b) At stationary points, $\frac{dy}{dx} = 0$. If $a = 0.95$ to 2 decimal places, then $0.945 \le a < 0.955$. ***[1 mark]***
When $x = 0.945$, $\frac{dy}{dx} = 0.00215...$ (positive)
When $x = 0.955$, $\frac{dy}{dx} = -0.00572...$ (negative)
There is a change of sign between 0.945 and 0.955, so $a = 0.95$ to 2 decimal places. ***[1 mark]***
[2 marks available in total — as above]

c) When $x = 0.95$, $\frac{dy}{dx} = -0.00177...$ (negative)
so the root is between 0.945 and 0.95, i.e. 0.95 is an overestimate. ***[1 mark]***

14 a) $\frac{dV}{dt} = \frac{2}{Vt} \Rightarrow \int \frac{V}{2}\, dV = \int \frac{1}{t}\, dt$
$\Rightarrow \frac{V^2}{4} = \ln|t| + C = \ln t + C$ as $t > 0$
When $t = 1$, there are 4000 views so $V = 4$.
So $\frac{4^2}{4} = \ln 1 + C \Rightarrow C = 4 \Rightarrow \frac{V^2}{4} = \ln t + 4$
$\Rightarrow V^2 = 4(\ln t + 4) \Rightarrow V = 2\sqrt{\ln t + 4}$
At the end of day 7, $V = 2\sqrt{\ln 7 + 4} = 4.876...$
At the end of day 8, $V = 2\sqrt{\ln 8 + 4} = 4.931...$
So during day 8 it got $1000 \times (4.931... - 4.876...) = 54.457... = 54$ views.
[5 marks available — 1 mark for the correct method for integration, 1 mark for the correct integration, 1 mark for finding the value of C, 1 mark for a correct method to find the number of views on day 8, 1 mark for the correct answer]

b) E.g. The model is undefined when $\ln t < -4$ (i.e. $t < 0.0183...$). Theo could improve the model by giving a different equation for V during this time (e.g. $V = 0$ for $t < 0.0183...$).
[2 marks available — 1 mark for a limitation of the model, 1 mark for a suitable improvement to address the given limitation]
There are other limitations that you could mention — for example, the model suggests that the views will continue to increase forever, so an upper limit on t might be needed.

15 a) $\cos 2x = \cos^2 x - \sin^2 x$
$= (1 - \sin^2 x) - \sin^2 x = 1 - 2\sin^2 x$ ***[1 mark]***
$\Rightarrow \sin^2 x = \frac{1 - \cos 2x}{2}$
$\Rightarrow \sin^2 5x = \frac{1 - \cos 10x}{2} = \frac{1}{2}(1 - \cos 10x)$ ***[1 mark]***
$\int \sin^2 5x\, dx = \frac{1}{2}\int 1 - \cos 10x\, dx$
$= \frac{1}{2}\left(x - \frac{\sin 10x}{10}\right) + C$ or $\frac{x}{2} - \frac{\sin 10x}{20} + C$
[1 mark for $\frac{x}{2}$, 1 mark for $-\frac{\sin 10x}{20}$]
[4 marks available in total — as above]

b) Using integration by parts: $u = x$, $\frac{dv}{dx} = \sin^2 5x$
$\frac{du}{dx} = 1$ and $v = \frac{x}{2} - \frac{\sin 10x}{20}$
$\int_0^{\frac{2\pi}{5}} x\sin^2 5x\, dx = \left[x\left(\frac{x}{2} - \frac{\sin 10x}{20}\right)\right]_0^{\frac{2\pi}{5}} - \int_0^{\frac{2\pi}{5}} 1\left(\frac{x}{2} - \frac{\sin 10x}{20}\right) dx$
$= \left[\frac{x^2}{2} - \frac{x\sin 10x}{20} - \left(\frac{x^2}{4} + \frac{\cos 10x}{200}\right)\right]_0^{\frac{2\pi}{5}}$
$= \left[\frac{x^2}{4} - \frac{x\sin 10x}{20} - \frac{\cos 10x}{200}\right]_0^{\frac{2\pi}{5}}$
$= \left[\frac{\left(\frac{2\pi}{5}\right)^2}{4} - \frac{\frac{2\pi}{5}\sin\frac{10\times 2\pi}{5}}{20} - \frac{\cos\frac{10 \times 2\pi}{5}}{200}\right] - \left[\frac{0^2}{4} - \frac{0\sin(10\times 0)}{20} - \frac{\cos(10 \times 0)}{200}\right]$
$= \left[\frac{\pi^2}{25} - 0 - \frac{1}{200}\right] - \left[0 - 0 - \frac{1}{200}\right] = \frac{\pi^2}{25}$
[5 marks available — 1 mark for attempting to use integration by parts, 1 mark for applying the integration by parts formula correctly, 1 mark for the correct integral, 1 mark for substituting in limits of the integral, 1 mark for the correct answer]

