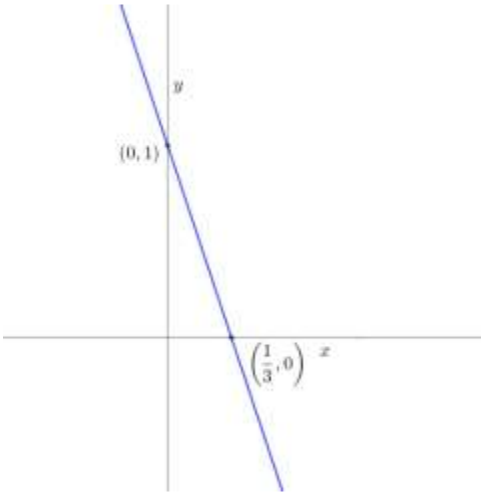
Section 1: Functions

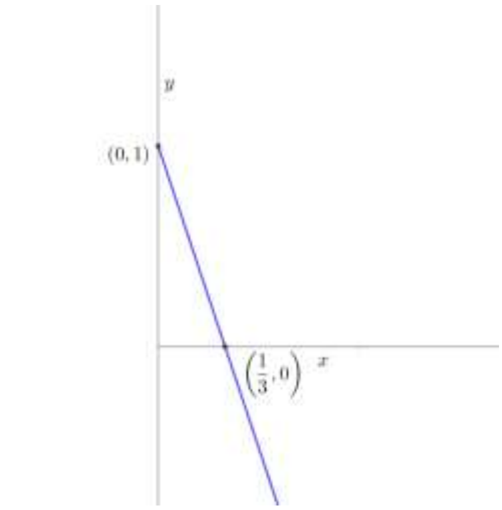
Solutions to Exercise

1. (i) $y = 1 - 3x$ where x can take any value
When $x = 0$, $y = 1$
When $y = 0$, $1 - 3x = 0 \quad \Rightarrow x = \frac{1}{3}$



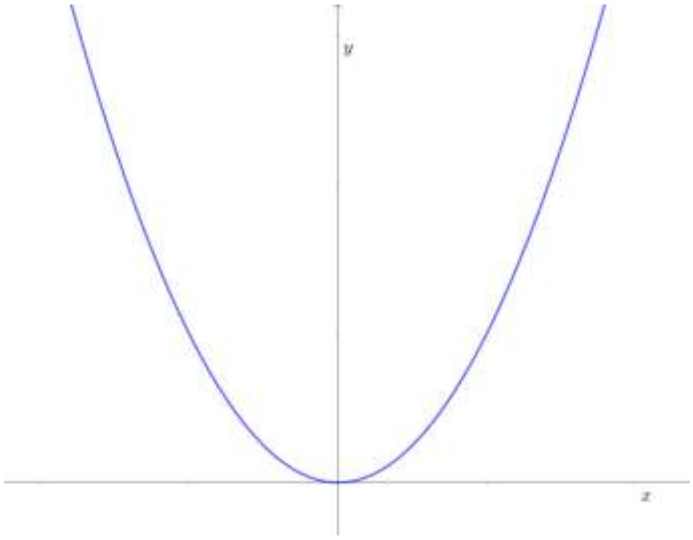
From the graph we can see the range is $f(x) \in \mathbb{R}$.

(ii) $y = 1 - 3x$ where $x > 0$
When $x = 0$, $y = 1$
When $y = 0$, $1 - 3x = 0 \quad \Rightarrow x = \frac{1}{3}$



