AQA Level 2 Further Mathematics Algebra II

Section 1: Factorising, algebraic fractions and formulae

Solutions to Exercise

1. (i) $10ab + 5ac = 5a(2b + c)$

   (ii) $2x^2 + 4xy - 8xz = 2x(x + 2y - 4z)$

   (iii) $3s^2t - 9s^3t + 12s^2t^2 = 3s^2t(1 - 3s + 4t)$

   (iv) $3(b - c) - 2a(b - c) = (b - c)(3 - 2a)$

2. (i) $$\begin{aligned} x^2 + 5x + 6 &= x^2 + 3x + 2x + 6 \\ &= x(x + 3) + 2(x + 3) \\ &= (x + 2)(x + 3) \end{aligned}$$

   (ii) $$\begin{aligned} x^2 + x - 12 &= x^2 + 4x - 3x - 12 \\ &= x(x + 4) - 3(x + 4) \\ &= (x - 3)(x + 4) \end{aligned}$$

   (iii) $x^2 - 9 = (x + 3)(x - 3)$

   (iv) $$\begin{aligned} x^2 - 6xy + 8y^2 &= x^2 - 2xy - 4xy + 8y^2 \\ &= x(x - 2y) - 4y(x - 2) \\ &= (x - 4y)(x - 2y) \end{aligned}$$

   (v) $$\begin{aligned} 2x^2 + 3xy + y^2 &= 2x^2 + xy + 2xy + y^2 \\ &= x(2x + y) + y(2x + y) \\ &= (x + y)(2x + y) \end{aligned}$$

   (vi) $$\begin{aligned} 3x^2 + x - 2 &= 3x^2 + 3x - 2x - 2 \\ &= 3x(x + 1) - 2(x + 1) \\ &= (3x - 2)(x + 1) \end{aligned}$$

   (vii) $$\begin{aligned} 4x^2 - 8x + 3 &= 4x^2 - 2x - 6x + 3 \\ &= 2x(2x - 1) - 3(2x - 1) \\ &= (2x - 3)(2x - 1) \end{aligned}$$

   (viii) $4x^2 - 25y^2 = (2x + 5y)(2x - 5y)$

(ix) $6x^2 - xy - 12y^2 = 6x^2 + 8xy - 9xy - 12y^2$

$= 2x(3x+4y) - 3y(3x+4y)$

$= (2x-3y)(3x+4y)$

3. (i) Using the difference of two squares:

$(x+4)^2 - (x-3)^2 = ((x+4)+(x-3))((x+4)-(x-3))$

$= (x+4+x-3)(x+4-x+3)$

$= (2x+1)\times 7$

$= 7(2x+1)$

(ii) Using the difference of two squares:

$(2x-y)^2 - (x+3y)^2 = \big((2x-y)+(x+3y)\big)\big((2x-y)-(x+3y)\big)$

$= \big(2x-y+x+3y\big)\big(2x-y-x-3y\big)$

$= (3x+2y)(x-4y)$

4. (i) $\frac{2a^2b}{4ab^2} = \frac{\not 2 \times \not a \times a \times \not b}{{}^2\not 4 \times \not a \times \not b \times b} = \frac{a}{2b}$

(ii) $\frac{12p^2qr^3}{9pq^2r} = \frac{{}^4\not{12} \times \not p \times p \times \not q \times \not r \times r \times r}{{}^3\not 9 \times \not p \times \not q \times q \times \not r} = \frac{4pr^2}{3q}$

(iii) $\frac{x^2y + xy^2}{x+y} = \frac{xy(\not{x+y})}{\not{x+y}} = xy$

(iv) $\frac{a}{2b} \times \frac{3bc}{a^2} \times \frac{a}{6c} = \frac{\not a \times \not 3 \times \not b \times \not c \times \not a}{2 \times \not b \times \not a \times \not a \times {}^2\not 6 \times \not c} = \frac{1}{4}$

5. (i) $\frac{x^2+x-6}{x^2-x-2} = \frac{(x+3)(\not{x-2})}{(\not{x-2})(x+1)} = \frac{x+3}{x+1}$

(ii) $\frac{x^2-4x+4}{x^2+x-6} = \frac{(x-2)^{\not 2}}{(x+3)(\not{x-2})} = \frac{x-2}{x+3}$

(iii) $\frac{x^2+x-2}{x^2+4x+3} = \frac{(x+2)(x-1)}{(x+3)(x+1)}$ - cannot be simplified.