Set 2 Paper 2 — Pure Mathematics 2

1 a) $\overrightarrow{AB} = \begin{pmatrix}-4\\1\\1\end{pmatrix} - \begin{pmatrix}4\\1\\5\end{pmatrix} = \begin{pmatrix}-8\\0\\-4\end{pmatrix}$ ***[1 mark]***
$|\overrightarrow{AB}| = \sqrt{(-8)^2 + 0^2 + (-4)^2}$ ***[1 mark]*** $= \sqrt{80} = 4\sqrt{5}$ ***[1 mark]***
[3 marks available in total — as above]

b) $\overrightarrow{AC} = 3 \times \begin{pmatrix}-8\\0\\-4\end{pmatrix} = \begin{pmatrix}-24\\0\\-12\end{pmatrix}$ ***[1 mark]***
$\overrightarrow{OC} = \begin{pmatrix}4\\1\\5\end{pmatrix} + \begin{pmatrix}-24\\0\\-12\end{pmatrix} = \begin{pmatrix}-20\\1\\-7\end{pmatrix}$ ***[1 mark]***
[2 marks available in total — as above]

2 $64^a \times \left(\frac{1}{16}\right)^b \div \sqrt[c]{32} = (2^6)^a \times (2^{-4})^b \div (2^5)^{\frac{1}{c}}$
$= 2^{6a} \times 2^{-4b} \div 2^{\frac{5}{c}} = 2^{6a - 4b - \frac{5}{c}}$
So $d = 6a - 4b - \frac{5}{c}$
[3 marks available — 1 mark for correctly rewriting one term as a power of 2, 1 mark for expressing all terms as powers of 2, 1 mark for the correct answer]

3 a) £5069 is the amount of money that the farmer would make if she sold her maize crop when $t = 0$ (on 1st July). ***[1 mark]***

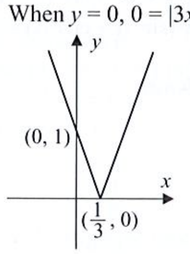
b) $\frac{dP}{dt} = -2t + 66$ ***[1 mark]*** At stationary points, $\frac{dP}{dt} = 0$
$\Rightarrow 0 = -2t + 66 \Rightarrow t = 33$ ***[1 mark]***
Since P is a quadratic with a negative coefficient of t^2, the turning point is a maximum ***[1 mark]***, so the optimum selling date is 33 days after 1st July, which is 3rd August. ***[1 mark]***
[4 marks available in total — as above]
You could also have justified your answer by finding the second derivative and showing it's negative at t = 33, so it's a maximum.

c) Substituting $t = 33$ into the equation for P gives $P = -(33^2) + (66 \times 33) + 5069 = £6158$. ***[1 mark]***

d) For sufficiently large t, e.g. $t = 200$, P is negative which doesn't make sense as P is the amount she sells the crop for. The value of t could be restricted in order to improve the model, e.g. by making $0 \le t \le 111$ as $t = 111$ is the last day where P is positive.
[2 marks available — 1 mark for any suitable limitation, 1 mark for a sensible suggestion for how it can be improved]

4 $4\cos x - 11 = \frac{\sin^2 x - 3}{\cos x}$
$\Rightarrow 4\cos^2 x - 11\cos x = \sin^2 x - 3$ ***[1 mark]***
$\Rightarrow 4\cos^2 x - 11\cos x = (1 - \cos^2 x) - 3$ ***[1 mark]***
$\Rightarrow 5\cos^2 x - 11\cos x + 2 = 0$ ***[1 mark]***
$\Rightarrow (5\cos x - 1)(\cos x - 2) = 0$ ***[1 mark]***
$\Rightarrow \cos x = 0.2$ ***[1 mark]*** ($\cos x \ne 2$ as $-1 \le \cos x \le 1$)
$\Rightarrow x = \cos^{-1}(0.2) = 78.46...° = 78.5°$ (1 d.p.)
and $x = 360° - 78.46...° = 281.53...° = 281.5°$ (1 d.p.)
[1 mark for both]
[6 marks available in total — as above]

