AQA Level 2 Further Mathematics Algebra IV

Section 3: The factor theorem

Solutions to Exercise

1. (i) $f(x) = x^3 - 4x^2 + x + 6$

$f(-1) = (-1)^3 - 4(-1)^2 + (-1) + 6 = -1 - 4 - 1 + 6 = 0$

so by the factor theorem, $x + 1$ is a factor.

(ii) $x^3 - 4x^2 + x + 6 = (x+1)(x^2 - 5x + 6)$

$= (x+1)(x-2)(x-3)$

2. (i) $f(x) = x^3 + ax^2 - 4x + 12$

$f(2) = 2^3 + a \times 2^2 - 4 \times 2 + 12$

$= 8 + 4a - 8 + 12$

$= 4a + 12$

$x - 2$ is a factor so by the factor theorem $f(2) = 0$

$4a + 12 = 0$

$a = -3$

(ii) $x^3 - 3x^2 - 4x + 12 = (x-2)(x^2 - x - 6)$

$= (x-2)(x+2)(x-3)$

3. (i) $f(x) = x^3 - 2x^2 - 11x + 12$

$f(1) = 1 - 2 - 11 + 12 = 0$ so $(x - 1)$ is a factor

$x^3 - 2x^2 - 11x + 12 = 0$

$(x-1)(x^2 - x - 12) = 0$

$(x-1)(x+3)(x-4) = 0$

$x = 1, x = -3, x = 4$

(ii) $f(x) = x^3 + 4x^2 - 3x - 18$

$f(1) = 1 + 4 - 3 - 18 \neq 0$

$f(-1) = -1 + 4 + 3 - 18 \neq 0$

$f(2) = 8 + 16 - 6 - 18 = 0$

so $(x - 2)$ is a factor

$x^3 + 4x^2 - 3x - 18 = 0$

$(x-2)(x^2 + 6x + 9) = 0$

$(x-2)(x+3)^2 = 0$

$x = 2$ or $x = -3$

(iii) $f(x)=x^3-19x-30$

$f(1)=1-19-30\neq 0$

$f(2)=8-38-30\neq 0$

$f(-2)=-8+38-30=0$

so $x+2$ is a factor

$x^3-19x-30=0$

$(x+2)(x^2-2x-15)=0$

$(x+2)(x+3)(x-5)=0$

$x=-2$ or $x=-3$ or $x=5$

4. (i) $f(x)=2x^3+5x^2+5x+3$

$f(-\frac{3}{2})=2(-\frac{3}{2})^3+5(-\frac{3}{2})^2+5(-\frac{3}{2})+3$

$=-\frac{27}{4}+\frac{45}{4}-\frac{15}{2}+3$

$=\frac{9}{2}-\frac{15}{2}+3$

$=0$

$f(-\frac{3}{2})=0$ so $(x+\frac{3}{2})$ is a factor of $f(x)$ so $(3x+2)$ is a factor of $f(x)$.

(ii) $2x^3+5x^2+5x+3=0$

$(2x+3)(x^2+x+1)=0$

The discriminant of x^2+x+1 is $1-4\times1\times1$ which is less than 0, so the quadratic $x^2+x+1=0$ has no real roots. So the only root is $x=-\frac{3}{2}$.

5. (i) $f(x)=12x^3-4x^2-3x+1$

$f(1)=12-4-3+1=6$ so $(x-1)$ is not a factor

$f(-1)=-12-4+3+1=-12$ so $(x+1)$ is not a factor

(ii) Since the constant term is 1, all factors must be of the form $(ax\pm1)$ so all roots must be of the form $\pm\dfrac{1}{a}$. Since we have shown that $(x-1)$ and $(x+1)$ are not factors, the value of a for all the roots must be greater than 1, so the roots cannot be integers.

(iii) $f(\frac{1}{2})=12(\frac{1}{2})^3-4(\frac{1}{2})^2-3(\frac{1}{2})+1=\frac{3}{2}-1-\frac{3}{2}+1=0$

$f(\frac{1}{2})=0$ so $(x-\frac{1}{2})$ is a factor of $f(x)$ so $(2x-1)$ is a factor of $f(x)$.

(iv) $12x^3-4x^2-3x+1=0$

$(2x-1)(6x^2+x-1)=0$

$(2x-1)(2x+1)(3x-1)=0$

$x = \frac{1}{2}, -\frac{1}{2}, \frac{1}{3}$

6. If Bob is right, then $x = 1$, $x = 2$ and $x = -5$ would all make $x^3 - 4x^2 - 7x + 10$ be zero.

<u>$x=1$</u>

$x^3 - 4x^2 - 7x + 10 = 1 - 4 - 7 + 10 = 0$ so $(x - 1)$ is a factor.

<u>$x=2$</u>

$x^3 - 4x^2 - 7x + 10 = 8 - 16 - 14 + 10 = -12 \neq 0$ so $(x - 2)$ is not a factor.

7. $xy - 9 = 15$

$2x + 2y = 20$

$y = 10 - x$

$x(10 - x) = 24$

$10x - x^2 = 24$

$x^2 - 10x + 24 = 0$

$(x - 6)(x - 4) = 10$

$x = 6$ and $y = 4$ (or $x = 4$ and $y = 6$)