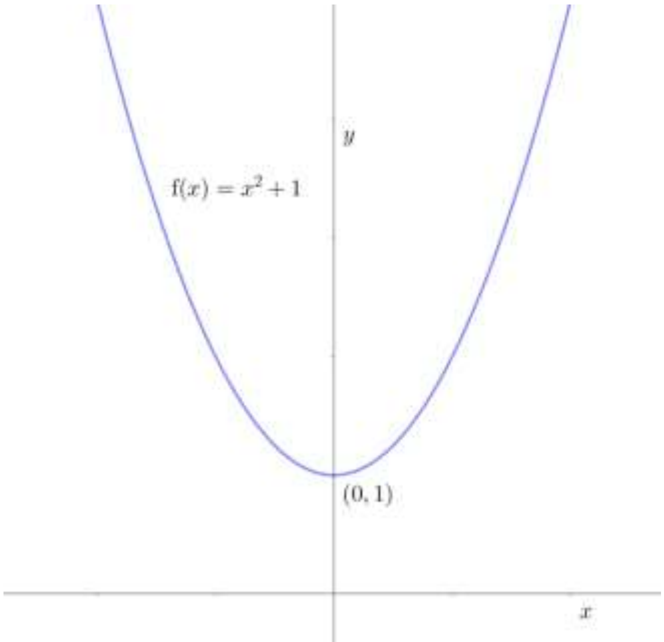
From the graph we can see the range is $f(x) < 1$

(iii) $y = x^2$ where x can take any value
When $x = 0$, $y = 0$
The graph is a positive quadratic with minimum at (0, 0)



The range is $f(x) \geq 0$.

(iv) $f(x) = x^2 + 1$ where x can take any value
When $x = 0$, $y = 1$

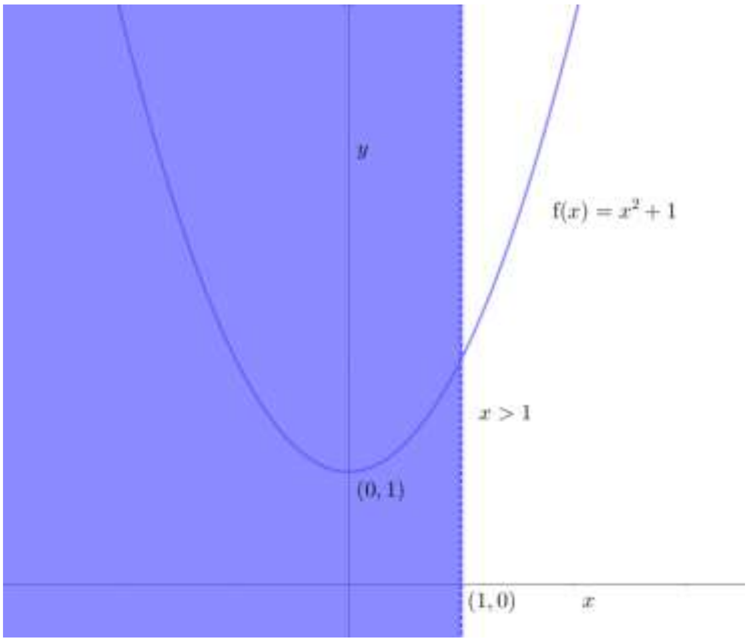


From the graph, we can see the range is $f(x) \geq 1$

(v)

