$\frac{d^2y}{dx^2} = v\frac{du}{dx} + u\frac{dv}{dx} = \sin^2 x \times -3\sin x + 3\cos x \times 2\sin x\cos x$
$= -3\sin^3 x + 6\sin x\cos^2 x$

When $x = \frac{\pi}{2}$, $\frac{d^2y}{dx^2} = -3\sin^3\left(\frac{\pi}{2}\right) + 6\sin\left(\frac{\pi}{2}\right)\cos^2\left(\frac{\pi}{2}\right)$
$= -3(1) + 6(0) = -3$

So the stationary point is a maximum.

[5 marks available — 1 mark for using a suitable method to find the first derivative, 1 mark for finding the first derivative correctly, 1 mark for using a suitable method to find the second derivative, 1 mark for finding the second derivative correctly, 1 mark for the correct answer and conclusion]

There are other methods you could have used to differentiate, such as using trig identities on the expressions first. You might get a $\frac{d^2y}{dx^2}$ that looks different to the one above, but it should still give you a value of –3 at the stationary point.

12 a) Area of minor sector $AOB = \frac{1}{2}r^2\theta$

Area of triangle $AOB = \frac{1}{2}r^2\sin\theta$

Area of $S_1 = \frac{1}{2}r^2\theta - \frac{1}{2}r^2\sin\theta$ ***[1 mark]***

Area of whole circle $= \pi r^2$

Now $S_1 : S_2 = 2 : 7 \Rightarrow S_1 :$ whole circle $= 2 : 9$

$\Rightarrow S_1 = \frac{2}{9} \times$ area of whole circle ***[1 mark]***

$\Rightarrow \frac{1}{2}r^2\theta - \frac{1}{2}r^2\sin\theta = \frac{2}{9} \times \pi r^2$ ***[1 mark]***

$\Rightarrow \frac{1}{2}\theta - \frac{1}{2}\sin\theta = \frac{2}{9}\pi$

$\Rightarrow \theta - \sin\theta = \frac{4\pi}{9}$

$\Rightarrow \theta - \sin\theta - \frac{4\pi}{9} = 0$ as required ***[1 mark]***

[4 marks available in total — as above]

You could also have found an expression for the area of S_2 and used the ratio to relate it to S_1 or the area of the whole circle.

b) $\theta_0 = \frac{\pi}{2} \Rightarrow \theta_1 = \sin\frac{\pi}{2} + \frac{4\pi}{9} = 2.3962...$

$\theta_2 = 2.0744...$ $\theta_3 = 2.2720...$ $\theta_4 = 2.1602...$ $\theta_5 = 2.2274...$

So $\theta = 2.2$ radians (2 s.f.) because θ_4 and θ_5 round to the same number to 2 s.f.

[2 marks available — 1 mark for using the iterative formula correctly, 1 mark for the correct answer with justification]

13 a) $\frac{-x-8}{x^2+6x+8} = \frac{-x-8}{(x+4)(x+2)} \equiv \frac{C}{x+4} + \frac{D}{x+2}$ ***[1 mark]***

$\Rightarrow -x - 8 \equiv C(x+2) + D(x+4)$

When $x = -4$: $4 - 8 = C(-4+2) \Rightarrow -4 = -2C \Rightarrow C = 2$

When $x = -2$: $2 - 8 = D(-2+4) \Rightarrow -6 = 2D \Rightarrow D = -3$

[1 mark for the correct value of C or D]

$\frac{-x-8}{(x+4)(x+2)} = \frac{2}{x+4} - \frac{3}{x+2}$ ***[1 mark]***

$= 2(x+4)^{-1} - 3(x+2)^{-1}$

$= 2(4^{-1})\left(\frac{x}{4}+1\right)^{-1} - 3(2^{-1})\left(\frac{x}{2}+1\right)^{-1}$ ***[1 mark]***

$2(4^{-1})\left(\frac{x}{4}+1\right)^{-1} = \frac{1}{2}\left(1 - \frac{x}{4} + \frac{-1\times-2}{1\times2}\left(\frac{x}{4}\right)^2 + ...\right)$

