Section 3: Exponentials functions

Exercise solutions

1.

(i)

x	0	1	2
$y=a^x$	1	2	4

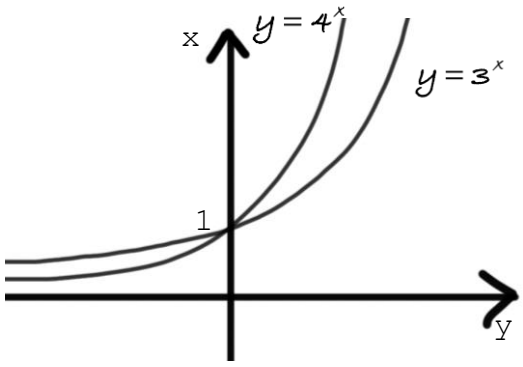
(ii) since $y=a^1=2$, $a=2$

(iii) at x=4, $y=2^4=16$

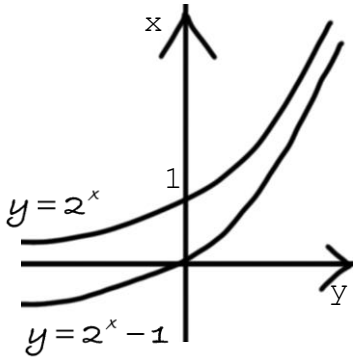
at x=-2, $y=2^{-2}=\frac{1}{2^2}=\frac{1}{4}$

2.

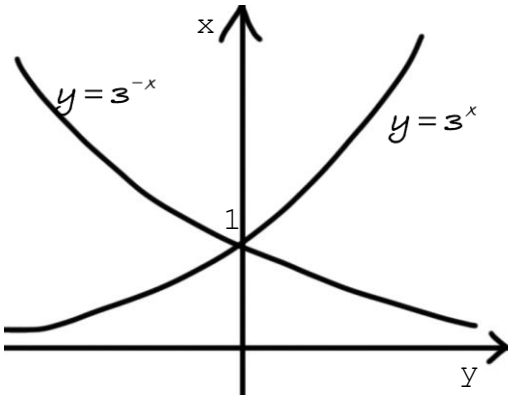
(i) Both curves must go through (0,1), which is labelled.
Neither graph touches or crosses the x-axis.
$y=4^x$ is steeper than $y=3^x$ and both curves are labelled



(ii)
$y=2^x$ crosses the y-axis at 1.
$y=2^x-1$ passes through the origin.
The graphs should not meet.
Both curves are labelled



(iii)
The graphs should look symmetric across the x axis.
Both pass through (0,1), which is labelled.



Neither graph touches or crosses the x-axis.
Both curves are labelled.

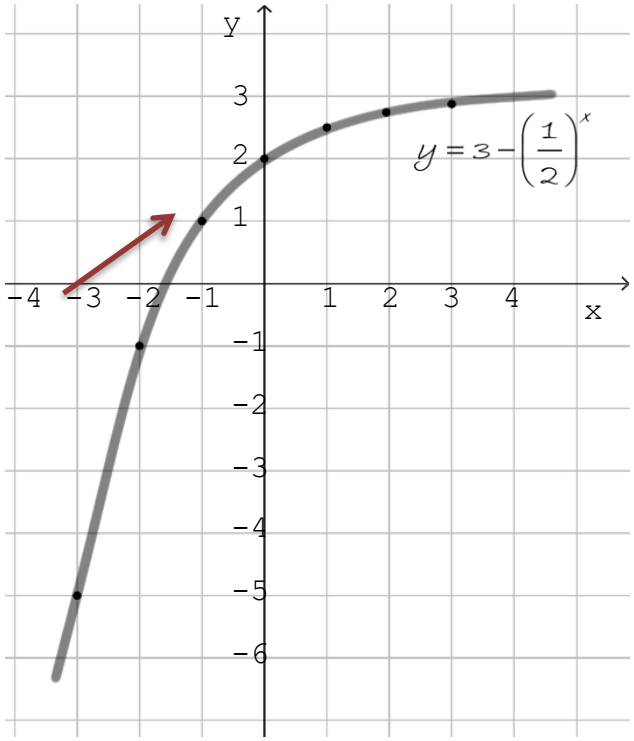
3.