(iv) $\dfrac{4x^2-1}{4x^2-4x-3}=\dfrac{\cancel{(2x+1)}(2x-1)}{\cancel{(2x+1)}(2x-3)}=\dfrac{2x-1}{2x-3}$

(v) $\dfrac{2x+3}{3x+1}\times\left(3x^2-2x-1\right)=\dfrac{2x+3}{\cancel{3x+1}}\times\cancel{(3x+1)}(x-1)=(2x+3)(x-1)$

(vi) $\dfrac{x+2}{2x^2-x-1}\div\dfrac{x^2-x-6}{2x+1}=\dfrac{x+2}{(2x+1)(x-1)}\div\dfrac{(x-3)(x+2)}{2x+1}$

$$=\frac{\cancel{x+2}}{\cancel{(2x+1)}(x-1)}\times\frac{\cancel{2x+1}}{(x-3)\cancel{(x+2)}}$$

$$=\frac{1}{(x-1)(x-3)}$$

6. (i) $\dfrac{2x}{5}+\dfrac{3x}{2}=\dfrac{4x}{10}+\dfrac{15x}{10}$

$$=\frac{19x}{10}$$

(ii) $\dfrac{3a}{4}-\dfrac{2b}{3}=\dfrac{9a}{12}-\dfrac{8b}{12}$

$$=\frac{9a-8b}{12}$$

(iii) $\dfrac{2x+1}{12}-\dfrac{x-2}{8}=\dfrac{2(2x+1)}{24}-\dfrac{3(x-2)}{24}$

$$=\frac{4x+2-3x+6}{24}$$

$$=\frac{x+8}{24}$$

(iv) $\dfrac{3x+4}{2x}-\dfrac{5x+6}{3x}=\dfrac{3(3x+4)}{6x}-\dfrac{2(5x+6)}{6x}$

$$=\frac{9x+12-10x-12}{6x}$$

$$=\frac{-x}{6x}$$

$$=-\frac{1}{6}$$

(v) $\frac{1}{p}+\frac{1}{q}=\frac{q}{pq}+\frac{p}{pq}$
$=\frac{q+p}{pq}$

(vi) $\frac{a}{2b}+\frac{5b}{3a}=\frac{3a^2}{6ab}+\frac{10b^2}{6ab}$
$=\frac{3a^2+10b^2}{6ab}$

(vii) $\frac{3}{2x+1}-\frac{2}{x-1}=\frac{3(x-1)}{(2x+1)(x-1)}-\frac{2(2x+1)}{(x-1)(2x+1)}$
$=\frac{3x-3-4x-2}{(2x+1)(x-1)}$
$=\frac{-x-5}{(2x+1)(x-1)}$

(viii) $\frac{2x}{x-2}-\frac{x+1}{x+3}=\frac{2x(x+3)}{(x-2)(x+3)}-\frac{(x+1)(x-2)}{(x+3)(x-2)}$
$=\frac{2x^2+6x-(x^2-x-2)}{(x-2)(x+3)}$
$=\frac{x^2+7x+2}{(x-2)(x+3)}$

7. (i) $ax+b=c$
$ax=c-b$
$x=\frac{c-b}{a}$

(ii) $p-qx^2=r$
$p=r+qx^2$
$p-r=qx^2$
$\frac{p-r}{q}=x^2$
$x=\sqrt{\frac{p-r}{q}}$

(iii) $\sqrt{\frac{x}{s}} = t$

$\frac{x}{s} = t^2$

$x = st^2$

(iv) $a - \frac{b}{x} = c$

$a = c + \frac{b}{x}$

$a - c = \frac{b}{x}$

$x(a-c) = b$

$x = \frac{b}{a-c}$

(v) $px + q = a - bx$

$px + bx + q = a$

$px + bx = a - q$

$x(p+b) = a - q$

$x = \frac{a-q}{p+b}$

(vi) $y = \frac{1}{w(z-x^2)}$

$wy(z-x^2) = 1$

$z - x^2 = \frac{1}{wy}$

$z = \frac{1}{wy} + x^2$

$z - \frac{1}{wy} = x^2$

$x = \sqrt{z - \frac{1}{wy}}$

8. Method 1

$$\frac{x^2+6x+8}{2x^2+7x-4}=3$$

$$\frac{(x+4)(x+2)}{(x+4)(2x-1)}=3$$

$$\frac{x+2}{2x-1}=3$$

$$x+2=3(2x-1)$$

$$x+2=6x-3$$

$$5=5x$$

$$x=1$$

Check in original equation: $\frac{1+6+8}{2+7-4}=\frac{15}{5}=3$

Method 2

$$\frac{x^2+6x+8}{2x^2+7x-4}=3$$

$$x^2+6x+8=3(2x^2+7x-4)$$

$$x^2+6x+8=6x^2+21x-12$$

$$0=5x^2+15x-20$$

$$x^2+3x-4=0$$

$$(x+4)(x-1)=0$$

$$x=1,-4$$

Check in original equation. -4 leads to 0/0 so x=1.

9. Method 1

$$(m+n)^2+m^2-n^2=(m+n)^2+(m+n)(m-n)$$
$$=(m+n)(m+n+m-n)$$
$$=(m+n)(2m)$$

This has 2 as a factor so it must be even.

Method 2

$$(m+n)^2+m^2-n^2=m^2+2mn+n^2+m^2-n^2$$
$$=2m^2+2mn$$
$$=2m(m+n)$$

This has 2 as a factor so it must be even.