$= \frac{1}{2} - \frac{x}{8} + \frac{x^2}{32} + ...$ ***[1 mark]***

$3(2^{-1})\left(\frac{x}{2}+1\right)^{-1} = \frac{3}{2}\left(1 - \frac{x}{2} + \frac{-1\times-2}{1\times2}\left(\frac{x}{2}\right)^2 + ...\right)$

$= \frac{3}{2} - \frac{3x}{4} + \frac{3x^2}{8} + ...$ ***[1 mark]***

So $2(x+4)^{-1} - 3(x+2)^{-1} \approx \frac{1}{2} - \frac{x}{8} + \frac{x^2}{32} - \left(\frac{3}{2} - \frac{3x}{4} + \frac{3x^2}{8}\right)$

$= -1 + \frac{5x}{8} - \frac{11x^2}{32}$ ***[1 mark]***

[7 marks available in total — as above]

b) The expansion of $\left(\frac{x}{4}+1\right)^{-1}$ is valid when:

$\left|\frac{x}{4}\right| < 1 \Rightarrow -1 < \frac{x}{4} < 1 \Rightarrow -4 < x < 4$ ***[1 mark]***

The expansion of $\left(\frac{x}{2}+1\right)^{-1}$ is valid when:

$\left|\frac{x}{2}\right| < 1 \Rightarrow -1 < \frac{x}{2} < 1 \Rightarrow -2 < x < 2$ ***[1 mark]***

So the expansion of f(x) is valid when both of these are satisfied, i.e. when $-2 < x < 2$. In set notation, this is: $\{x : -2 < x < 2\}$ or $\{x : x > -2\} \cap \{x : x < 2\}$ ***[1 mark]***

[3 marks available in total — as above]

c) When $x = 0.1$, $\frac{-0.1-8}{0.1^2+6(0.1)+8} = -0.9407...$

and $-1 + \frac{5(0.1)}{8} - \frac{11(0.1)^2}{32} = -0.9409...$ ***[1 mark for both]***

Percentage error $= \frac{-0.9409... - -0.9407...}{-0.9407...} \times 100\%$

$= 0.018\%$ (2 s.f.) ***[1 mark]***

[2 marks available in total — as above]

14 a) E.g. Using $\tan^2\theta + 1 = \sec^2\theta$ and $\cos 2\theta = \cos^2\theta - \sin^2\theta$

$\frac{1-\tan^2\theta}{1+\tan^2\theta} = \frac{1-\tan^2\theta}{\sec^2\theta}$ ***[1 mark]*** $= \frac{1}{\sec^2\theta} - \frac{\tan^2\theta}{\sec^2\theta}$ ***[1 mark]***

$= \cos^2\theta - \frac{\sin^2\theta}{\cos^2\theta} \div \frac{1}{\cos^2\theta} = \cos^2\theta - \frac{\sin^2\theta}{\cos^2\theta} \times \cos^2\theta$ ***[1 mark]***

$= \cos^2\theta - \sin^2\theta$ ***[1 mark]*** $= \cos 2\theta$

[4 marks available in total — as above]

b) From part a), $\frac{1-\tan^2\left(\beta+\frac{\pi}{2}\right)}{1+\tan^2\left(\beta+\frac{\pi}{2}\right)} = \cos 2\left(\beta + \frac{\pi}{2}\right)$

$\Rightarrow \cos(2\beta+\pi) - 0.5\sec(2\beta+\pi) = 0$ ***[1 mark]***

$\Rightarrow \cos(2\beta+\pi) - \frac{0.5}{\cos(2\beta+\pi)} = 0$

$\Rightarrow \cos^2(2\beta+\pi) - 0.5 = 0$ ***[1 mark]***

$\Rightarrow \cos(2\beta+\pi) = \pm\sqrt{0.5}$ ***[1 mark]***

$\Rightarrow 2\beta + \pi = \cos^{-1}(\pm\sqrt{0.5})$ and $0 < \beta < \pi \Rightarrow \pi < 2\beta + \pi < 3\pi$

$\Rightarrow 2\beta + \pi = \frac{5\pi}{4}, \frac{7\pi}{4}, \frac{9\pi}{4}, \frac{11\pi}{4}$ ***[1 mark]***

