Section 2: Simultaneous Equations

Solutions to Exercise

1. (i) $2x+5y=11$ (1)

$2x-y=5$ (2)

Subtracting: $6y=6$

$y=1$

Substituting into (1): $2x+5\times1=11$

$2x=6$

$x=3$

The solution is $x=3$, $y=1$. Check: $2x+5y=2\times3+5\times1=11$

$2x-y=2\times3-1=5$

(ii) $x+2y=6$ (1)$\times4$ $4x+8y=24$

$4x+3y=4$ (2) $4x+3y=4$

Subtracting: $5y=20$

$y=4$

Substituting into (1): $x+2\times4=6$

$x=-2$

The solution is $x=-2$, $y=4$. Check: $x+2y=-2+8=6$

$4x+3y=-8+12=4$

(iii) $3a-2b=4$ (1)$\times2$ $6a-4b=8$

$5a+4b=3$ (2) $5a+4b=3$

Adding: $11a=11$

$a=1$

Substituting into (1): $3\times1-2b=4$

$-2b=1$

$b=-\frac{1}{2}$

The solution is $a=1$, $b=-\frac{1}{2}$. Check: $3a-2b=3+1=4$

$5a+4b=5-2=3$

(iv) $2p-5q=5$ (1)$\times3$ $6p-15q=15$

$3p-2q=-9$ (2)$\times2$ $6p-4q=-18$

Subtracting: $-11q=33$

$q=-3$

Substituting into (1): $2p - 5 \times -3 = 5$

$2p = -10$

$p = -5$

The solution is $p = -5$, $q = -3$. Check: $2p - 5q = -10 + 15 = 5$

$3p - 2q = -15 + 6 = -9$

(v) $5x + 3y = 9$ (1)

$y = 3x - 4$ (2)

Substituting (2) into (1): $5x + 3(3x - 4) = 9$

$5x + 9x - 12 = 9$

$14x = 21$

$x = \frac{3}{2}$

Substituting into (1): $y = 3 \times \frac{3}{2} - 4 = \frac{9}{2} - 4 = \frac{1}{2}$

The solution is $x = \frac{3}{2}, y = \frac{1}{2}$. Check: $5x + 3y = \frac{15}{2} + \frac{3}{2} = 9$

(vi) $3a + 2b = 1$ (1) $\times 2$ $\quad 6a + 4b = 2$

$9a - 4b = 4$ (2) $\quad 9a - 4b = 4$

Adding: $15a = 6$

$a = \frac{2}{5}$

Substituting into (1): $3 \times \frac{2}{5} + 2b = 1$

$2b = 1 - \frac{6}{5} = -\frac{1}{5}$

$b = -\frac{1}{10}$

The solution is $a = \frac{2}{5}$, $b = -\frac{1}{10}$. Check: $3a + 2b = \frac{6}{5} - \frac{1}{5} = 1$

$9a - 4b = \frac{18}{5} + \frac{2}{5} = 4$

2. (i) $7x^2 + y^2 = 64$ (1)

$x + y = 4$ (2)

(2) $\Rightarrow y = 4 - x$

Substituting into (1): $7x^2 + (4 - x)^2 = 64$

$7x^2 + 16 - 8x + x^2 = 64$

$8x^2 - 8x - 48 = 0$

$x^2 - x - 6 = 0$

$(x - 3)(x + 2) = 0$

$x = 3$ or $x = -2$

When $x = 3$, $y = 4 - 3 = 1$

When $x = -2$, $y = 4 - (-2) = 6$

The solutions are $x = 3, y = 1$ and $x = -2, y = 6$

Check: $x = 3, y = 1 \Rightarrow 7x^2 + y^2 = 63 + 1 = 64$

$x = -2, y = 6 \Rightarrow 7x^2 + y^2 = 28 + 36 = 64$

(ii) $3x^2 - 2y^2 = -5$ (1)

$y - x = 1$ (2)

(2) $\Rightarrow y = 1 + x$

Substituting into (1): $3x^2 - 2(1 + x)^2 = -5$

$3x^2 - 2(1 + 2x + x^2) = -5$

$3x^2 - 2 - 4x - 2x^2 = -5$

$x^2 - 4x + 3 = 0$

$(x - 1)(x - 3) = 0$

$x = 1$ or $x = 3$

When $x = 1$, $y = 1 + 1 = 2$

When $x = 3$, $y = 1 + 3 = 4$

The solutions are $x = 1, y = 2$ and $x = 3, y = 4$

Check: $x = 1, y = 2 \Rightarrow 3x^2 - 2y^2 = 3 - 5 = -5$

$x = 3, y = 4 \Rightarrow 3x^2 - 2y^2 = 27 - 32 = -5$

(iii) $p^2 + pq = 2$ (1)

$q - p = 3$ (2)

(2) $\Rightarrow q = 3 + p$

Substituting into (1): $p^2 + p(3 + p) = 2$

$p^2 + 3p + p^2 = 2$

$2p^2 + 3p - 2 = 0$

$(2p - 1)(p + 2) = 0$

$p = \frac{1}{2}$ or $p = -2$

When $p = \frac{1}{2}$, $q = 3 + \frac{1}{2} = \frac{7}{2}$

When $p = -2$, $q = 3 - 2 = 1$

The solutions are $p = \frac{1}{2}, q = \frac{7}{2}$ and $p = -2, q = 1$.