$f(x) = x^2 + 1$ where $x > 1$

Using the same graph as part (iv), we shade out the values for which $x \leq 1$

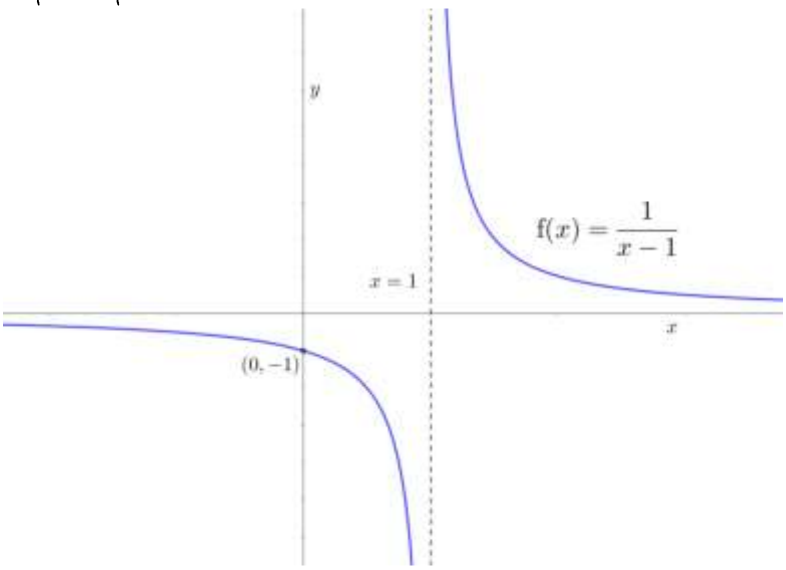


To find the range, look at where the line $x = 1$ meets $f(x) = x^2 + 1$

$f(1) = (1)^2 + 1 = 2$

Since $x > 1$ is a strict inequality, the range of the function is $f(x) > 2$

2. (i) $x = 1$ must be excluded from the domain, since the function is not defined for this value.



(ii)

(a) $f(2) = \frac{1}{2-1} = 1$

(b) $f(-3) = \frac{1}{-3-1} = -\frac{1}{4}$

(c) $f(0) = \frac{1}{0-1} = -1$

(iii) $f(x) = 2$

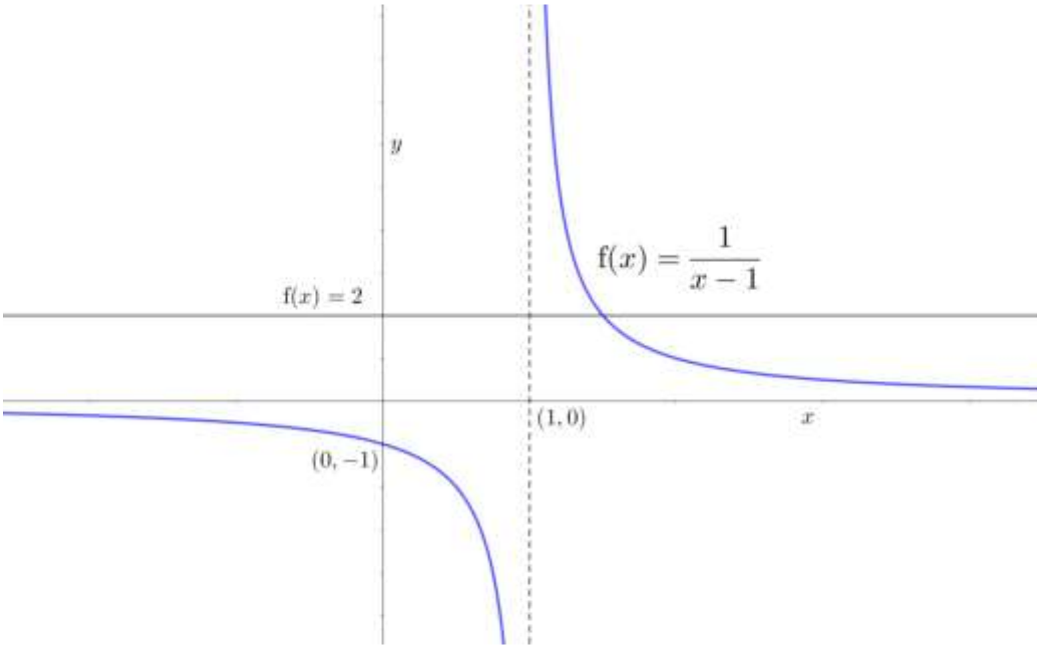
$\frac{1}{x-1} = 2$

$1 = 2(x-1)$

$1 = 2x - 2$

$2x = 3$

$x = \frac{3}{2}$



3. $f(x) = x + 1$
$g(x) = x^3$
$h(x) = \frac{1}{x}$

(i) $f(g(x)) = f(x^3) = x^3 + 1$
(ii) $g(f(x)) = g(x+1) = (x+1)^3 = x^3 + 3x^2 + 3x + 1$
(iii) $f(h(x)) = f\left(\frac{1}{x}\right) = \frac{1}{x} + 1$
(iv) $g(h(x)) = g\left(\frac{1}{x}\right) = \left(\frac{1}{x}\right)^3 = \frac{1}{x^3}$

4.