$\Rightarrow 2\beta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4} \Rightarrow \beta = \frac{\pi}{8}, \frac{3\pi}{8}, \frac{5\pi}{8}$ and $\frac{7\pi}{8}$ ***[1 mark]***

[5 marks available in total — as above]

15 a) $\frac{dC}{dt} = -kCt \Rightarrow \int \frac{1}{C}\,dC = \int -kt\,dt$

$\Rightarrow \ln|C| = -\frac{k}{2}t^2 + d$ ***[1 mark]***

$\Rightarrow C = e^{-\frac{k}{2}t^2+d} = Ae^{-\frac{k}{2}t^2}$ (where $A = e^d$) ***[1 mark]***

At the end of the holidays $t = 0$, so $A = 3600$.

The equation for C is $C = 3600e^{-\frac{k}{2}t^2}$ ***[1 mark]***.

[3 marks available in total — as above]

b) $k = 0.2$, so the equation for C becomes $C = 3600e^{-0.1t^2}$.

Solve the equation $300 = 3600e^{-0.1t^2}$:

$300 = 3600e^{-0.1t^2} \Rightarrow \frac{1}{12} = e^{-0.1t^2}$

$\Rightarrow \ln\frac{1}{12} = -0.1t^2 \Rightarrow t = \sqrt{-10\ln\frac{1}{12}} = 4.984...$

So $t = 5$ weeks (to the nearest whole week).

[3 marks available — 1 mark for substituting the values of k and C into the equation, 1 mark for rearranging and taking logs, 1 mark for the correct answer]

You can check that your answer is correct by putting $t = 4$ and $t = 5$ into the equation for C: $t = 4$ gives 726.827... customers, and $t = 5$ gives 295.505... This is less than 300, so the ice cream parlour will close after 5 weeks.

Set 2 Paper 3 — Statistics and Mechanics

1 a) Since values are given correct to the nearest knot, speeds in this class are actually between 4.5 kn and 8.5 kn, giving a class width of 4 kn. ***[1 mark]***

b) $n = 17 + 14 + 7 + 2 = 40$, so Q_1 is in the $\frac{40}{4} = 10^{th}$ position and Q_3 is in the $\frac{40\times3}{4} = 30^{th}$ position. From the frequency table, Q_1 is in the 0 – 4 class and Q_3 is in the 5 – 8 class. ***[1 mark for both]***

Using linear interpolation:

$Q_1 \approx 0 + 4.5 \times \frac{10-0}{17} = 2.6470... = 2.65$ kn (3 s.f.) ***[1 mark]***

$Q_3 \approx 4.5 + 4 \times \frac{30-17}{14} = 4.5 + 3.7142... = 8.2142...$
$= 8.21$ kn (3 s.f.) ***[1 mark]***

IQR ≈ 8.2142... – 2.6470... = 5.5672...
= 5.57 kn (3 s.f.) ***[1 mark]***

[4 marks available in total — as above]

c) Lower boundary = 2.6470... – (1.5 × 5.5672...) = –5.7037...
The lowest data value, 1, is greater than the lower boundary, so it is not an outlier. ***[1 mark]***
Upper boundary = 8.2142... + (1.5 × 5.5672...) = 16.565...
The highest data value, 18, is greater than the upper boundary, so it is an outlier. ***[1 mark]***

[2 marks available in total — as above]

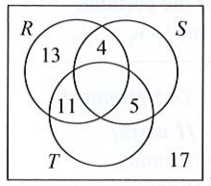
d) E.g. The median, as it is not as heavily affected by outliers. ***[1 mark]***

You could also argue that the mean is better because it uses all of the values so may be more representative of the data — as long as you justify your answer, you'll get the mark.

e) E.g. He has only looked at data for part of each country — the average for the whole country might be different / he has only looked at data for part of 1987 — the weather patterns may have changed since then / he has only looked at windspeed — other data may contradict this / he has not done a hypothesis test to see whether the higher mean is just down to random chance / he has not considered any other measures of location or variation (e.g. median, interquartile range, etc) / he has not considered other factors that could impact windspeed (e.g. how close the location is to the coast) / etc.

