**Answer ALL the questions.**

**Write your answers in the spaces provided.**

Leave blank

1 Point $A$ has position vector $\begin{pmatrix} 4 \\ 1 \\ 5 \end{pmatrix}$ and point $B$ has position vector $\begin{pmatrix} -4 \\ 1 \\ 1 \end{pmatrix}$.

**a)** Find the distance between points $A$ and $B$, giving your answer as a fully simplified surd. **(3)**

**b)** Given that $\overrightarrow{AC} = 3\overrightarrow{AB}$, find the position vector of point $C$. **(2)**

2 Given that:

$$64^a \times \left(\frac{1}{16}\right)^b \div \sqrt[c]{32} = 2^d,$$

express $d$ in terms of $a$, $b$ and $c$. **(3)**





Leave blank

3 A farmer believes that the following formula models the relationship between the amount of money, $P$, in pounds, she can make from selling her maize crop $t$ days after 1st July:

$$P = -t^2 + 66t + 5069$$

**a)** With reference to the model, interpret the significance of 5069 in this formula. **(1)**

**b)** Use calculus to find the date on which she should sell the crop to make as much money as possible, fully justifying your answer. **(4)**

**c)** How much money can she make from the crop if she sells on this date? **(1)**

**d)** Describe a limitation of this model and suggest one way in which it could be improved. **(2)**

4 Solve the equation:

$$4\cos x - 11 = \frac{\sin^2 x - 3}{\cos x}, \quad \cos x \neq 0$$

for $0° \leq x < 360°$. Give your answers correct to 1 decimal place. **(6)**

Leave blank

5 a) Sketch the graph of $y = |3x - 1|$ below, clearly stating the coordinates of any points at which the graph touches or intersects the axes. **(3)**





b) The equation $|3x - 1| = 2x + 5$ has two solutions. Use this information to help you solve the inequality $|3x - 1| \leq 2x + 5$. **(3)**

Leave blank

6 Sarah runs a weekly maths competition. The number of prizes given out in consecutive competitions forms an arithmetic sequence.

After 20 competitions, a total of 1390 prizes had been given out.
After 30 competitions, a total of 3135 prizes had been given out.

How many prizes were given out at the 10th competition? **(5)**

**7** A curve is defined by the parametric equations:

$$x = 3t + 1 \qquad y = (t + 3)^3 - 5.$$

**a)** Verify that the curve can be written in Cartesian form as $y = \left(\frac{x+8}{3}\right)^3 - 5$. **(2)**

**b)** Caleb wants to evaluate $\int_4^{10} y \, dx$.

He decides to integrate parametrically, and works out that $\int_4^{10} y \, dx = \int_4^{10} ((t+3)^3 - 5) \, dt$.

Identify two errors that Caleb has made. **(2)**

**c)** By correcting Caleb's mistakes, use parametric integration to evaluate $\int_4^{10} y \, dx$. **(3)**

Leave blank




**8** A scientist is studying a newly-formed ant colony. After observing the colony for several days, the scientist decides that the population can be modelled by the equation $y = at^b$, where $y$ is the population, $t$ is the time that the colony has existed for, measured in days, and $a$ and $b$ are constants.

**a)** Find a linear equation for $\log_{10} y$ in terms of $\log_{10} t$. **(2)**

The scientist plots his observed values of $\log_{10} y$ against $\log_{10} t$, then draws a line of best fit as shown.