(i) Let y be the output from f

So, $y = 4x + 2$

$4x = y - 2$ and

$x = \dfrac{y-2}{4}$

Replace x with $f^{-1}(x)$ and y with x

So, $f^{-1}(x) = \dfrac{y-2}{4}$

(ii) Let y be the output from f

So,

$y = \dfrac{x+7}{3}$

$x + 7 = 3y$

$x = 3y - 7$

Replace x with $f^{-1}(x)$ and y with x

So, $f^{-1}(x) = 3x - 7$

5. $f(x) = (x - 3)^4$

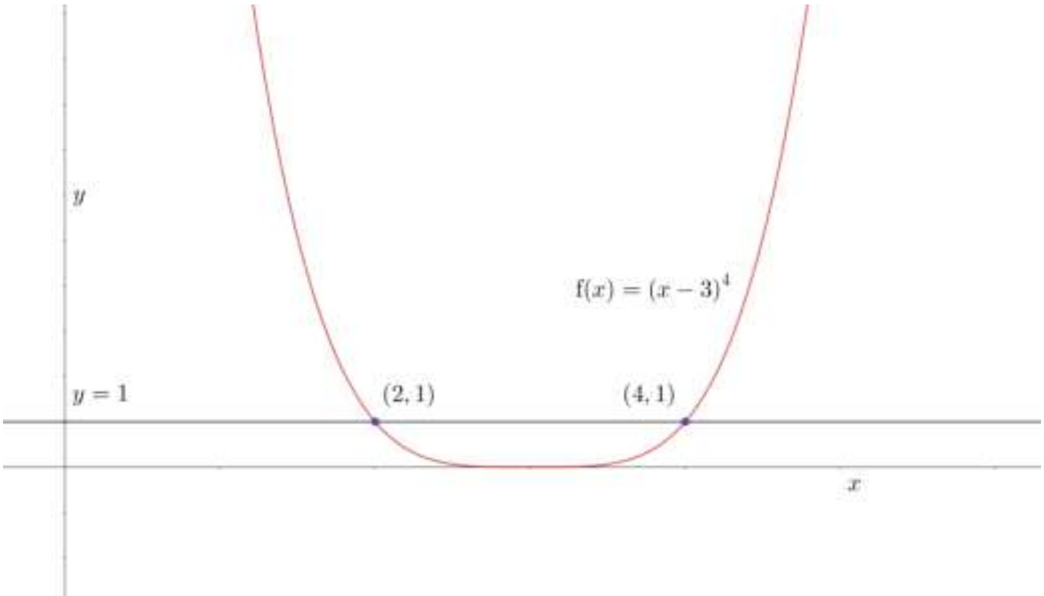
When $x = 4$

$f(4) = (4 - 3)^4 = 1^4 = 1$

When $x = 2$

$f(2) = (2 - 3)^4 = (-1)^4 = 1$

Both $x = 4$ and $x = 2$ are mapped to $y = 1$ so this is a many – to – one function as many values for x are mapped to one y value.



An inverse can't exist when $x \in \mathbb{R}$ because there are two possible values it should take when x=1 making it a one – to - many function. So, we need to restrict the domain.

Method 1

Find the inverse function:

$y = (x-3)^4$

$x - 3 = \sqrt[4]{y}$

$x = 3 + \sqrt[4]{y}$

$f^{-1}(x) = 3 + \sqrt[4]{x}$

Taking fourth root of a negative value will not give you a real number, so x can't be negative. This means the domain for $f^{-1}(x)$ is $x \geq 0$

Method 2

The range of a function is the domain of its inverse function. The range of f(x) is $f(x) \geq 0$ so the domain for $f^{-1}(x)$ is $x \geq 0$