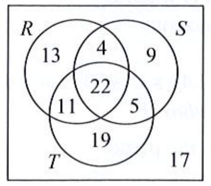
[2 marks available — 1 mark for each sensible criticism]

2 a) Draw a Venn diagram with three overlapping circles, then fill in the values of n(only R), n(don't like any), n(only R and T), n(only S and T) and n(only S and R):



[1 mark]

n(R and S) = 26, so n(R and S and T) = 26 – 4 = 22 ***[1 mark]***
n(S) = 40, so n(only S) = 40 – (4 + 5 + 22) = 9 ***[1 mark]***
100 students were surveyed, so n(only T)
= 100 – (13 + 4 + 9 + 11 + 22 + 5 + 17) = 19 ***[1 mark]***
So the completed Venn diagram is:



[4 marks available in total — as above]

b) i) P(likes at least one) = 1 – P(doesn't like any)
$= 1 - \frac{17}{100} = \frac{83}{100}$

[2 marks available — 1 mark for the correct numerator, 1 mark for the correct denominator]

You could also add up all of the numbers inside the circles (you should get 83) and divide it by the total (100).

ii) $P(T \mid S) = \frac{5+22}{9+5+4+22} = \frac{27}{40}$

[2 marks available — 1 mark for the correct numerator, 1 mark for the correct denominator]

iii) $P((R \cap S') \mid T) = \frac{11}{19+11+22+5} = \frac{11}{57}$

[2 marks available — 1 mark for the correct numerator, 1 mark for the correct denominator]

c) If the teacher's claim is true, then the events T and S are independent, i.e. $P(T) \times P(S) = P(T \cap S)$.

$P(T) \times P(S) = \frac{11+22+5+19}{100} \times \frac{4+22+5+9}{100} = \frac{57}{250}$

$P(T \cap S) = \frac{27}{100}$ so the results of the survey do not support the teachers claim.

[3 marks available — 1 mark for using appropriate probability law for determining whether events are dependent, 1 mark for correctly calculating relevant probabilities, 1 mark for correct conclusion]

There are other ways you could check whether these are independent — for example, you could compare P(T|S) to P(T).

3 a) E.g. The graph shows weak positive correlation between the variables. ***[1 mark]***

b) $H_0: \rho = 0$, $H_1: \rho \neq 0$ ***[1 mark]***
This is a two-tailed test at the 10% significance level, so use the 0.05 column in the critical value table.
$n = 10$, so critical value = 0.5494. ***[1 mark]***
0.5494 > 0.3850, so there is not enough evidence at the 10% level to reject H_0 and conclude that the product moment correlation coefficient is non-zero. ***[1 mark]***

[3 marks available in total — as above]

c) The remaining points would lie closer to a straight line, so the value of r would increase. ***[1 mark]***

4 a) Any one of: there are a fixed number of trials (packets of sweets) / each trial is independent of the others (whether one packet contains a gold-wrapped sweet does not affect whether another packet does) / there are only two possible outcomes (a packet either contains a gold-wrapped sweet or doesn't) / the probability remains constant from trial to trial. ***[1 mark]***

$n = 50$, $p = \frac{9}{20} = 0.45$ ***[1 mark for both]***

[2 marks available in total — as above]

b) $H_0: p = 0.45$, $H_1: p < 0.45$ ***[1 mark]*** $G \sim B(50, 0.45)$
Using the binomial tables: $P(G \leq 16) = 0.0427$ ***[1 mark]***
0.0427 < 0.05, so the result is significant. ***[1 mark]***
There is enough evidence at the 5% level to reject H_0 and conclude that the company has put fewer gold-wrapped sweets in their packets of sweets than they said they did. ***[1 mark]***

[4 marks available in total — as above]

5 a) i) P(positive test result and carrier)
= 0.98 × 0.03 = 0.0294 ***[1 mark]***
P(positive test result and non-carrier)
= (1 – 0.95) × (1 – 0.03) = 0.0485 ***[1 mark]***
P(positive test result) = 0.0294 + 0.0485 = 0.0779 ***[1 mark]***
P(carrier given a positive test result)
= 0.0294 ÷ 0.0779 = 0.37740... = 0.377 (3 s.f.) ***[1 mark]***

