Further Maths GCSE
Factor Theorem Answers

1. Let $x=2$ then $x^3 + 8x^2 + x - 42$
$$= 8 + 32 + 2 - 42$$
$$= 0 \qquad \text{so } (x-2) \text{ is a factor.}$$

2. $2x^2 - 8x + 9 = 2[x^2 - 4x + 4.5]$
$$= 2[(x-2)^2 + 0.5]$$
All parts are >0 $\therefore 2x^2 - 8x + 9 > 0$

3. $x^3 + ax^2 + bx + 150 = (x+c)^2(x+d)$
$$= (x^2 + 2xc + c^2)(x+d)$$
$$= x^3 + (2c+d)x^2 + (2dc + c^2)x + c^2d$$

So $c^2d = 150$ $c \neq 1$ only other square factor of 150
is 25 so $c^2 = 25$ so $c = 5$, $d = 6$.

Compare x^2 terms $a = 2c+d = 16$
x terms $b = 2dc + c^2 = 85$.

4. If $x+3$ is a factor $f(-3) = 0$
$$\Rightarrow -27 + 54 - 3a - 12 = 0 \Rightarrow a = 5.$$
$x^3 + 6x^2 + 5x - 12 = (x+3)(x^2 + bx - 4)$

comparing x^2 terms $6 = b + 3 \Rightarrow b = 3$

$$= (x+3)(x^2 + 3x - 4)$$
$$= (x+3)(x+4)(x-1)$$

5. $f(5) = 0 \Rightarrow 125 - 150 + 5a - 20 = 0$
$$\text{so} \qquad a = 9.$$

6. $f(a) = 0 \Rightarrow 2a^3 - 7a^2 + 3a = 0$
$$a(2a^2 - 7a + 3) = 0$$
$$a(2a - 1)(a - 3)$$
So $a = 0, \tfrac{1}{2}, 3$ largest value $= 3$.

7. $f(1) = 1^3 - 21 \times 1 + 20 = 0$ so $x - 1$ is a factor
$f(4) = 4^3 - 21 \times 4 + 20 = 0$ so $x - 4$ is a factor.

8. $f(2) = 0 \quad 8 + 4a + 2b + 24 = 0$
$$4a + 2b = -32 \Rightarrow 2a + b = -16$$
$f(-3) = 0 \quad -27 + 9a - 3b + 24 = 0$ $\qquad +$
$$9a - 3b = 3 \Rightarrow 3a - b = 1$$
$$5a = -15$$
$$a = -3$$
$$b = -10.$$

9. $f(1) = 1 - 10 + 29 - 20 = 0$ so $x - 1$ is a factor
$f(4) \quad 64 - 160 + 116 - 20 = 0 \quad x - 4$ is a factor.