Section 1: Factorising, algebraic fractions and formulae

Exercise

1. Factorise the following expressions:
   (i) $10ab + 5ac$
   (ii) $2x^2 + 4xy - 8xz$
   (iii) $3s^2t - 9s^3t + 12s^2t^2$
   (iv) $3(b - c) - 2a(b - c)$

2. Factorise these quadratic expressions.
   (i) $x^2 + 5x + 6$ (ii) $x^2 + x - 12$ (iii) $x^2 - 9$
   (iv) $x^2 - 6xy + 8y^2$ (v) $2x^2 + 3xy + y^2$ (vi) $3x^2 + x - 2$
   (vii) $4x^2 - 8x + 3$ (viii) $4x^2 - 25y^2$ (ix) $6x^2 - xy - 12y^2$

3. Factorise these expressions.
   (i) $(x+4)^2 - (x-3)^2$
   (ii) $(2x-y)^2 - (x+3y)^2$

4. Simplify the following as much as possible:
   (i) $\dfrac{2a^2b}{4ab^2}$ (ii) $\dfrac{12p^2qr^3}{9pq^2r}$
   (iii) $\dfrac{x^2y + xy^2}{x+y}$ (iv) $\dfrac{a}{2b} \times \dfrac{3bc}{a^2} \times \dfrac{a}{6c}$

5. Simplify these expressions where possible.
   (i) $\dfrac{x^2+x-6}{x^2-x-2}$ (ii) $\dfrac{x^2-4x+4}{x^2+x-6}$
   (iii) $\dfrac{x^2+x-2}{x^2+4x+3}$ (iv) $\dfrac{4x^2-1}{4x^2-4x-3}$
   (v) $\dfrac{2x+3}{3x+1} \times \left(3x^2-2x-1\right)$ (vi) $\dfrac{x+2}{2x^2-x-1} \div \dfrac{x^2-x-6}{2x+1}$

6. Write as single fractions:
   (i) $\dfrac{2x}{5} + \dfrac{3x}{2}$ (ii) $\dfrac{3a}{4} - \dfrac{2b}{3}$
   (iii) $\dfrac{2x+1}{12} - \dfrac{x-2}{8}$ (iv) $\dfrac{3x+4}{2x} - \dfrac{5x+6}{3x}$
   (v) $\dfrac{1}{p} + \dfrac{1}{q}$ (vi) $\dfrac{a}{2b} + \dfrac{5b}{3a}$
   (vii) $\dfrac{3}{2x+1} - \dfrac{2}{x-1}$ (viii) $\dfrac{2x}{x-2} - \dfrac{x+1}{x+3}$

7. Make x the subject of each of these formulae:
   (i) $ax + b = c$ (ii) $p - qx^2 = r$

(iii) $\sqrt{\frac{x}{s}} = t$

(iv) $a - \frac{b}{x} = c$

(v) $px + q = a - bx$

(vi) $y = \frac{1}{w(z - x^2)}$

8. Solve

$$\frac{x^2 + 6x + 8}{2x^2 + 7x - 4} = 3$$

9. m and n are both positive integers. Prove that $(m + n)^2 + m^2 - n^2$ is even.