Write your name here

Surname	Other names

Pearson Edexcel GCE

Centre Number

Candidate Number

A level Further Mathematics
Further Statistics 1
Practice Paper 2

You must have:
Mathematical Formulae and Statistical Tables (Pink)

Total Marks

Instructions

- Use black ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- Fill in the boxes at the top of this page with your name, centre number and candidate number.
- Answer all the questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – there may be more space than you need.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are **7** questions in this question paper. The total mark for this paper is **75**.
- The marks for each question are shown in brackets – use this as a guide as to how much time to spend on each question.
- Calculators must not be used for questions marked with a * sign.

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

1. Accidents occur randomly at a road junction at a rate of 18 every year.

The random variable X represents the number of accidents at this road junction in the next 6 months.

(a) Write down the distribution of X.

(2)

(b) Find $P(X > 7)$.

(2)

(c) Show that the probability of at least one accident in a randomly selected month is 0.777 (correct to 3 decimal places).

(3)

(d) Find the probability that there is at least one accident in exactly 4 of the next 6 months.

(3)

(Total 10 marks)

[Mark scheme for Question 1](#)

[Examiner comment](#)

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2. (a) Define

(i) a Type I error,

(ii) a Type II error.

(2)

Rolls of material, manufactured by a machine, contain defects at a mean rate of 6 per roll.

The machine is modified. A single roll is selected at random and a test is carried out to see whether or not the mean number of defects per roll has decreased. The significance level is chosen to be as close as possible to 5%.

(b) Calculate the probability of a Type I error for this test.

(3)

(c) Given that the true mean number of defects per roll of material made by the machine is now 4, calculate the probability of a Type II error.

(2)

(Total 7 marks)

[Mark scheme for Question 2](#)

[Examiner comment](#)

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3. (a) Write down the two conditions needed to approximate the binomial distribution by the Poisson distribution.

(2)

A machine which manufactures bolts is known to produce 3% defective bolts. The machine breaks down and a new machine is installed. A random sample of 200 bolts is taken from those produced by the new machine and 12 bolts are defective.

(b) Using a suitable approximation, test at the 5% level of significance whether or not the proportion of defective bolts is higher with the new machine than with the old machine. State your hypotheses clearly.

(7)

(Total 9 marks)

Mark scheme for Question 3

Examiner comment

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4. An archer shoots at a target until he hits it. The random variable S is the number of shots needed by the archer to hit the target.

(a) State a suitable distribution to model S.

(1)

Given that the mean of S is 8, calculate the probability of the archer

(b) hitting the target for the first time on his 5th shot,

(3)

(c) taking at least 3 shots to hit the target for the first time.

(3)

(d) State any assumptions you have made in using this model.

(2)

(Total 9 marks)

Mark scheme for Question 4

Examiner comment

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5. A total of 100 random samples of 6 items are selected from a production line n a factory and the number of defective items in each sample is recorded. The results are summarised in the table below.

Number of defective items	0	1	2	3	4	5	6
Number of samples	6	16	20	23	17	10	8

(a) Show that the mean number of defective items per sample is 2.91.

(2)

A factory manager suggests that the data can be modelled by a binomial distribution with $n = 6$. He uses the mean from the sample above and calculates expected frequencies as shown in the table below.

Number of defective items	0	1	2	3	4	5	6
Expected frequency	1.87	10.54	24.82	a	22.01	8.29	b

(b) Calculate the value of a and the value of b, giving your answers to 2 decimal places.

(4)

(c) Test, at the 5% level, whether or not the binomial distribution is a suitable model for the number of defective items in samples of 6 items. State your hypotheses clearly.

(8)

(Total 14 marks)

Mark scheme for Question 5

Examiner comment

6. The probability generating function of the random variable X is given by

$$G_x(t) = k(1 + 2t + 2t^2)^2.$$

(a) Show that $k = \dfrac{1}{25}$.

(2)

(b) Find $P(X = 2)$.

(2)

(c) Calculate E(X) and Var(X).

(8)

(d) Write down the probability generating function of $2X + 1$.

(2)

(Total 14 marks)

Mark scheme for Question 6

Examiner comment

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7. The random variable Y is the number of times a biased coin is tossed until 3 heads have occurred. The variance of Y is 60.

(a) Find the probability of obtaining a head.

(5)

(b) Find $P(Y = 8)$.

(2)

(c) Find $P(Y \leq 10 \mid \text{the first head was gained on the second toss})$.

(5)

(Total 12 marks)

Mark scheme for Question 7

Examiner comment

TOTAL FOR PAPER: 75 MARKS

A level Further Mathematics – Further Statistics 1 – Practice Paper 02 – Mark scheme –

Mark scheme for Question 1 (Examiner comment) (Return to Question 1)

Question	Scheme	Marks
1(a)	$X \sim \text{Po}(9)$	M1A1
		(2)
(b)	$\text{P}(X > 7) = 1 - \text{P}(X \le 7)$	M1
	$= [1 - 0.3239] = 0.6761$	A1
		(2)
(c)	[Y = no. of accidents in a month] $Y \sim \text{Po}(1.5)$	B1
	$\text{P}(Y \ge 1) = 1 - \text{P}(Y = 0)$	M1
	$= [1 - 0.2231] = 0.7769$ (= 0.777 (3dp))*	A1cso
		(3)
(d)	[A = no. of months with at least one accident] $A \sim \text{B}(6, 0.777)$	M1
	$\text{P}(A = 4) = \binom{6}{4}(0.777)^4(0.223)^2$	M1
	$= 0.2719\ldots$ awrt **0.272**	A1
		(3)
		(10 marks)

