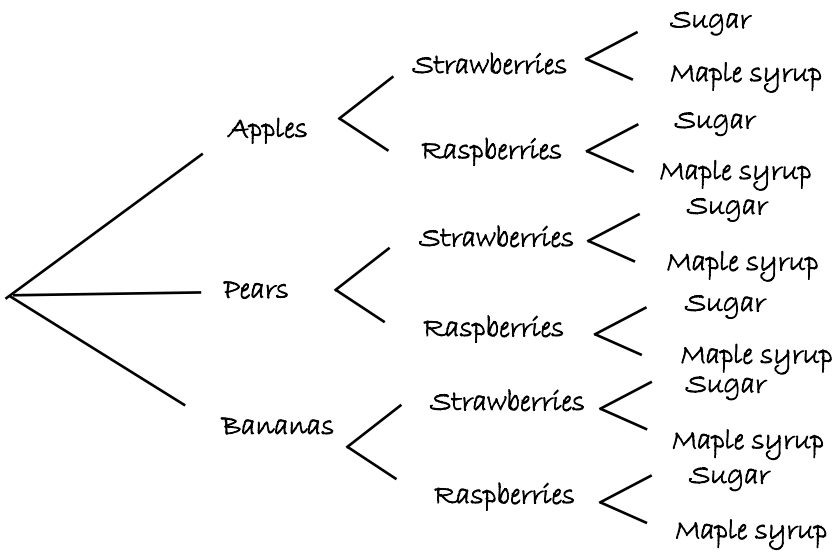
Section 4: The product rule for counting

Exercise solutions

1. There are 26 letters in the alphabet. Since repeated letters are allowed there are 26 ways to choose the first letter, 26 ways to choose the second letter and 26 ways to choose the third letter. So, the number of strings is:
   $26 \times 26 \times 26 = 26^3 = 17{,}576$

2. There are 8 ways to choose the shirt and 6 ways to choose the tie so the number of sets is:
   $8 \times 6 = 48$

3. There are 3 ways to choose the first fruit, two ways to choose the second fruit and two ways to choose a topping. So, the number of different pancakes that can be made is:
   $3 \times 2 \times 2 = 12$
   Using a tree diagram:



So, the list of all different pancakes is:

	Fruit 1	Fruit 2	Topping
1	Apple	Strawberries	Sugar
2	Apple	Strawberries	Maple syrup
3	Apple	Raspberries	Sugar
4	Apple	Raspberries	Maple syrup
5	Pear	Strawberries	Sugar
6	Pear	Strawberries	Maple syrup
7	Pear	Raspberries	Sugar
8	Pear	Raspberries	Maple syrup
9	Bananas	Strawberries	Sugar
10	Bananas	Strawberries	Maple syrup
11	Bananas	Raspberries	Sugar
12	Bananas	Raspberries	Maple syrup

4. (a) If repeated digits and letters are allowed there are 10 ways to choose each of the 3 numbers so 10^3 choices in total. There are 26 ways to choose each of the 3 letters so 26^3 choices in total. So, the number of different number plates is:
$10^3 \times 26^3 = 17{,}576{,}000$
(b) If no repeated digits or repeated letters are allowed there are 10 ways to choose the first number, 9 ways to choose the second and 8 ways to choose the third. There are 26 ways to choose the first letter, 25 ways to choose the second and 24 ways to choose the third. So, the total number of different number plates is:
$10 \times 9 \times 8 \times 26 \times 25 \times 24 = 11{,}232{,}000$
(c) If the first digit cannot be zero then there are 9 ways to choose the first number but the rest of the digits/letters are chosen in the same way as part (a) since repeated digits/letters are allowed. So, the number of different number plates is:
$9 \times 10 \times 10 \times 26^3 = 15{,}818{,}400$

5. The probability of picking the red ball first is 1/6
The probability of picking the blue ball second is 1/5
The probability of picking the yellow ball third is 1/4
a) So, the probability of picking red, blue, yellow in this order is:

$$\frac{1}{6} \times \frac{1}{5} \times \frac{1}{4} = \frac{1}{120}$$

b) We need to consider all the different orders the balls could be removed in:

Red	Blue	Yellow
Red	Yellow	Blue
Blue	Red	Yellow
Blue	Yellow	Red
Yellow	Red	Blue
Yellow	Blue	Red

There are 3 ways to choose the first ball, 2 ways to choose the second and one way to choose the third so the number of different arrangements is:

$3 \times 2 \times 1 = 6$

We can then multiply the probability from part a) by the number of arrangements to get the probability of removing the three balls in any order:

$$\frac{1}{120} \times 6 = \frac{6}{120} = \frac{1}{20}$$

6.

1	2	3	4	5

The five spaces on the bench each need to be occupied by one of the five people

You can put any of the people in position 1 so there are 5 different ways of making this first selection.

You can then put any of the four-remaining people in position 2 so there are then four different ways of making this second selection.

There are 3 different ways to put someone in position 3

There are 2 different ways to put someone in position 4

At this point, all the other spaces are occupied leaving you with one space and only one person left so there is only 1 way to put someone in position 5

This gives you:

$5 \times 4 \times 3 \times 2 \times 1 = 120$ ways to arrange five people.

The question asks for the number of arrangements different to the one already stated so there are $120 - 1 = 119$ other different arrangements in which they can sit.

7. There are 10 ways to choose the first question

9 ways to choose the second

8 ways to choose the third

7 ways to choose the fourth

6 ways to choose the fifth

5 ways to choose the sixth

and 4 ways to choose the seventh.

So, the number of different seven question tests that can be made from 10 questions is:

$10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 = 604{,}800$