Answer ALL the questions.

Write your answers in the spaces provided.

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1 Given that $f(x) = 3x^2 + 12x + 19$:

a) find the value of the discriminant and explain what it tells you about the graph of $y = f(x)$. **(2)**

b) write $f(x)$ in the form $p(x + q)^2 + r$, where p, q and r are integers to be found. **(3)**

c) sketch the graph of $y = f(x)$, showing clearly the coordinates of any points of intersection with the axes and any stationary points. **(3)**

Leave blank

d) Describe a combination of two transformations that would map the graph of $y = 3x^2 + 12x + 19$ onto the graph $y = x^2$. **(2)**

2 The organisers of a car show want to rope off a rectangular area of floor space.
They have 40 m of rope available and need to have at least 60 m² of space roped off.
They want the length of the rectangle to be exactly twice its width.

Find the difference between the maximum and minimum width of the rectangle.

Give your answer in metres, in the form $\frac{a-\sqrt{b}}{c}$, where a, b and c are integers. **(4)**

3 a) Show that when $y = e^{2x}$, y is directly proportional to $\frac{dy}{dx}$. **(2)**

Leave blank

b) The line l is the tangent to the curve $y = e^{2x}$ at the point $(2, e^4)$.
Find, using exact values, the coordinates of the point where l crosses the x-axis. **(2)**

4 Freya wants to prove that $3\sin x + x^2 + 3 > 0$ for all values of x.

a) Freya makes the following claim, "Because $\sin x$ repeats every 2π, I only need to prove it for $0 \leq x \leq 2\pi$." Is she correct? Explain your answer. **(2)**

b) Prove that $3\sin x + x^2 + 3 > 0$ for all values of x. **(3)**

5 The graph of $y = f(x)$ intersects the x-axis at $x = -2$. Given that:

$$f(x) = x^3 - bx^2 + 2x + 40$$

find the value of b and express $f(x)$ as the product of 3 linear factors. **(6)**

Leave blank

6 A rubber ball is dropped from a height of 6 m.
It falls vertically before hitting the ground and bouncing straight back up.
After the first time it hits the ground, it ascends to a height of 5.52 m.
The maximum heights that the ball reaches after each bounce form a geometric sequence.

Leave blank

a) Show algebraically that the first time the ball bounces to a maximum height of less than 1 metre is after the 22nd bounce.

(5)

b) Eventually the ball comes to rest. Estimate the total distance that the ball travelled.

(2)

7 A square has vertices at (0, 4), (4, 0), (0, –4) and (–4, 0) as shown in Figure 1.

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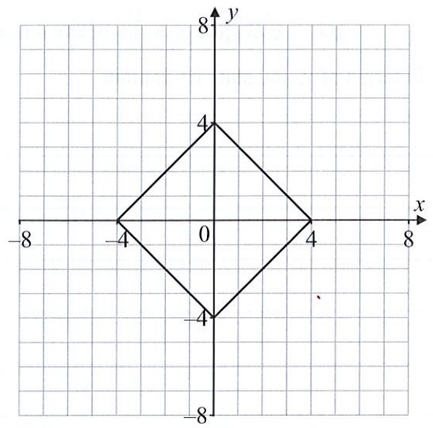

Figure 1

A circle of radius 5 units has its centre at the point (–2, 3).
Find the coordinates of the points of intersection of the circle and the square.
Give your answers correct to 2 decimal places where appropriate.

(7)

8 Given that:

$$f^{-1}(x) = \frac{2x-5}{x},\ x \neq 0 \qquad g(x) = \sqrt{2x-k},\ x \geq \frac{k}{2},\ \text{where } k \text{ is a positive constant}$$

Leave blank

a) find $fg(x)$, giving your answer in terms of x, and state its domain. **(3)**

b) If $gg(10) = 2$, find the value of k. **(3)**

9 A curve has equation $3y^2 - 4y = 4 - 2x^3$.

Leave blank

a) Find $\frac{dy}{dx}$ in terms of x and y in its simplest form. **(3)**

b) Find the exact distance between the two stationary points on the curve. **(3)**

10 By using the substitution $x = \frac{2}{3}\sin\theta$, find:

$$\int \frac{1}{\sqrt{4-9x^2}}\,dx$$

giving your answer in terms of x.

(5)

Leave blank

11 a) $5\sin\theta + 7\cos\theta$ can be written in the form $R\sin(\theta + \alpha)$ where α is an acute angle.

Find R, giving your answer in surd form, and find α, giving your answer in degrees to the nearest whole number.

(3)

Leave blank

b) Two boats sail from the same harbour on the same bearing.
Boat A sails at 5 mph and boat B sails at 7 mph.

After an hour, boat A sails due west until it is directly north of where it started and boat B sails due south until it is directly east of where it started. Once both boats are in these positions, the total distance they have sailed is 18 miles.

Find the bearing on which the boats both sailed on the first stage of their journey, giving your answer to the nearest whole degree.

(4)

12 A sum of money is invested in a savings account with a compound interest rate of $r\%$, paid annually. T is the number of years taken for the investment to be worth twice the sum invested.

Leave blank

a) Express T in the form $\log_a(b)$, where b is an integer and a is given in terms of r. **(3)**

b) It takes exactly 15 years for an investment to double in an account offering compound interest. Find the interest rate, giving your answer as a percentage correct to 2 decimal places. **(2)**

c) Money invested in a simple interest account with annual interest rate $p\%$ takes twice as long to double in value as money in a compound interest account with annual interest rate $q\%$.

Show that $q = 100(2^{\frac{p}{50}} - 1)$. **(4)**

13 A curve is given by the equation $y = \frac{x^2 \cos x}{3 \sin x}$, $\sin x \neq 0$.

Leave blank

a) Show that $\frac{dy}{dx} = \frac{x(\sin 2x - x)}{3 \sin^2 x}$. **(5)**

b) The curve has one turning point between $x = 0$ and $x = 1$.
The x-coordinate of this turning point is a.

i) Verify that $a = 0.95$ to 2 decimal places. **(2)**

ii) Determine whether 0.95 is an overestimate or an underestimate for the value of a. **(1)**

14 Theo models the total number of views (V, in thousands) that each of his videos has t days after he uploads it to his channel using the following equation:

$$\frac{dV}{dt} = \frac{2}{Vt}, \quad t > 0$$

1 day after being uploaded, a particular video has 4000 views.

a) Use the model to estimate how many views the video would get during the 8th day after it was uploaded. **(5)**

Leave blank

b) Explain one limitation of this model, and suggest a possible improvement that Theo could make to the model in order to address it. **(2)**

15 **a)** Find $\int \sin^2 5x \, dx$. **(4)**

Leave blank

b) Hence, find the exact value of $\int_0^{\frac{2\pi}{5}} x \sin^2 5x \, dx$. **(5)**

END OF QUESTIONS