5 a) When $x = 0$, $y = |3(0) - 1| = 1$
When $y = 0$, $0 = |3x - 1| \Rightarrow 3x = 1 \Rightarrow x = \frac{1}{3}$



[3 marks available — 1 mark for v-shaped graph above the x-axis, 1 mark for a graph touching the x-axis at $x = \frac{1}{3}$, 1 mark for a graph crossing the y-axis at y = 1]
You could also start with the graph y = 3x – 1, which has gradient 3 and y-intercept –1. Then any point below the x-axis gets reflected in the x-axis to produce the graph of y = |3x – 1|.

b) First, solve the equation $|3x - 1| = 2x + 5$:
When $3x - 1 \ge 0$, the equation becomes
$3x - 1 = 2x + 5 \Rightarrow x = 6$ ***[1 mark]***
When $3x - 1 < 0$, the equation becomes
$-3x + 1 = 2x + 5 \Rightarrow -5x = 4 \Rightarrow x = -0.8$ ***[1 mark]***
Since it is given that the equation has 2 solutions, they must be at $x = 6$ and $x = -0.8$
As the graph of $y = |3x - 1|$ is v-shaped, the inequality $|3x - 1| \le 2x + 5$ will be satisfied between the two solutions, i.e. when $-0.8 \le x \le 6$ ***[1 mark]***
[3 marks available in total — as above]

6 $S_{20} = 1390$ and $S_{30} = 3135$
Using the formula $S_n = \frac{1}{2}n[2a + (n - 1)d]$:
(1): For S_{20}, $1390 = 10(2a + 19d) = 20a + 190d$ ***[1 mark]***
(2): For S_{30}, $3135 = 15(2a + 29d) = 30a + 435d$ ***[1 mark]***
(1) × 3: $4170 = 60a + 570d$
(2) × 2: $6270 = 60a + 870d$
Now subtract one equation from the other to give:
$2100 = 300d \Rightarrow d = 7$ ***[1 mark]*** Substituting back into (1) gives:
$1390 = 20a + 190 \times 7 \Rightarrow 60 = 20a \Rightarrow a = 3$ ***[1 mark]***
Now if $a = 3$ and $d = 7$, $u_{10} = 3 + (9 \times 7) = 66$ prizes ***[1 mark]***
[5 marks available in total — as above]

7 a) Rearranging $x = 3t + 1$ to make t the subject gives $\frac{x-1}{3} = t$.
Substituting this into $y = (t + 3)^3 - 5$ gives
$y = \left(\frac{x-1}{3} + 3\right)^3 - 5 = \left(\frac{x-1}{3} + \frac{9}{3}\right)^3 - 5 = \left(\frac{x+8}{3}\right)^3 - 5$
[2 marks available — 1 mark for rearranging to make t the subject, 1 mark for substituting and rearranging to give the required result]

b) Caleb hasn't changed the limits of the integration — he needs to change them to be in terms of t ***[1 mark]***.
He also hasn't multiplied by $\frac{dx}{dt}$ ***[1 mark]***.
[2 marks available in total — as above]

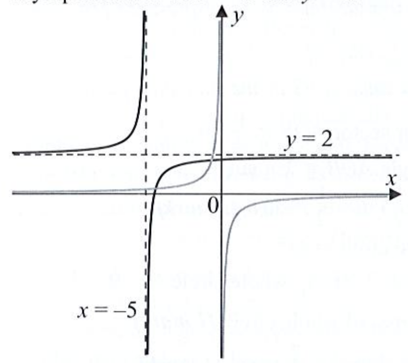
c) The limits of the integration become:
$x = 4 \Rightarrow 4 = 3t + 1 \Rightarrow t = 1$
$x = 10 \Rightarrow 10 = 3t + 1 \Rightarrow t = 3$
$\frac{dx}{dt} = 3$ and $y = (t + 3)^3 - 5$
So $\int_4^{10} y\, dx = 3\int_1^3 ((t + 3)^3 - 5)\, dt = 3\left[\frac{1}{4}(t + 3)^4 - 5t\right]_1^3$
$= 3\left[\frac{6^4}{4} - 15 - \left(\frac{4^4}{4} - 5\right)\right] = 750$
[3 marks available — 1 mark for correcting both of Caleb's errors (i.e. converting the limits and finding $\frac{dx}{dt}$), 1 mark for integrating correctly, 1 mark for substituting in the limits to obtain the correct final answer]