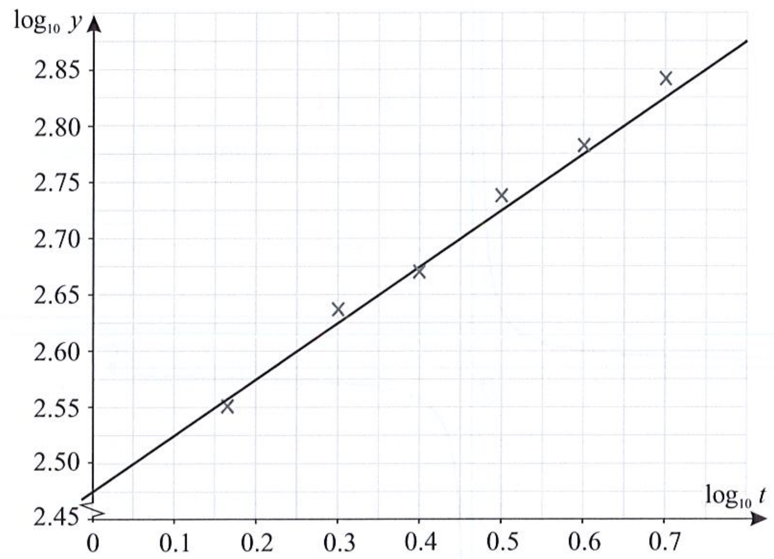


**Figure 1**

**b)** By finding the equation of the line of best fit, estimate how many days the colony will have existed for when the population reaches 1000. Give your answer to the nearest whole day. **(4)**

Leave blank

**c)** Give one limitation of this model, explaining your answer in context of the question. **(1)**

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**9** Figure 2 shows a sketch of the graph of $y = \frac{-1}{x}$.

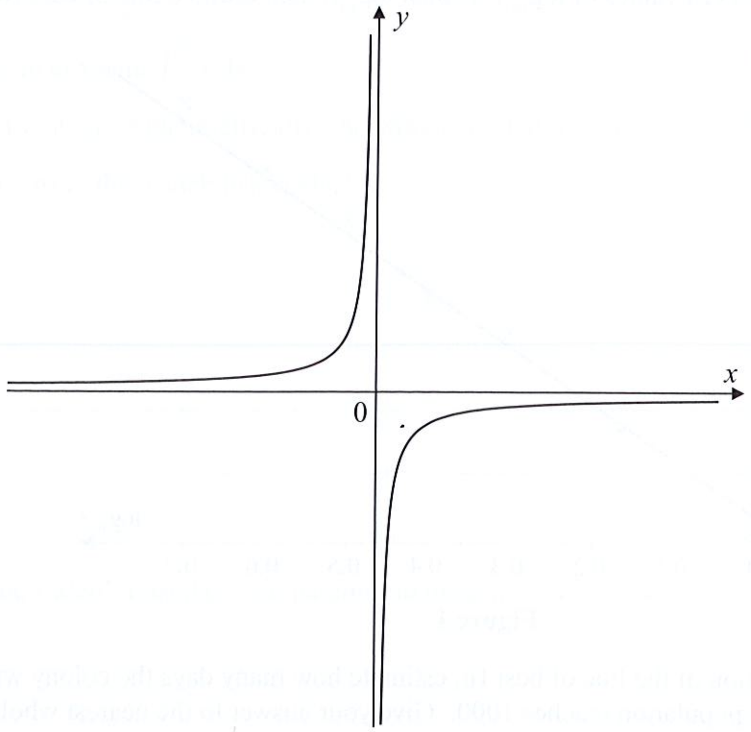


**Figure 2**

**a)** Sketch the graph of $y = \frac{-1}{x+5} + 2$ on the axes above.
Clearly draw and label any asymptotes with their equations. **(2)**

**b)** Describe the transformations which would take the graph of $y = \frac{-1}{x+5} + 2$ onto the graph of $y = \frac{-3}{x+5} + 8$. **(2)**




**10** Figure 3 shows part of the curve with equation $y = x^3 - 5x^2 + 2x + 8$ and the line $AB$.
$A$ has coordinates (0, 0). $B$ has coordinates (1, 6) and lies on the curve.
(–1, 0) and $C$ are two points of intersection between the curve and the $x$-axis.