(i)

x	-3	-2	-1	0	1	2	3
$y=3-\left(\frac{1}{2}\right)^x$	-5	-1	1	2	2.5	2.75	2.875

(ii)

Points plotted
Graph labelled
Smooth curve drawn



(iii)

Some indication of having read from the graph.

Solution in range

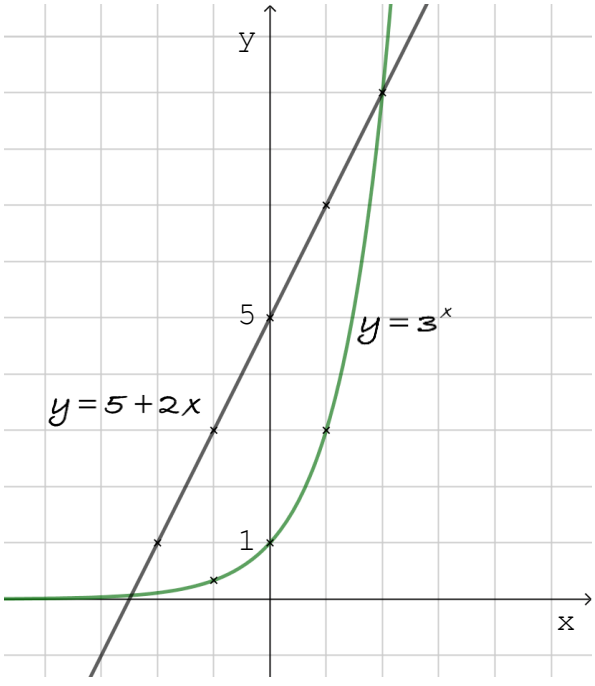
$-1.8 \leq x \leq -1.3$

(More accuracy than 1dp suggests solving by a different method
e.g. x=-1.58496...)

4.

(i)

Graph shows:
-Line passing through (0,5).
-Curve passes through (0,1).



(ii)

As the graphs intersect twice, there will be 2 solutions.

5.

(i)

$y = a \times b^x$

at $x=-1$, $y=5$, so:

$$5 = a \times b^{-1}$$

$$5 = \frac{a}{b}$$

$$a = 5b$$

also, at $x=5$, $y=3645$

$$3645 = a \times b^5$$

$$3645 = 5b \times b^5$$

$$\frac{3645}{5} = b^6$$

$$729 = b^6$$

$$b = \sqrt[6]{729}$$

$$b = 3$$

since $a=5b$,

$$a=15$$

(ii)

$y=135$ where $x=c$:

$$y = 15 \times 3^x$$

$$135 = 15 \times 3^c$$

$$\frac{135}{15} = 3^c$$

$$9 = 3^c$$

$$c = 2$$

6.

(i)

$P(t) = 40000 \times (1.2)^t$

(ii)

$P(5) = 40000 \times (1.2)^5 = 99533$

7.

(i) $P(324) = 1013 \times 0.88^{(324/1000)} = 972$ millibars

(ii) $P(1345) = 1013 \times 0.88^{(1345/1000)} = 853$ millibars

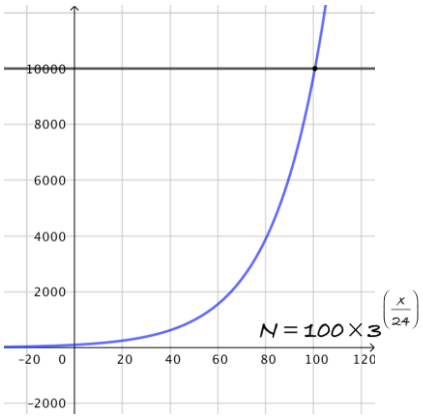
8.

(i) a $x=0$, $N = 100 \times 3^{\left(\frac{0}{24}\right)} = 100 \times 3^{0} = 100 \times 1 = 100$

(ii) After 2 days, 48 hours have passed.

At $x=48$, $N = 100 \times 3^{\left(\frac{48}{24}\right)} = 100 \times 3^{2} = 100 \times 9 = 900$

(iii) Plot a graph of $N = 100 \times 3^{\left(\frac{x}{24}\right)}$



Reading off the graph at 10 000 gives an answer of approx. 100.

At $x=100$, $N = 100 \times 3^{\left(\frac{100}{24}\right)} = 9728$

At $x=101$, $N = 100 \times 3^{\left(\frac{101}{24}\right)} = 10183$

We would expect it to take 4 days and 5 hours to reach 10 000 bacteria.