Further Maths GCSE
Trigonometric Equations and Identities Answers

1. $\tan^2 x - 1 \equiv \frac{\sin^2 x}{\cos^2 x} - 1$

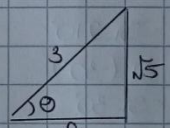
$\equiv \frac{\sin^2 x - \cos^2 x}{\cos^2 x}$

[but $\sin^2 x + \cos^2 x \equiv 1$, so $\sin^2 x \equiv 1 - \cos^2 x$]

$\equiv \frac{(1 - \cos^2 x) - \cos^2 x}{\cos^2 x}$

$\equiv \frac{1 - 2\cos^2 x}{\cos^2 x}$ QED. Proved

2. $\sin\theta = \frac{o}{h} = \frac{\sqrt{5}}{3}$



Using Pythagoras $a^2 = 3^2 - (\sqrt{5})^2$

$a = \pm 2$

so $\cos\theta = \frac{a}{h} = \frac{-2}{3}$ (since θ is obtuse)

Alternative $\sin^2\theta = \frac{5}{9}$ $\cos^2\theta = 1 - \sin^2\theta$

$= 4/9$

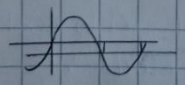
$\cos\theta = -2/3$ since θ is obtuse

3. $(2s+1)(s-1) = 2s^2 - s - 1$

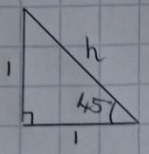
$2\sin^2\theta - \sin\theta + 1 = 0 \Rightarrow (2\sin\theta + 1)(\sin\theta - 1) = 0$

so $\sin\theta = -1/2$ or $\sin\theta = 1$

$\Rightarrow \theta = 30, 210, 330$ $\theta = 90°$



so $\theta = 90°, 210°, 330°$

4. 

Using Pythagoras $h = \sqrt{(1^2 + 1^2)} = \sqrt{2}$

so $\sin 45 = \frac{o}{h} = \frac{1}{\sqrt{2}} \left(\times \frac{\sqrt{2}}{\sqrt{2}}\right) = \frac{\sqrt{2}}{2}$

5. $5\sin^2 x - 2\sin x = 0$

$\sin x(5\sin x - 2) = 0$

so $\sin x = 0$ $\sin x = 2/5$

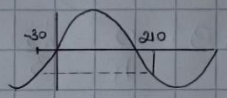
$x = 0, 180, 360$ $x = 23.6°, 156.4°$

6. $f(210°) = \sin 210°$

$= \sin(-30°)$

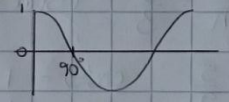
$= -\sin 30°$

$= -1/2$



$-1 \leq f(x) \leq 1$

$g(x) = \cos x$



$\theta = 90°$ to keep range positive.

7. $1 - \tan\theta \sin\theta \cos\theta$

$\equiv 1 - \frac{\sin\theta}{\cos\theta} \sin\theta \cos\theta$

$\equiv 1 - \sin^2\theta$ but $\sin^2\theta + \cos^2\theta = 1$

$\equiv \cos^2\theta$

8. $3\cos^2\theta - 1 = 0$

$\cos^2\theta = 1/3$

$\cos\theta = \pm 1/\sqrt{3}$

$\cos\theta = 1/\sqrt{3}$ or $\cos\theta = -1/\sqrt{3}$

$\theta = 54.7°$ $\theta = 125.3°$

9. $\tan\theta + \frac{1}{\tan\theta} \equiv \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$

$\equiv \frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}$

$\equiv \frac{1}{\sin\theta\cos\theta}$ Proved

10. $\tan^2\theta + 3\tan\theta = 0$

$\tan\theta(\tan\theta + 3) = 0$

$\tan\theta = 0$ or $\tan\theta = -3$

$\theta = 0°, 360°, 180°$ $\theta = -71.6, 108.4, 288.4$

$+180$ $+180$

so $\theta = 0, 180°, 360°, 108.4, 288.4$