8 a) $y = at^b \Rightarrow \log_{10} y = \log_{10}(at^b) \Rightarrow \log_{10} y = \log_{10} a + \log_{10} t^b$
$\Rightarrow \log_{10} y = \log_{10} a + b\log_{10} t$
[2 marks available — 1 mark for taking logs of both sides, 1 mark for using laws of logs to simplify]

b) Use the graph to find the values of $\log_{10} a$ and b:
$\log_{10} a$ is the vertical intercept, which is 2.475 ***[1 mark]***.
$b = \text{gradient} = \frac{\text{change in } \log_{10} y}{\text{change in } \log_{10} t} = \frac{2.60 - 2.55}{0.25 - 0.15} = 0.5$ ***[1 mark]***
So the equation of the line of best fit is:
$\log_{10} y = 2.475 + 0.5\log_{10} t$
When $y = 1000$: $\log_{10} 1000 = 2.475 + 0.5\log_{10} t$ ***[1 mark]***
$\Rightarrow 0.5\log_{10} t = 3 - 2.475 \Rightarrow \log_{10} t = 1.05$
$\Rightarrow t = 11.220... = 11$ days (nearest whole day) ***[1 mark]***
[4 marks available in total — as above]
You could also work out the value of a and use the original equation (y = 298.53... × $t^{0.5}$) to find the value of t.

c) E.g. The observed pattern might not continue — the rate of change could increase if the ants breed faster or more join the colony, or decrease if the breeding rate or number of ants joining the colony slows. / The colony might reach a certain size then remain at that size (e.g. due to restrictions on space or resources). / Once it gets to a certain size, ants might leave the colony to form a new one, so the number of ants could decrease. ***[1 mark for a sensible limitation linked to the number of ants in the colony]***

9 a) The graph of $y = \frac{-1}{x+5} + 2$ will be a translation of $y = \frac{-1}{x}$ 5 units to the left and 2 units up. So the equations of the asymptotes will be $x = -5$ and $y = 2$.



[2 marks available — 1 mark for drawing the graph in the correct position, 1 mark for drawing and labelling the asymptotes correctly]

b) A stretch, scale factor 3, in the y-direction ***[1 mark]***
followed by a translation by the vector $\begin{pmatrix}0\\2\end{pmatrix}$ ***[1 mark]***.
[2 marks available in total — as above]
You could also have described it as a translation by the vector $\begin{pmatrix}0\\ \frac{2}{3}\end{pmatrix}$ followed by a stretch in the y-direction by scale factor 3.

10 The curve has a root at –1 so $(x + 1)$ is a factor. ***[1 mark]***
$x^3 - 5x^2 + 2x + 8 = (x + 1)(x^2 - 6x + 8)$ ***[1 mark]***
$= (x + 1)(x - 4)(x - 2)$
So the x-value at point C must be $x = 2$ ***[1 mark]***
Shaded area $= \left(\frac{1}{2} \times 1 \times 6\right) + \int_1^2 (x^3 - 5x^2 + 2x + 8)\, dx$ ***[1 mark]***
$= 3 + \left[\frac{x^4}{4} - \frac{5x^3}{3} + x^2 + 8x\right]_1^2$ ***[1 mark]***
$= 3 + \left[\frac{16}{4} - \frac{40}{3} + 4 + 16\right] - \left[\frac{1}{4} - \frac{5}{3} + 1 + 8\right]$ ***[1 mark]***
$= 3 + \frac{32}{3} - \frac{91}{12} = \frac{73}{12}$ ***[1 mark]***
[7 marks available in total — as above]

11 a) Let $f(x) = x^3$ then
$f'(x) = \lim_{h\to 0}\left(\frac{(x + h)^3 - x^3}{h}\right)$ ***[1 mark]***
$= \lim_{h\to 0}\left(\frac{(x^2 + 2xh + h^2)(x + h) - x^3}{h}\right)$
$= \lim_{h\to 0}\left(\frac{(x^3 + 3x^2h + 3xh^2 + h^3) - x^3}{h}\right)$ ***[1 mark]***
$= \lim_{h\to 0}\left(\frac{3x^2h + 3xh^2 + h^3}{h}\right)$
$= \lim_{h\to 0}(3x^2 + 3xh + h^2)$ ***[1 mark]***
As $h \to 0$, $3x^2 + 3xh + h^2 \to 3x^2$, so $f'(x) = 3x^2$ ***[1 mark]***
[4 marks available in total — as above]

b) Let $u = \sin x \Rightarrow \frac{du}{dx} = \cos x$
$y = \sin^3 x = u^3$ so $\frac{dy}{du} = 3u^2 = 3\sin^2 x$
Then $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = 3\cos x \sin^2 x$
Use the product rule where $u = 3\cos x$ and $v = \sin^2 x$:
$\Rightarrow \frac{du}{dx} = -3\sin x$ and $\frac{dv}{dx} = 2\sin x \cos x$

