Section 3: Trig graphs, identities and equations

Solutions to Exercise

1. (i) $\dfrac{\sqrt{1-\cos^2 x}}{\tan x} = \dfrac{\sqrt{\sin^2 x}}{\tan x}$

$= \sin x \times \dfrac{\cos x}{\sin x} = \cos x$

(ii) $\dfrac{\sin x}{\sqrt{1-\sin^2 x}} = \dfrac{\sin x}{\sqrt{\cos x}}$

$= \dfrac{\sin x}{\cos x} = \tan x$

(iii) $\dfrac{\cos^2 x}{1+\sin x} = \dfrac{1-\sin^2 x}{1+\sin x}$

$= \dfrac{(1+\sin x)(1-\sin x)}{1+\sin x}$

$= 1-\sin x$

2. (i) $\sin x = 0.3$

Solutions are in the first and second quadrants

$x = 17.5°$ or $180° - 17.5°$

$x = 17.5°$ or $162.5°$

(ii) $\tan x = 1.5$

Solutions are in the first and third quadrants

$x = 56.3°$ or $180° + 56.3°$

$x = 56.3°$ or $236.3°$

(iii) $\cos x = -0.7$

Solutions are in the second and third quadrant

$x = 180° - 45.6°$ or $180° + 45.6°$

$x = 134.4°$ or $225.6°$

(iv) $\sin x = -0.6$

Solutions are in the third and fourth quadrant

$x = 180° + 36.9°$ or $360° - 36.9°$

$x = 216.9°$ or $323.1°$

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3. (i) $\sin x = 0.6$

Solutions are in the first and second quadrants

$x = 36.9°$ or $180° - 36.9°$

$x = 36.9°$ or $143.1°$

(ii) $\cos x = 0.8$

Solutions are in the first and fourth quadrants

$x = 36.9°$ or $360° - 36.9°$

$x = 36.9°$ or $323.1°$

(iii) $\tan x = -0.6$

Solutions are in the second and fourth quadrants

$x = 180° - 31.0°$ or $360° - 31.0°$

$x = 149.0°$ or $329.0°$

(iv) $\cos x = -0.3$

Solutions are in the second and third quadrants

$x = 180° - 72.5°$ or $180° + 72.5°$

$x = 107.5°$ or $252.5°$

4. (i) $\sin x = \frac{1}{\sqrt{2}}$

$\sin 45° = \frac{1}{\sqrt{2}}$, and solutions are in the first and second quadrants

so $x = 45°$ or $180° - 45°$

$x = 45°$ or $135°$

(ii) $\sin x = -\frac{1}{2}$

$\sin 30° = \frac{1}{2}$, and solutions are in the third and fourth quadrants

so $x = 180° + 30°$ or $360° - 30°$

$x = 210°$ or $330°$

(iii) $\cos x = \frac{\sqrt{3}}{2}$

$\cos 30° = \frac{\sqrt{3}}{2}$, and solutions are in the first and fourth quadrants

so $x = 30°$ or $360° - 30°$

$x = 30°$ or $330°$

(iv) $\cos x = -\frac{1}{\sqrt{2}}$

$\cos 45^\circ = \frac{1}{\sqrt{2}}$, and solutions are in the second and third quadrants

