Further Maths GCSE
Trigonometry Answers

1. $\tan x = \dfrac{12 - 5\sqrt{3}}{4\sqrt{3} - 5}$ rationalise denominator

$\dfrac{(12 - 5\sqrt{3})}{(4\sqrt{3} + 5)} \times \dfrac{(4\sqrt{3} + 5)}{(4\sqrt{3} + 5)} = \dfrac{48\sqrt{3} - 25\sqrt{3} - 60 + 60}{48 - 25}$

$= \dfrac{23\sqrt{3}}{23} = \sqrt{3}$

$\tan x = \sqrt{3} \quad \Rightarrow \quad x = 60°$
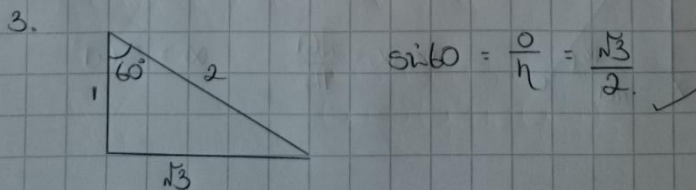
2. Area of triangle $= \frac{1}{2}(4\sqrt{3} - 6)(2 - \sqrt{3}) \sin A$

$= \frac{1}{2}[8\sqrt{3} + 6\sqrt{3} - 12 - 12] \times \frac{1}{4}$

$= \quad 14\sqrt{3}$

2. $\dfrac{\sin A}{2 - \sqrt{3}} = \dfrac{\sin B}{4\sqrt{3} - 6}$

$\sin B = \dfrac{\frac{1}{4}(4\sqrt{3} - 6)}{2 - \sqrt{3}} \times \dfrac{(2 + \sqrt{3})}{(2 + \sqrt{3})}$

$= \dfrac{\frac{1}{4}(2\sqrt{3})}{1} = \dfrac{\sqrt{3}}{2}$

So $\sin B = \dfrac{\sqrt{3}}{2}$ so $B = 60°$

3.

$\sin 60 = \dfrac{o}{h} = \dfrac{\sqrt{3}}{2}$ ✓

4. $\hat{DCA} = 120°$ so $\hat{DAC} = 15°$

so $AC \times \sin 60 = 3\sqrt{2} \times \sin 45$ [$= AB$]

$\dfrac{\sqrt{3}}{2} \cdot AC = 3\sqrt{2} \times \dfrac{1}{\sqrt{2}}$

so $AC = 2\sqrt{3}$.   [could use sine rule]
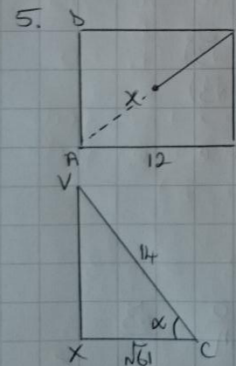
$BC = AC \times \cos 60 = 2\sqrt{3} \times \frac{1}{2} = \sqrt{3}$.

In ABD using Pythagoras:

$(3\sqrt{2})^2 = (x + \sqrt{3})^2 + 3^2$

$18 = x^2 + 2\sqrt{3}x + 3 + 9$

$x^2 + 2\sqrt{3}x - 6 = 0$

$x = \dfrac{-2\sqrt{3} \pm \sqrt{(12 + 24)}}{2} = -\sqrt{3} + 3$ [$x > 0$]

5.

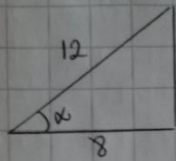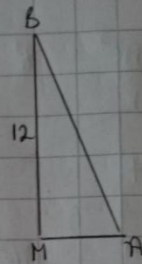$CX = \sqrt{10^2 + 12^2} \div 2$

$= \sqrt{61}$

$VX = \sqrt{14^2 - (\sqrt{61})^2}$

$= \sqrt{135} = 3\sqrt{15} = 11.6 cm$

$\cos \alpha = \dfrac{\sqrt{61}}{14}$ $\alpha = 56.1°$

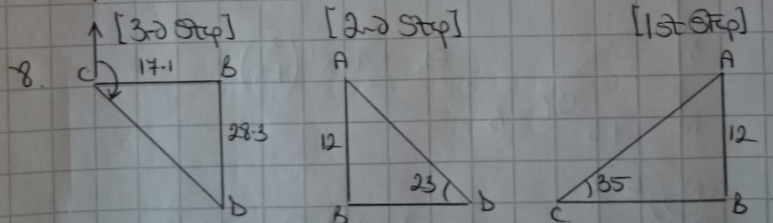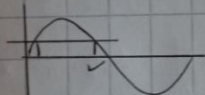**6.** Let M = midpoint of AC.

$\cos x = 8/12 \Rightarrow x = 48.2°$

7. $\frac{1}{2}ab\sin C = \text{Area}$

$\frac{1}{2} \times 10 \times 18 \times \sin P = 27$

$\sin P = 0.3$

$P = \sin^{-1}(0.3) = 17.5°$ but P is obtuse

so $P = 180 - 17.5$

$= 162.5°$

8.

[3rd Step]   [2nd Step]   [1st Step]

$CD = 33.0m$

$BD = \dfrac{12}{\tan 23} = 28.3$ $CB = \dfrac{12}{\tan 35} = 17.1$

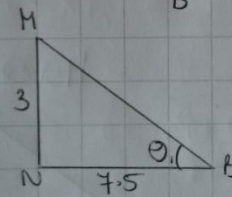Bearing of D from C $= 90 + \tan^{-1}\dfrac{28.3}{17.1}$

$= 149°$

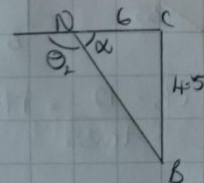**9.** $AC^2 = 8^2 + 14^2 \Rightarrow AC = 2\sqrt{65}$

$\theta = \tan^{-1}\dfrac{7}{2\sqrt{65}} = 23.5°$

10.
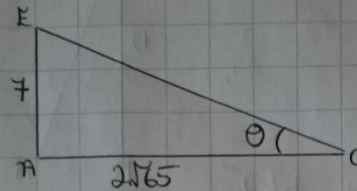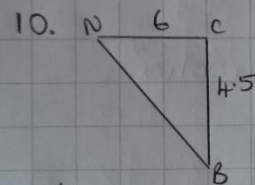
$BN = \sqrt{(4.5^2 + 6^2)}$
$= 7.5$
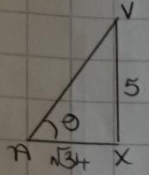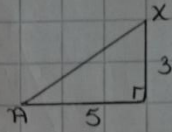
$\theta_1 = \tan^{-1}\left(\dfrac{3}{7.5}\right) =$

$\alpha = \tan^{-1}\left(\dfrac{4.5}{6}\right) = 36.9°$
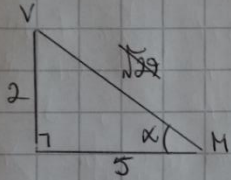
so $\theta_2 = 180° - 36.9° = 143°$

11. $(AX)^2 = 5^2 + 3^2$ $AX = \sqrt{34}$.

$\theta = \tan^{-1}\left(\dfrac{5}{\sqrt{34}}\right) = 40.6°$

let M = midpoint of RQ

$\alpha = \tan^{-1}\left(\dfrac{2}{5}\right) = 21.8°$