$\frac{d^2y}{dx^2} = v\frac{du}{dx} + u\frac{dv}{dx} = \sin^2 x \times -3\sin x + 3\cos x \times 2\sin x\cos x$
$= -3\sin^3 x + 6\sin x\cos^2 x$

When $x = \frac{\pi}{2}$, $\frac{d^2y}{dx^2} = -3\sin^3\left(\frac{\pi}{2}\right) + 6\sin\left(\frac{\pi}{2}\right)\cos^2\left(\frac{\pi}{2}\right)$
$= -3(1) + 6(0) = -3$

So the stationary point is a maximum.

[5 marks available — 1 mark for using a suitable method to find the first derivative, 1 mark for finding the first derivative correctly, 1 mark for using a suitable method to find the second derivative, 1 mark for finding the second derivative correctly, 1 mark for the correct answer and conclusion]

There are other methods you could have used to differentiate, such as using trig identities on the expressions first. You might get a $\frac{d^2y}{dx^2}$ that looks different to the one above, but it should still give you a value of –3 at the stationary point.

12 a) Area of minor sector $AOB = \frac{1}{2}r^2\theta$

Area of triangle $AOB = \frac{1}{2}r^2\sin\theta$

Area of $S_1 = \frac{1}{2}r^2\theta - \frac{1}{2}r^2\sin\theta$ ***[1 mark]***

Area of whole circle $= \pi r^2$

Now $S_1 : S_2 = 2:7 \Rightarrow S_1$: whole circle $= 2:9$

$\Rightarrow S_1 = \frac{2}{9} \times$ area of whole circle ***[1 mark]***

$\Rightarrow \frac{1}{2}r^2\theta - \frac{1}{2}r^2\sin\theta = \frac{2}{9} \times \pi r^2$ ***[1 mark]***

$\Rightarrow \frac{1}{2}\theta - \frac{1}{2}\sin\theta = \frac{2}{9}\pi$

$\Rightarrow \theta - \sin\theta = \frac{4\pi}{9}$

$\Rightarrow \theta - \sin\theta - \frac{4\pi}{9} = 0$ as required ***[1 mark]***

[4 marks available in total — as above]

You could also have found an expression for the area of S_2 and used the ratio to relate it to S_1 or the area of the whole circle.

b) $\theta_0 = \frac{\pi}{2} \Rightarrow \theta_1 = \sin\frac{\pi}{2} + \frac{4\pi}{9} = 2.3962...$

$\theta_2 = 2.0744...$ $\theta_3 = 2.2720...$ $\theta_4 = 2.1602...$ $\theta_5 = 2.2274...$

So $\theta = 2.2$ radians (2 s.f.) because θ_4 and θ_5 round to the same number to 2 s.f.

[2 marks available — 1 mark for using the iterative formula correctly, 1 mark for the correct answer with justification]

13 a) $\frac{-x-8}{x^2+6x+8} \equiv \frac{-x-8}{(x+4)(x+2)} \equiv \frac{C}{x+4} + \frac{D}{x+2}$ ***[1 mark]***

$\Rightarrow -x - 8 \equiv C(x+2) + D(x+4)$

When $x = -4$: $4 - 8 = C(-4+2) \Rightarrow -4 = -2C \Rightarrow C = 2$

When $x = -2$: $2 - 8 = D(-2+4) \Rightarrow -6 = 2D \Rightarrow D = -3$

[1 mark for the correct value of C or D]

$\frac{-x-8}{(x+4)(x+2)} = \frac{2}{x+4} - \frac{3}{x+2}$ ***[1 mark]***
$= 2(x+4)^{-1} - 3(x+2)^{-1}$
$= 2(4^{-1})\left(\frac{x}{4}+1\right)^{-1} - 3(2^{-1})\left(\frac{x}{2}+1\right)^{-1}$ ***[1 mark]***

$2(4^{-1})\left(\frac{x}{4}+1\right)^{-1} = \frac{1}{2}\left(1 - \frac{x}{4} + \frac{-1\times-2}{1\times2}\left(\frac{x}{4}\right)^2 + ...\right)$
$= \frac{1}{2} - \frac{x}{8} + \frac{x^2}{32} + ...$ ***[1 mark]***

$3(2^{-1})\left(\frac{x}{2}+1\right)^{-1} = \frac{3}{2}\left(1 - \frac{x}{2} + \frac{-1\times-2}{1\times2}\left(\frac{x}{2}\right)^2 + ...\right)$
$= \frac{3}{2} - \frac{3x}{4} + \frac{3x^2}{8} + ...$ ***[1 mark]***

