AQA Level 2 Further Mathematics Coordinate geometry

Section 2: Circles

Solutions to Exercise

1. (i) $x^2 + y^2 = 36$

   (ii) $(x-3)^2 + (y-1)^2 = 25$

   (iii) $(x+2)^2 + (y-5)^2 = 1$

   (iv) $(x-0)^2 + (y+4)^2 = 9$
   $x^2 + (y+4)^2 = 9$

2. (i) $x^2 + y^2 = 100 = 10^2$
   Centre = (0, 0), radius = 10.

   (ii) $(x-2)^2 + \left(y-7\right)^2 = 16 = 4^2$
   Centre = (2, 7), radius = 4

   (iii) $(x+3)^2 + \left(y-4\right)^2 = 4 = 2^2$
   Centre = (-3, 4), radius = 2

   (iv) $(x+4)^2 + \left(y+5\right)^2 = 20$
   Centre = (-4, -5), radius $= \sqrt{20}$

3. (i) The centre of the circle moves from (0, 0) to (5, -2), so the transformation is a translation through $\begin{pmatrix} 5 \\ -2 \end{pmatrix}$.

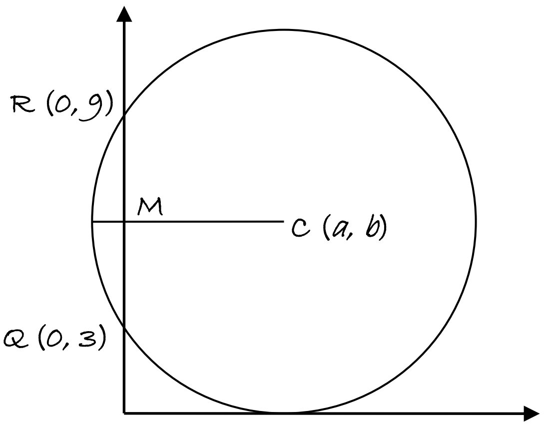
   (ii) The centre of the circle moves from (-1, 3) to (0, 0), so the transformation is a translation through $\begin{pmatrix} 1 \\ -3 \end{pmatrix}$.

4. Radius of circle $= \sqrt{(6-4)^2 + (3-(-2))^2} = \sqrt{4+25} = \sqrt{29}$
   Equation of circle is $(x-4)^2 + (y+2)^2 = 29$

5. Centre of circle C is the midpoint of AB.
   $$C = \left(\frac{2+6}{2}, \frac{0+4}{2}\right) = (4, 2)$$

Radius of circle is distance AC $= \sqrt{(2-4)^2 + (0-2)^2} = \sqrt{4+4} = \sqrt{8}$
Equation of circle is $(x-4)^2 + (y-2)^2 = 8$

6.



The midpoint M of QR is (0, 6).
Since a diameter which passes through M is perpendicular to QR, then the line CM must be horizontal, and therefore b = 6.

Since the circle touches the x-axis, the radius of the circle must be b, i.e. 6.

The equation of the circle is therefore $(x-a)^2 + (y-6)^2 = 6^2$
The circle passes through (0, 3), so $(0-a)^2 + (3-6)^2 = 6^2$

$$a^2 + 9 = 36$$
$$a^2 = 27$$
$$a = \pm\sqrt{27} = \pm 3\sqrt{3}$$

The equation of the circle is either $(x-3\sqrt{3})^2 + (y-6)^2 = 36$
or $(x+3\sqrt{3})^2 + (y-6)^2 = 36$.

7. (i) $x^2 + y^2 = 8$
Substituting in $y = 4 - x$ gives $x^2 + (4-x)^2 = 8$

$$x^2 + 16 - 8x + x^2 = 8$$
$$2x^2 - 8x + 8 = 0$$
$$x^2 - 4x + 4 = 0$$
$$(x-2)^2 = 0$$

The line meets the circle at just one point, so the line touches the circle and is therefore a tangent.

(ii) $x^2 + y^2 = 25$

Substituting in $4y = 3x - 25 \quad \Rightarrow y = \frac{3x-25}{4}$

gives $x^2 + y^2 = 25$

$$x^2 + \left(\frac{3x-25}{4}\right)^2 = 25$$

$$x^2 + \frac{(3x-25)^2}{16} = 25$$

$$16x^2 + 9x^2 - 150x + 625 = 400$$

$$25x^2 - 150x + 225 = 0$$

$$x^2 - 6x + 9 = 0$$

$$(x-3)^2 = 0$$

The line meets the circle at just one point, so the line touches the circle and is therefore a tangent.

8. $x^2 + y^2 = 65$

$2y + x = 10 \quad \Rightarrow x = 10 - 2y$

Substituting in: $(10-2y)^2 + y^2 = 65$

$$100 - 40y + 4y^2 + y^2 = 65$$

$$5y^2 - 40y + 35 = 0$$

$$y^2 - 8y + 7 = 0$$

$$(y-1)(y-7) = 0$$

$$y = 1 \text{ or } y = 7$$

When $y = 1$, $x = 10 - 2 \times 1 = 8$

When $y = 7$, $x = 10 - 2 \times 7 = -4$

so P is (8, 1) and Q is (-4, 7)

Length PQ $= \sqrt{(8-(-4))^2 + (1-7)^2} = \sqrt{144+36} = \sqrt{180}$

9. (i) Gradient of PR $= \frac{7-6}{5-(-2)} = \frac{1}{7}$

Gradient of QR $= \frac{7-0}{5-6} = \frac{7}{-1} = -7$

Gradient of PR × gradient of QR $= \frac{1}{7} \times -7 = -1$

so PR and QR are perpendicular.

(ii) The angle in a semicircle is 90°, so PQ must be a diameter.