[4 marks available in total — as above]

ii) E.g. Joey is a random member of the UK population and isn't more likely to have the disease just because he takes the test. ***[1 mark]***

b) P(correct test result) = P(positive test result and carrier) + P(negative test result and not carrier) ***[1 mark]***
= (0.98 × 0.03) + (0.95 × 0.97) = 0.0294 + 0.9215
= 0.9509 or 95.09% so Hiroshi is not correct. ***[1 mark]***

[2 marks available in total — as above]

c) E.g. No, because the majority of positive results are for people who don't carry the disease (i.e. most of the time a positive result is a false positive). ***[1 mark]***

You could argue either yes or no for this question, as long as you fully explain your reasoning. E.g. you could say yes because only 2% of people who carry the disease will not be identified.

6 a) Let L be the length of a pencil in cm. Then $L \sim N(18, 0.1^2)$.
P(pencil will fit in box) = $P(L < 18.2)$ ***[1 mark]***
From your calculator: $P(L < 18.2) = 0.9772...$ ***[1 mark]***
P(box will not be damaged) = $(0.9772...)^5 = 0.8913...$ ***[1 mark]***
P(box will be damaged) = 1 – 0.8913...
= 0.1086... = 0.109 (3 s.f.) ***[1 mark]***

[4 marks available in total — as above]

You could also have used a binomial distribution for the number of pencils in a box that are longer/shorter than 18.2 cm.

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b) Let X be the number of boxes that are damaged in one day.
Then $X \sim B(1000, 0.1086...)$ ***[1 mark]***
$\Rightarrow \mu = 1000 \times 0.1086... = 108.69...$
and $\sigma^2 = 1000 \times 0.1086... \times (1 - 0.1086...) = 96.87...$
So X is approximated by $Y \sim N(108.69..., 96.87...)$ ***[1 mark]***
$P(X > 125) \approx P(Y > 125.5)$ ***[1 mark]***
From your calculator:
$P(Y > 125.5) = 0.043842... = 0.0438$ (3 s.f.) ***[1 mark]***
[4 marks available in total — as above]

7 a) Resolving forces vertically: $R = W + \frac{5}{8}W = \frac{13}{8}W$
[2 marks available — 1 mark for a correct method to resolve forces vertically, 1 mark for the correct answer]

b) Let x be the distance from A to the centre of mass of the plank.
E.g. equating moments about the centre of the plank:
Clockwise moments = anticlockwise moments
$\frac{5}{8}W \times 1 = W \times (2 - x)$
$\Rightarrow \frac{5}{8} = 2 - x \Rightarrow x = 2 - \frac{5}{8} = 1\frac{3}{8}$ m $= 1.375$ m
[3 marks available — 1 mark for equating moments, 1 mark for a correct equation, 1 mark for the correct answer]

8 a) $\mathbf{s} = \mathbf{s}$, $\mathbf{u} = (1.5\mathbf{i} - 2\mathbf{j})$ ms^{-1}, $\mathbf{a} = 0$ ms^{-2}, $t = 7$ seconds
$\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2 = 7(1.5\mathbf{i} - 2\mathbf{j}) + 0 = (10.5\mathbf{i} - 14\mathbf{j})$ m
[2 marks available — 1 mark for a correct method, 1 mark for the correct answer]

b) i) They collide after $t = 7 + 6 = 13$ seconds, so the position of the skateboarder (and where they collide) is:
$\mathbf{s} = 13(1.5\mathbf{i} - 2\mathbf{j}) + 0 = (19.5\mathbf{i} - 26\mathbf{j})$
[2 marks available — 1 mark for a correct method, 1 mark for the correct answer]