So $2(x+4)^{-1} - 3(x+2)^{-1} \approx \frac{1}{2} - \frac{x}{8} + \frac{x^2}{32} - \left(\frac{3}{2} - \frac{3x}{4} + \frac{3x^2}{8}\right)$
$= -1 + \frac{5x}{8} - \frac{11x^2}{32}$ ***[1 mark]***

[7 marks available in total — as above]

b) The expansion of $\left(\frac{x}{4}+1\right)^{-1}$ is valid when:

$\left|\frac{x}{4}\right| < 1 \Rightarrow -1 < \frac{x}{4} < 1 \Rightarrow -4 < x < 4$ ***[1 mark]***

The expansion of $\left(\frac{x}{2}+1\right)^{-1}$ is valid when:

$\left|\frac{x}{2}\right| < 1 \Rightarrow -1 < \frac{x}{2} < 1 \Rightarrow -2 < x < 2$ ***[1 mark]***

So the expansion of f(x) is valid when both of these are satisfied, i.e. when $-2 < x < 2$. In set notation, this is: $\{x : -2 < x < 2\}$ or $\{x : x > -2\} \cap \{x : x < 2\}$ ***[1 mark]***

[3 marks available in total — as above]

c) When $x = 0.1$, $\frac{-0.1-8}{0.1^2 + 6(0.1) + 8} = -0.9407...$

and $-1 + \frac{5(0.1)}{8} - \frac{11(0.1)^2}{32} = -0.9409...$ ***[1 mark for both]***

Percentage error $= \frac{-0.9409... - -0.9407...}{-0.9407...} \times 100\%$
$= 0.018\%$ (2 s.f.) ***[1 mark]***

[2 marks available in total — as above]

14 a) E.g. Using $\tan^2\theta + 1 = \sec^2\theta$ and $\cos 2\theta = \cos^2\theta - \sin^2\theta$

$\frac{1-\tan^2\theta}{1+\tan^2\theta} = \frac{1-\tan^2\theta}{\sec^2\theta}$ ***[1 mark]*** $= \frac{1}{\sec^2\theta} - \frac{\tan^2\theta}{\sec^2\theta}$ ***[1 mark]***

$= \cos^2\theta - \frac{\sin^2\theta}{\cos^2\theta} \div \frac{1}{\cos^2\theta} = \cos^2\theta - \frac{\sin^2\theta}{\cos^2\theta} \times \cos^2\theta$ ***[1 mark]***

$= \cos^2\theta - \sin^2\theta$ ***[1 mark]*** $= \cos 2\theta$

[4 marks available in total — as above]

b) From part a), $\frac{1-\tan^2\left(\beta+\frac{\pi}{2}\right)}{1+\tan^2\left(\beta+\frac{\pi}{2}\right)} = \cos 2\left(\beta + \frac{\pi}{2}\right)$

$\Rightarrow \cos(2\beta+\pi) - 0.5\sec(2\beta+\pi) = 0$ ***[1 mark]***

$\Rightarrow \cos(2\beta+\pi) - \frac{0.5}{\cos(2\beta+\pi)} = 0$

$\Rightarrow \cos^2(2\beta+\pi) - 0.5 = 0$ ***[1 mark]***

$\Rightarrow \cos(2\beta+\pi) = \pm\sqrt{0.5}$ ***[1 mark]***

$\Rightarrow 2\beta + \pi = \cos^{-1}(\pm\sqrt{0.5})$ and $0 < \beta < \pi \Rightarrow \pi < 2\beta + \pi < 3\pi$

$\Rightarrow 2\beta + \pi = \frac{5\pi}{4}, \frac{7\pi}{4}, \frac{9\pi}{4}, \frac{11\pi}{4}$ ***[1 mark]***

$\Rightarrow 2\beta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4} \Rightarrow \beta = \frac{\pi}{8}, \frac{3\pi}{8}, \frac{5\pi}{8}$ and $\frac{7\pi}{8}$ ***[1 mark]***

[5 marks available in total — as above]

15 a) $\frac{dC}{dt} = -kCt \Rightarrow \int \frac{1}{C}\,dC = \int -kt\,dt$

$\Rightarrow \ln|C| = -\frac{k}{2}t^2 + d$ ***[1 mark]***

$\Rightarrow C = e^{-\frac{k}{2}t^2 + d} = Ae^{-\frac{k}{2}t^2}$ (where $A = e^d$) ***[1 mark]***

At the end of the holidays $t = 0$, so $A = 3600$.