so $x = 180^\circ - 45^\circ$ or $180^\circ + 45^\circ$

$x = 135^\circ$ or 225°

5. (i) $3\sin x = 4\cos x$

$\tan x = \frac{4}{3}$

$x = 53.1^\circ$ or $180^\circ + 53.1^\circ$

$x = 53.1^\circ$ or 233.1°

Using $\tan\theta = \frac{\sin\theta}{\cos\theta}$

(ii) $2\cos x = -3\sin x$

$\tan x = -\frac{2}{3}$

$x = 180^\circ - 33.7^\circ$ or $360^\circ - 33.7^\circ$

$x = 146.3^\circ$ or 326.3°

Using $\tan\theta = \frac{\sin\theta}{\cos\theta}$

6. (i) $4\cos^2\theta = 3$

$\cos^2\theta = \frac{3}{4}$

$\cos\theta = \pm\frac{\sqrt{3}}{2}$

$\cos\theta = \frac{\sqrt{3}}{2} \quad \Rightarrow \theta = 30^\circ$ or 330°

$\cos\theta = -\frac{\sqrt{3}}{2} \quad \Rightarrow \theta = 150^\circ$ or 210°

$\theta = 30^\circ, 150^\circ, 210, 330^\circ$

(ii) $2\cos^2\theta = \cos\theta$

$2\cos^2\theta - \cos\theta = 0$

$\cos\theta(2\cos\theta - 1) = 0$

$\cos\theta = 0$ or $\cos\theta = \frac{1}{2}$

$\cos\theta = 0 \quad \Rightarrow \theta = 90^\circ$ or 270°

$\cos\theta = \frac{1}{2} \quad \Rightarrow \theta = 60^\circ$ or 300°

$\theta = 60^\circ, 90^\circ, 270, 300^\circ$

(iii) $4\sin\theta\cos\theta = \sin\theta$

$4\sin\theta\cos\theta - \sin\theta = 0$

$\sin\theta(4\cos\theta - 1) = 0$

$\sin\theta = 0$ or $\cos\theta = \frac{1}{4}$

$\sin\theta = 0 \quad \Rightarrow \theta = 0^\circ$ or 180° or 360°

AQA FM Geometry I 3 Exercise solutions

$\cos\theta = \frac{1}{4} \quad \Rightarrow \theta = 75.5^\circ \text{ or } 284.5^\circ$

$\theta = 0^\circ, 75.5^\circ, 180^\circ, 284.5, 360^\circ$

(iv) $\cos^2\theta - \cos\theta - 2 = 0$

$(\cos\theta - 2)(\cos\theta + 1) = 0$

$\cos\theta = 2 \text{ or } \cos\theta = -1$

There are no real values of θ for which $\cos\theta = 2$

$\cos\theta = -1 \quad \Rightarrow \theta = 180^\circ$

(v) $3\sin^2\theta + 5\cos\theta - 1 = 0$

$3(1-\cos^2\theta) + 5\cos\theta - 1 = 0$

$3 - 3\cos^2\theta + 5\cos\theta - 1 = 0$

$3\cos^2\theta - 5\cos\theta - 2 = 0$

$(3\cos\theta + 1)(\cos\theta - 2) = 0$

$\cos\theta = -\frac{1}{3} \text{ or } \cos\theta = 2$

There are no real values of θ for which $\cos\theta = 2$

$\cos\theta = -\frac{1}{3} \quad \Rightarrow \theta = 109.5^\circ \text{ or } 250.5^\circ$

(vi) $3\tan\theta - 2\cos\theta = 0$

$\frac{3\sin\theta}{\cos\theta} - 2\cos\theta = 0$

Using $\tan\theta = \frac{\sin\theta}{\cos\theta}$

$3\sin\theta - 2\cos^2\theta = 0$

$3\sin\theta - 2(1-\sin^2\theta) = 0$

$2\sin^2\theta + 3\sin\theta - 2 = 0$

Using $\sin^2\theta + \cos^2\theta = 1$

$(2\sin\theta - 1)(\sin\theta + 2) = 0$

$\sin\theta = \frac{1}{2} \text{ or } \sin\theta = -2$

There are no real values of θ for which $\sin\theta = -2$

$\sin\theta = \frac{1}{2} \quad \Rightarrow \theta = 30^\circ \text{ or } 150^\circ$

7. Any three angles which are a multiple of 180° either more or less than 132°.

8. $$\frac{1}{1+\sin x} + \frac{1}{1-\sin x} = \frac{1-\sin x + 1 + \sin x}{(1-\sin x)(1+\sin x)}$$

$$= \frac{2}{1-\sin^2 x}$$

$$= \frac{2}{\cos^2 x}$$