(iii) Since PQ is a diameter, the centre C of the circle is the midpoint of PQ.

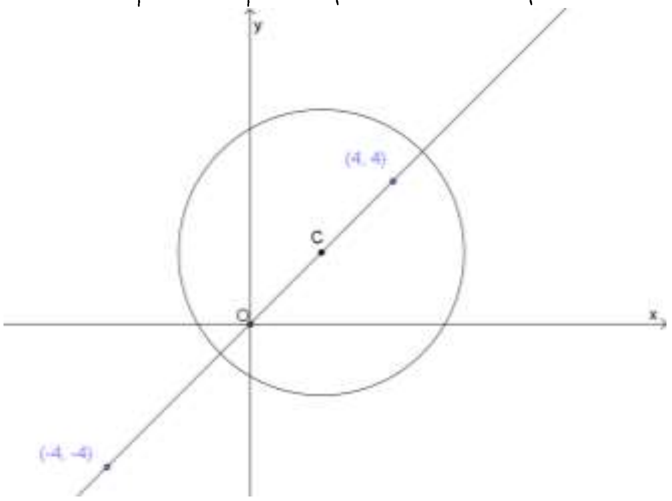
$$C = \left(\frac{-2+6}{2}, \frac{6+0}{2}\right) = (2,3)$$

Radius of circle = length CQ $= \sqrt{(6-2)^2 + (0-3)^2}$

$$= \sqrt{16+9} = \sqrt{25} = 5$$

Equation of circle is $(x-2)^2 + (y-3)^2 = 25$.

10. The radius of the original circle, $(x-2)^2 + (y-2)^2 = 16$, is 4.
    The new circle touches the x-axis so its centre is a distance 4 from the x-axis.
    The centre of the new circle lies on OC. Point C is (2, 2) so OC is $y = x$.
    There are two possible places for the centre of the new circle: (4, 4) and (-4, -4)



The equation of the new circle is either $(x-4)^2 + (y-4)^2 = 16$ or $(x+4)^2 + (y+4)^2 = 16$.

11. <u>Method 1</u>
    The centre of the circle is at (7, 3). The radius of the circle is 5.
    The equation of the circle is $(x-7)^2 + (y-3)^2 = 25$
    C is where this circle crosses the line $2y - x = 4$.
    On the line, $x = 2y - 4$.
    Substituting into the equation of the circle:

$$(2y-11)^2 + (y-3)^2 = 25$$

$$4y^2 - 44y + 121 + y^2 - 6y + 9 = 25$$

$$5y^2 - 50y + 105 = 0$$

$$y^2 - 10y + 21 = 0$$

$$(y-7)(y-3) = 0$$

$y = 7, \quad y = 3$

$y = 3$ at point A. At point C, $y = 7$.

$x = 2y - 4 = 14 - 4 = 10$

C has coordinates (10, 7)

<u>Method 2</u>

Angle ABC is an angle in a semicircle so is 90^0.
BC is perpendicular to AC.
AC has gradient ½. BC has gradient -2.
The equation of BC is $(y-3)=-2(x-12)$

$$y-3=-2x+24$$
$$y=-2x+27$$

C is where this line crosses $2y-x=4$.

$$2y-x=4$$
$$y+2x=27$$

Multiplying the first equation by 2:

$$4y-2x=8$$
$$y+2x=27$$
$$5y=35$$
$$y=7$$
$$x=2y-4=14-4=10$$

C has coordinates (7, 10)

12. (i) Mid point of OA $=\left(\dfrac{0+3}{2},\dfrac{0+1}{2}\right)=(1.5,0.5)$

Gradient of OA $=\dfrac{1-0}{3-0}=\dfrac{1}{3}$

Equation of perpendicular bisector

$$y-0.5=-3(x-1.5)$$
$$y=-3x+5$$

(ii)

$$2=-3x+5$$
$$3x=3$$
$$x=1$$

Coodinates = (1, 2)

13. (i) Gradient of $AC=\dfrac{1-0}{4-1}=\dfrac{1}{3}$, Gradient of $BC=\dfrac{4-1}{5-4}=3$

Equation of tangent at A

$$y-0=-3(x-1)$$
$$y=-3x+3$$

Equation of tangent at B

AQA FM Coordinate geometry 2 Exercise

$$y-4=-\frac{1}{3}(x-5)$$
$$y=-\frac{1}{3}x+\frac{17}{3}$$

(ii) Solve Simultaneously:

$$-3x+3=-\frac{1}{3}x+\frac{17}{3}$$
$$-9x+9=-x+17$$
$$8x=-8$$
$$x=-1$$

$$y=-3(-1)+3=6$$

Intersection of the tangents (-1,6)

14. (i) Midpoint of AB = $\left(\frac{-2+6}{2},\frac{2+2}{2}\right)=(2,2)$

Gradient of AB = $\frac{2-2}{6--2}=0$

Perp-bisector of AB = vertical line through (2,2)

so the equation is $x=2$

(ii) Midpoint of BD = $\left(\frac{0+6}{2},\frac{-4+2}{2}\right)=(3,-1)$

Gradient of BD = $\frac{2--4}{6-0}=\frac{6}{6}=1$

Perp-bisector of BD = line through (3,-1) with grad -1

so the equation is

$$y--1=-1(x-3)$$
$$y=-x+2$$

(iii) Solving simultaneously: x=2, $y=-(2)+2=0$

Centre of circle = (2, 0)

(iv) Radius = distance from D to centre

$$\sqrt{(2-0)^2+(0--4)^2}$$
$$\sqrt{(2)^2+(4)^2}$$
$$\sqrt{4+16}=\sqrt{20}$$

Equation of circle : $(x-2)^2+y^2=20$