The equation for C is $C = 3600e^{-\frac{k}{2}t^2}$ ***[1 mark]***.

[3 marks available in total — as above]

b) $k = 0.2$, so the equation for C becomes $C = 3600e^{-0.1t^2}$.

Solve the equation $300 = 3600e^{-0.1t^2}$:

$300 = 3600e^{-0.1t^2} \Rightarrow \frac{1}{12} = e^{-0.1t^2}$

$\Rightarrow \ln\frac{1}{12} = -0.1t^2 \Rightarrow t = \sqrt{-10\ln\frac{1}{12}} = 4.984...$

So $t = 5$ weeks (to the nearest whole week).

[3 marks available — 1 mark for substituting the values of k and C into the equation, 1 mark for rearranging and taking logs, 1 mark for the correct answer]

You can check that your answer is correct by putting t = 4 and t = 5 into the equation for C: t = 4 gives 726.827... customers, and t = 5 gives 295.505... This is less than 300, so the ice cream parlour will close after 5 weeks.

Set 2 Paper 3 — Statistics and Mechanics

1 a) Since values are given correct to the nearest knot, speeds in this class are actually between 4.5 kn and 8.5 kn, giving a class width of 4 kn. ***[1 mark]***

b) $n = 17 + 14 + 7 + 2 = 40$, so Q_1 is in the $\frac{40}{4} = 10^{th}$ position and Q_3 is in the $\frac{40\times3}{4} = 30^{th}$ position. From the frequency table, Q_1 is in the 0 – 4 class and Q_3 is in the 5 – 8 class. ***[1 mark for both]***

Using linear interpolation:

$Q_1 \approx 0 + 4.5 \times \frac{10-0}{17} = 2.6470... = 2.65$ kn (3 s.f.) ***[1 mark]***

$Q_3 \approx 4.5 + 4 \times \frac{30-17}{14} = 4.5 + 3.7142... = 8.2142...$
$= 8.21$ kn (3 s.f.) ***[1 mark]***

IQR ≈ 8.2142... – 2.6470... = 5.5672...
= 5.57 kn (3 s.f.) ***[1 mark]***

[4 marks available in total — as above]

c) Lower boundary = 2.6470... – (1.5 × 5.5672...) = –5.7037...
The lowest data value, 1, is greater than the lower boundary, so it is not an outlier. ***[1 mark]***
Upper boundary = 8.2142... + (1.5 × 5.5672...) = 16.565...
The highest data value, 18, is greater than the upper boundary, so it is an outlier. ***[1 mark]***

[2 marks available in total — as above]

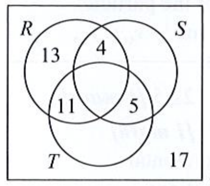
d) E.g. The median, as it is not as heavily affected by outliers. ***[1 mark]***

You could also argue that the mean is better because it uses all of the values so may be more representative of the data — as long as you justify your answer, you'll get the mark.

e) E.g. He has only looked at data for part of each country — the average for the whole country might be different / he has only looked at data for part of 1987 — the weather patterns may have changed since then / he has only looked at windspeed — other data may contradict this / he has not done a hypothesis test to see whether the higher mean is just down to random chance / he has not considered any other measures of location or variation (e.g. median, interquartile range, etc) / he has not considered other factors that could impact windspeed (e.g. how close the location is to the coast) / etc.

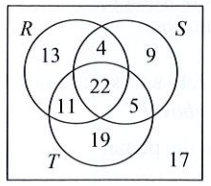
[2 marks available — 1 mark for each sensible criticism]

2 a) Draw a Venn diagram with three overlapping circles, then fill in the values of n(only R), n(don't like any), n(only R and T), n(only S and T) and n(only S and R):



[1 mark]

n(R and S) = 26, so n(R and S and T) = 26 – 4 = 22 ***[1 mark]***
n(S) = 40, so n(only S) = 40 – (4 + 5 + 22) = 9 ***[1 mark]***
100 students were surveyed, so n(only T)
= 100 – (13 + 4 + 9 + 11 + 22 + 5 + 17) = 19 ***[1 mark]***
So the completed Venn diagram is:



[4 marks available in total — as above]

b) i) P(likes at least one) = 1 – P(doesn't like any)
$= 1 - \frac{17}{100} = \frac{83}{100}$

[2 marks available — 1 mark for the correct numerator, 1 mark for the correct denominator]

You could also add up all of the numbers inside the circles (you should get 83) and divide it by the total (100).

ii) $P(T \mid S) = \frac{5+22}{9+5+4+22} = \frac{27}{40}$

[2 marks available — 1 mark for the correct numerator, 1 mark for the correct denominator]

iii) $P((R \cap S') \mid T) = \frac{11}{19+11+22+5} = \frac{11}{57}$

[2 marks available — 1 mark for the correct numerator, 1 mark for the correct denominator]

c) If the teacher's claim is true, then the events T and S are independent, i.e. $P(T) \times P(S) = P(T \cap S)$.