ii) For the cyclist's motion between $t = 7$ s and $t = 13$ s:
$\mathbf{s} = \mathbf{s}$, $\mathbf{u} = (-4.5\mathbf{i} + 6\mathbf{j})$ ms^{-1}, $\mathbf{a} = (-0.3\mathbf{i} + 0.4\mathbf{j})$ ms^{-2},
$t = (13 - 7) = 6$ s. Now, using $\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$
$\mathbf{s} = (-4.5\mathbf{i} + 6\mathbf{j}) \times 6 + \frac{1}{2} \times (-0.3\mathbf{i} + 0.4\mathbf{j}) \times 6^2$ ***[1 mark]***
$= (-27\mathbf{i} + 36\mathbf{j}) + (-5.4\mathbf{i} + 7.2\mathbf{j})$
$= (-32.4\mathbf{i} + 43.2\mathbf{j})$ m ***[1 mark]***
So the initial position of the cyclist is:
$(19.5\mathbf{i} - 26\mathbf{j}) - (-32.4\mathbf{i} + 43.2\mathbf{j})$
$= (51.9\mathbf{i} - 69.2\mathbf{j})$ m ***[1 mark]***
[3 marks available in total — as above]

9 a) Resolving forces vertically on Q: $4g - T = 4a$ (1) ***[1 mark]***
The force pulling P downwards parallel to the plane is $2g\sin 15°$. ***[1 mark]*** So resolving forces parallel to the plane on P: $T - 2g \sin 15° = 2a$ (2) ***[1 mark]***
Adding equations (1) and (2) gives:
$4g - T + T - 2g \sin 15° = 4a + 2a$
$4g - 2g \sin 15° = 6a$ ***[1 mark]***
$a = 34.127... \div 6 = 5.6878... = 5.7$ ms^{-2} (2 s.f.) ***[1 mark]***
[5 marks available in total — as above]
You could have used cos 75° instead of sin 15°.

b) Substituting $a = 5.6878...$ ms^{-2} into e.g. equation (2) gives:
$T = 2g\sin 15° + 2(5.6878...) = 5.0728... + 11.375...$
$= 16.448...$ N ***[1 mark]***
Vertical component of $F = T + T\sin 15°$ ***[1 mark]***
$= 16.448... + 16.448... \sin 15°$
$= 20.705... = 21$ N (2 s.f.) ***[1 mark]***
[3 marks available in total — as above]

10 a) $\mathbf{r} = \int \mathbf{v}\, dt$ ***[1 mark]*** $= \int \begin{pmatrix} 5t - 6t^2 \\ 6 - t^3 \end{pmatrix} dt = \begin{pmatrix} \frac{5t^2}{2} - 2t^3 \\ 6t - \frac{t^4}{4} \end{pmatrix} + \mathbf{C}$ ***[1 mark]***

At $t = 0$, $\mathbf{r} = \begin{pmatrix} \frac{5(0)^2}{2} - 2(0)^3 \\ 6(0) - \frac{(0)^4}{4} \end{pmatrix} + \mathbf{C} = \begin{pmatrix} 6 \\ 2 \end{pmatrix} \Rightarrow \mathbf{C} = \begin{pmatrix} 6 \\ 2 \end{pmatrix}$ ***[1 mark]***

At $t = 3$, $\mathbf{r} = \begin{pmatrix} \frac{5(3)^2}{2} - 2(3)^3 + 6 \\ 6(3) - \frac{(3)^4}{4} + 2 \end{pmatrix} = \begin{pmatrix} -25.5 \\ -0.25 \end{pmatrix}$ m ***[1 mark]***

[4 marks available in total — as above]

b) $\mathbf{a} = \frac{d\mathbf{v}}{dt} = \begin{pmatrix} 5 - 12t \\ -3t^2 \end{pmatrix}$ ***[1 mark]***

At $t = 1$, $\mathbf{a} = \begin{pmatrix} 5 - 12(1) \\ -3(1)^2 \end{pmatrix} = \begin{pmatrix} -7 \\ -3 \end{pmatrix}$ ms^{-2} ***[1 mark]***

$|\mathbf{a}| = \sqrt{(-7)^2 + (-3)^2}$ ***[1 mark]***
$= \sqrt{49 + 9} = \sqrt{58} = 7.62$ ms^{-2} (3 s.f.) ***[1 mark]***
[4 marks available in total — as above]

c) Using $\mathbf{F}_{net} = m\mathbf{a}$: $\begin{pmatrix} -20 \\ k \end{pmatrix} = 5\begin{pmatrix} 5 - 12t \\ -3t^2 \end{pmatrix}$ ***[1 mark]***
$-20 = 5(5 - 12t) \Rightarrow -4 = 5 - 12t$
$\Rightarrow 12t = 9 \Rightarrow t = 0.75$ seconds ***[1 mark]***
So $k = 5(-3t^2) = -15(0.75^2) = -15 \times 0.5625 = -8.4375$ ***[1 mark]***
[3 marks available in total — as above]