Check: $p = \frac{1}{2}, q = \frac{7}{2} \Rightarrow p^2 + pq = \frac{1}{4} + \frac{7}{4} = 2$

$p = -2, q = 1 \Rightarrow p^2 + pq = 4 - 2 = 2$

(iv) $8a^2 - b^2 = 2 \quad (1)$

$2a + b = 1 \quad (2)$

$(2) \Rightarrow b = 1 - 2a$

Substituting into (1): $8a^2 - (1-2a)^2 = 2$

$8a^2 - (1 - 4a + 4a^2) = 2$

$8a^2 - 1 + 4a - 4a^2 = 2$

$4a^2 + 4a - 3 = 0$

$(2a + 3)(2a - 1) = 0$

$a = -\frac{3}{2}$ or $a = \frac{1}{2}$

When $a = -\frac{3}{2}, b = 1 - 2 \times -\frac{3}{2} = 1 + 3 = 4$

When $a = \frac{1}{2}, b = 1 - 2 \times \frac{1}{2} = 1 - 1 = 0$

The solutions are $a = -\frac{3}{2}, b = 4$ and $a = \frac{1}{2}, b = 0$.

Check: $a = -\frac{3}{2}, b = 4 \Rightarrow 8a^2 - b^2 = 8 \times \frac{9}{4} - 16 = 18 - 16 = 2$

$a = \frac{1}{2}, b = 0 \Rightarrow 8a^2 - b^2 = 8 \times \frac{1}{4} - 0 = 2$

3. $xy - 9 = 15$

$2x + 2y = 20$

$y = 10 - x$

$x(10 - x) = 24$

$10x - x^2 = 24$

$x^2 - 10x + 24 = 0$

$(x - 6)(x - 4) = 10$

$x = 6$ and $y = 4$ (or $x = 4$ and $y = 6$)

4. (i)

(1) $x - y + z = 4$

(2) $3x + y + 3z = -4$

(3) $x + y + 2z = -2$

eliminate x by combining equations

(3) − (1) $2y + z = -6$

3 * (3) − (2) $2y + 3z = -2$

eliminate y by combining equations

$2z = -2 - (-6)$

$z = 2$

Substitute in (1) and (3)

(1) $x - y + 2 = 4$

(3) $x + y + 4 = -2$

$2x + 6 = 2$

$2x = -4$

$x = -2$

Substitute in (3)

(3) $-2 + y + 4 = -2$

$y = -4$

so x= -2, y=-4 and z=2

(ii)

(1) $2x + 2y - z = 0$

(2) $8x - 12y - z = -32$

(3) $-2x - 4y - z = 6$

eliminate z by combining equations

(1)−(3) $4x + 6y = -6$

(2)−(1) $6x - 14y = -32$

eliminate x by combining equations

$6x + 9y = -9$

$6x - 14y = -32$

$9y + 14y = -9 - (-32)$

$23y = 23$

$y = 1$

Substitute in (1) and (3)

(1) $2x + 2 - z = 0$

(3) $-2x - 4 - z = 6$

$-2 - 2z = 6$

$2z = -8$

$z = -4$

Substitute in (1)

$2x + 2 + 4 = 0$

$2x = -6$

$x = -3$

so $x = -3$, $y = 1$ and $z = -4$

5. (i)

(1) $2x + y + 3z = 8$

(2) $-3x + 3y + 2z = 12$

(3) $3x + 2y - z = -4$

eliminate y by combining equations

$3*(1) - (2)$ $9x + 7z = 12$

$2*(1) - (3)$ $x + 7z = 20$

eliminate z by combining equations

$8x = -8$

$x = -1$

Substitute in (1) and (2) and combine

(1) $-2 + y + 3z = 8$

(2) $3 + 3y + 2z = 12$

$7z = 21$

$z = 3$

Substitute in (3)

$(1) \quad -2 + y + 9 = 8$

$y = 1$

so $x = -1$, $y = 1$ and $z = 3$

(ii)

(1) $x - 2y - 2z = 1$

(2) $2x - y - 14z = 3$

(3) $2x - 3y + 3z = -8$

eliminate x by combining equations

$(2) - (3)$ $2y - 17z = 11$

$2*(1) - (3)$ $-y - 7z = 10$

eliminate y by combining equations

$-31z = 31$

$z = -1$

Substitute in (1) and (2) and combine

(1) $x-2y+2=1$

(2) $2x-y+14=3$

$-3y=9$

$y=-3$

Substitute in (3)

(1) $x+6+2=1$

$x=-7$

so x= -7, y=-3 and z=-1

6. let x represent number of cupcakes, y be number of bags crisps and z be number of cans of soda

Abby: $16x+40y+24z=36$

Ben: $24x+20y+12z=30$

Carrie: $12x+10y+36z=30$

Simplify the equations:

(A): $4x+10y+6z=9$

(B): $12x+10y+6z=15$

(C): $6x+5y+18z=15$

eliminate y and z by combining equations

(B)−(A) $8x=6$

$$x=\frac{6}{8}=\frac{3}{4}=0.75$$

Substitute in (A) and (C) and combine

(A) $4(0.75)+10y+6z=9$

(C) $6(0.75)+5y+18z=15$

➔

(A) $10y+6z=6$

(C) $5y+18z=10.5$

$30z=15$

$$z=\frac{15}{30}=0.5$$

Substitute in (A)

(1) $10y+3=6$

$10y=3$

$$y=\frac{3}{10}=0.3$$

So the cupcakes were 75p each, the crisps were 30p a bag and the sodas were 50p a can.