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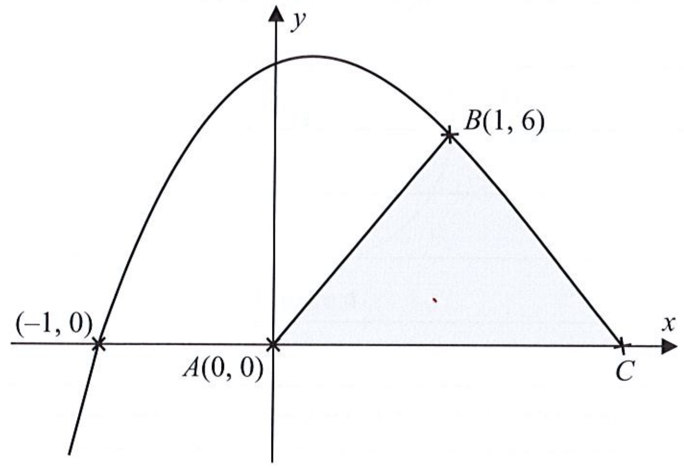


**Figure 3**

Find the exact area of the shaded region. **(7)**

**11 a)** Prove, from first principles, that the derivative of $x^3$ is $3x^2$. **(4)**

Leave blank

**b)** The curve $y = \sin^3 x$ has a stationary point at $x = \frac{\pi}{2}$.

By finding $\frac{d^2y}{dx^2}$, determine the nature of the stationary point. **(5)**





**12** Figure 4 shows a circle with centre $O$ and radius $r$, divided into two segments, $S_1$ and $S_2$, by the line $AB$.

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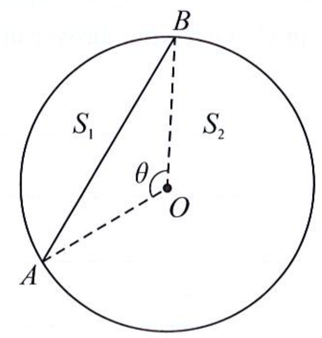


Diagram not drawn accurately.

**Figure 4**

$\theta$ is the obtuse angle $AOB$, given in radians.

**a)** Given that the ratio of the area of $S_1$ to the area of $S_2$ is $2:7$, show that:

$$\theta - \sin\theta - \frac{4\pi}{9} = 0$$

**(4)**

**b)** Using the iterative formula $\theta_{n+1} = \sin\theta_n + \frac{4\pi}{9}$ and $\theta_0 = \frac{\pi}{2}$,

find the value of $\theta$ correct to 2 significant figures, justifying your answer. **(2)**

**13 a)** By expressing $f(x) = \frac{-x-8}{x^2+6x+8}$ as partial fractions, find the binomial expansion of $f(x)$, in ascending powers of $x$, as far as the term in $x^2$. Give your answer in its fully simplified form. **(7)**

*Leave blank*





**b)** Find the range of values of $x$ for which this expansion is valid. Give your answer in set notation. **(3)**

*Leave blank*

**c)** Find the percentage error when your expansion is used to find an estimate for the value of $f(0.1)$. Give your answer correct to 2 significant figures. **(2)**

**14 a)** Prove that $\frac{1-\tan^2\theta}{1+\tan^2\theta} \equiv \cos 2\theta$. **(4)**

**b)** Hence find the exact solutions of the equation:

$$\frac{1-\tan^2\left(\beta+\frac{\pi}{2}\right)}{1+\tan^2\left(\beta+\frac{\pi}{2}\right)} - 0.5\sec(2\beta+\pi) = 0, \ \cos(2\beta+\pi) \neq 0, \ \text{for } 0 < \beta < \pi.$$ **(5)**

*Leave blank*





**15** The number of customers at an ice cream parlour decreases after the end of the summer holidays. The rate of change of the decrease can be modelled by the differential equation $\frac{dC}{dt} = -kCt$ $(k > 0)$, where $C$ is the number of customers per week, $t$ is the number of weeks after the end of the holidays, and $k$ is a constant.

*Leave blank*

**a)** At the end of the holidays, the ice cream parlour had 3600 customers per week. Use this information to solve the differential equation, giving your answer as a formula for $C$ in terms of $k$ and $t$. **(3)**

**b)** The ice cream parlour will close when the number of customers drops below 300 per week. Given that the value of $k$ is 0.2, calculate how many weeks after the end of the holidays the ice cream parlour will close. Give your answer to the nearest whole week. **(3)**

**END OF QUESTIONS**