11 a) Considering the vertical motion of the particle:
$s = -3.5$ m, $u_V = (203\sin\theta)$ ms^{-1}, $a = -g = -9.8$ ms^{-2},
$t = 5$ seconds. Using $s = ut + \frac{1}{2}at^2$:
$-3.5 = (203\sin\theta \times 5) + \left(\frac{1}{2} \times (-9.8) \times 5^2\right)$ ***[1 mark]***
$\Rightarrow -3.5 = 1015\sin\theta - 122.5 \Rightarrow 119 = 1015\sin\theta$
$\Rightarrow \sin\theta = \frac{119}{1015}$ ***[1 mark]***
$\Rightarrow \theta = \sin^{-1}(0.1172...) = 6.732...° = 6.73°$ (3 s.f.) ***[1 mark]***
[3 marks available in total — as above]
Upward was defined as positive here — if you'd taken downwards to be positive, then s and a would have been positive, and u would have been negative (you'd still get the same answer).

b) There is no horizontal acceleration, so:
$v_H = u_H = 203\cos\theta = 201.6$ ms^{-1} ***[1 mark]***
Considering the vertical motion of the particle:
$s = -3.5$ m, $u_V = 203\sin\theta = 23.8$ ms^{-1}, $v_V = v_V$,
$a = -g = -9.8$ ms^{-2}, $t = 5$ seconds.
Using $s = vt - \frac{1}{2}at^2$: $-3.5 = 5v_V + 122.5$ ***[1 mark]***
$\Rightarrow 5v_V = -126 \Rightarrow v_V = -25.2$ ms^{-1} ***[1 mark]***
Let α be the angle between the horizontal and the direction of the bullet's motion:
$\tan\alpha = \frac{v_V}{v_H} = \frac{25.2}{201.6}$
$\Rightarrow \alpha = \tan^{-1}(0.125) = 7.125... = 7.13°$ (3 s.f.) ***[1 mark]***
[4 marks available in total — as above]

c) v_H is constant, so minimum speed is when $v_V = 0$.
Using $v = u + at$ for the vertical motion:
$0 = 203\sin\theta - 9.8t$ ***[1 mark]***
$\Rightarrow 9.8t = 23.8 \Rightarrow t = 2.428... = 2.43$ s (3 s.f.) ***[1 mark]***
[2 marks available in total — as above]

12 a) Resolving forces perpendicular to the plane:
$R = 2g\cos 20°$ ***[1 mark]***
Resolving forces parallel to the plane: $F_{net} = ma$ ***[1 mark]***
$2g \sin 20° - F = 2 \times 0.15g$ ***[1 mark]***
$\Rightarrow F = 2g \sin 20° - 0.3g$
The particle is sliding, so $F = \mu R = \mu(2g \cos 20°)$ ***[1 mark]***
$\Rightarrow 2g \sin 20° - 0.3g = \mu(2g \cos 20°)$ ***[1 mark]***
$\mu = \frac{2\sin 20° - 0.3}{2\cos 20°} = 0.20434... = 0.20$ (2 d.p.) ***[1 mark]***
[6 marks available in total — as above]

b) Resolving parallel to the plane: $F = 2g \sin\theta$ ***[1 mark]***
Resolving perpendicular to the plane: $R = 2g\cos\theta$
Since the particle is not sliding, $F \leq \mu R$:
$2g \sin\theta \leq \mu(2g \cos\theta)$ ***[1 mark]*** $\Rightarrow \frac{2g\sin\theta}{2g\cos\theta} \leq 0.20434...$
$\Rightarrow \tan\theta \leq 0.20434...$ ***[1 mark]*** $\Rightarrow \theta \leq 11.549...°$ ***[1 mark]***
[4 marks available in total — as above]