$P(T) \times P(S) = \frac{11+22+5+19}{100} \times \frac{4+22+5+9}{100} = \frac{57}{250}$

$P(T \cap S) = \frac{27}{100}$ so the results of the survey do not support the teachers claim.

[3 marks available — 1 mark for using appropriate probability law for determining whether events are dependent, 1 mark for correctly calculating relevant probabilities, 1 mark for correct conclusion]

There are other ways you could check whether these are independent — for example, you could compare P(T|S) to P(T).

3 a) E.g. The graph shows weak positive correlation between the variables. ***[1 mark]***

b) $H_0: \rho = 0$, $H_1: \rho \neq 0$ ***[1 mark]***
This is a two-tailed test at the 10% significance level, so use the 0.05 column in the critical value table.
$n = 10$, so critical value = 0.5494. ***[1 mark]***
0.5494 > 0.3850, so there is not enough evidence at the 10% level to reject H_0 and conclude that the product moment correlation coefficient is non-zero. ***[1 mark]***

[3 marks available in total — as above]

c) The remaining points would lie closer to a straight line, so the value of r would increase. ***[1 mark]***

4 a) Any one of: there are a fixed number of trials (packets of sweets) / each trial is independent of the others (whether one packet contains a gold-wrapped sweet does not affect whether another packet does) / there are only two possible outcomes (a packet either contains a gold-wrapped sweet or doesn't) / the probability remains constant from trial to trial. ***[1 mark]***
$n = 50$, $p = \frac{9}{20} = 0.45$ ***[1 mark for both]***

[2 marks available in total — as above]

b) $H_0: p = 0.45$, $H_1: p < 0.45$ ***[1 mark]*** $G \sim B(50, 0.45)$
Using the binomial tables: $P(G \leq 16) = 0.0427$ ***[1 mark]***
0.0427 < 0.05, so the result is significant. ***[1 mark]***
There is enough evidence at the 5% level to reject H_0 and conclude that the company has put fewer gold-wrapped sweets in their packets of sweets than they said they did. ***[1 mark]***

[4 marks available in total — as above]

5 a) i) P(positive test result and carrier)
= 0.98 × 0.03 = 0.0294 ***[1 mark]***
P(positive test result and non-carrier)
= (1 – 0.95) × (1 – 0.03) = 0.0485 ***[1 mark]***
P(positive test result) = 0.0294 + 0.0485 = 0.0779 ***[1 mark]***
P(carrier given a positive test result)
= 0.0294 ÷ 0.0779 = 0.37740... = 0.377 (3 s.f.) ***[1 mark]***

[4 marks available in total — as above]

ii) E.g. Joey is a random member of the UK population and isn't more likely to have the disease just because he takes the test. ***[1 mark]***

b) P(correct test result) = P(positive test result and carrier) + P(negative test result and not carrier) ***[1 mark]***
= (0.98 × 0.03) + (0.95 × 0.97) = 0.0294 + 0.9215
= 0.9509 or 95.09% so Hiroshi is not correct. ***[1 mark]***

[2 marks available in total — as above]

c) E.g. No, because the majority of positive results are for people who don't carry the disease (i.e. most of the time a positive result is a false positive). ***[1 mark]***

You could argue either yes or no for this question, as long as you fully explain your reasoning. E.g. you could say yes because only 2% of people who carry the disease will not be identified.

6 a) Let L be the length of a pencil in cm. Then $L \sim N(18, 0.1^2)$.
P(pencil will fit in box) = $P(L < 18.2)$ ***[1 mark]***
From your calculator: $P(L < 18.2) = 0.9772...$ ***[1 mark]***
P(box will not be damaged) = $(0.9772...)^5 = 0.8913...$ ***[1 mark]***
P(box will be damaged) = 1 – 0.8913...
= 0.1086... = 0.109 (3 s.f.) ***[1 mark]***

[4 marks available in total — as above]

You could also have used a binomial distribution for the number of pencils in a box that are longer/shorter than 18.2 cm.

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