Mark scheme for Question 2 (Examiner comment) (Return to Question 2)

Question	Scheme	Marks
2(a)(i)	Type I - H_0 rejected when it is true	**B1**
(ii)	Type II - H_0 is accepted when it is false	**B1**
		(2)
(b)	$P(X < c \mid \lambda = 6) \approx 0.05$	**M1**
	$P(X \leq 2) = 0.0620$	
	$P(X \leq 1) = 0.0174$	
	Critical region = $X \leq 2$	**A1**
	P(Type 1 error) = $P(X \leq 2 \mid \lambda = 6) = 0.062$	**A1cao**
		(3)
(c)	P(Type 2 error) = P($X \geq 3$ | □□= 4)	**M1**
	$= 1 - 0.2381$	
	$= 0.7619$	**A1**
		(2)
		(7 marks)

Mark scheme for Question 3 (Examiner comment) (Return to Question 3)

Question	Scheme	Marks
3(a)	n – large/high/big/ $n > 50$	**B1**
	p – small/close to 0 / $p < 0.2$	**B1**
		(2)
(b)	$H_0 : p = 0.03 \quad H_1 : p > 0.03$	**B1B1**
	Po(6)	**B1**
	$P(X \geq 12) = 1 - P(X \leq 11)$ or $P(X \leq 10) = 0.9574$	**M1**
	$= 1 - 0.9799$ $P(X \geq 11) = 0.0426$	
	$= 0.0201$ CR $X \geq 11$	**A1**
	$(0.0201 < 0.05)$	
	Reject H_0 or Significant or 12 lies in the Critical region.	**M1d**
	There is evidence that the proportion of defective bolts has increased.	**A1ft**
		(7)
		(9 marks)

Mark scheme for Question 4 (Examiner comment) (Return to Question 4)

Question	Scheme	Marks
4(a)	Geometric	B1
		(1)
(b)	$p = \frac{1}{8}$	B1
	$P(S = 5) = \left(\frac{7}{8}\right)^4 \times \left(\frac{1}{8}\right)$	M1
	$= 0.073$	A1
		(3)
(c)	$P(S \geq 3) = (1 - p)^2$	M1 A1ft
	$= \left(\frac{7}{8}\right)^2 = \frac{49}{64}$ awrt 0.766	A1
		(3)
(d)	Assume shots are *independent* and *probability* of hits is *constant*	B1B1
		(2)
		(9 marks)

Mark scheme for Question 5 (Examiner comment) (Return to Question 5)

Question	Scheme	Marks
5(a)	Mean= $\frac{1\times16+2\times20+...+6\times8}{100}=2.91$ **ag**	M1A1
		(2)
(b)	$p=\frac{2.91}{6}=0.485$	B1
	$a=100\times C_3^6\times0.485^3\times0.515^3=31.17$	M1A1
	$b=100\times0.485^6=1.3(0)$	A1
		(4)
(c)	H_0 : Binomial is a good fit	
	H_1 : Binomial is a not a good fit	B1
	see table below	M1
	$\sum\frac{(O-E)^2}{E}=\frac{(22-12.41)^2}{12.41}+\frac{(20-24.82)^2}{24.82}+...+\frac{(18-9.59)^2}{9.59}=18.998...$ awrt 19.0	M1A1
	ν =5-2=3 degrees of freedom	B1
	$\chi_3^2(5\%)=7.815$	B1ft
	18.998...>7.815 so reject H_0	M1
	Binomial is a not a good fit (and is not a good model for the number of defective items in samples of size 6)	A1
		(8)
		(14 marks)

Number of defective items	0 or 1	2	3	4	5 or 6
O	22	20	23	17	18
E	12.41	24.82	31.17	22.01	9.59

Mark scheme for Question 6 (Examiner comment) (Return to Question 6)

Question	Scheme		Marks
6(a)	$G_x(1) = 1$		M1
	$k(1+2+2)^2 = 1 \quad \Rightarrow \quad k = \dfrac{1}{25}$ *AG	cso	A1
			(2)
(b)	$\dfrac{1}{25}(1+2t+2t^2)(1+2t+2t^2) = \dfrac{1}{25}(1+4t+8t^2+\ldots)$		M1
	$P(X=2) = \dfrac{8}{25}$		A1
			(2)
(c)	$G'_x(t) = \dfrac{2}{25}(1+2t+2t^2)(2+4t)$		M1A1
	$G'_x(t) = \dfrac{12}{5} \Rightarrow E(X) = \dfrac{12}{5}$		A1
	$G''_x(t) = \dfrac{2}{25}(2+4t)(2+4t) + \dfrac{2}{25}(1+2t+2t^2)\,.\,4$		M1A1
	$G''_x(t) = \dfrac{112}{25}$		A1
	$\text{Var}(X) = \dfrac{112}{25} + \dfrac{12}{5} - \left(\dfrac{12}{5}\right)^2 = \dfrac{28}{25}$		M1A1
			(8)
(d)	$\dfrac{t}{25}(1+2t^2+2t^4)^2$		B1B1
			(2)
			(14 marks)

Mark scheme for Question 7 (Examiner comment) (Return to Question 7)

Question	Scheme	Marks
7(a)	$60 = \dfrac{3(1-p)}{p^2}$	**M1A1**
	$20p = 1 - p$	**M1**
	$(5p-1)(4p+1) = 0$	**M1**
	$p = \frac{1}{5}$	**A1**
		(5)
(b)	$\mathrm{P}(Y = 8) \quad = \binom{7}{2}p^2\,(1-p)^5 \times p$	**M1**
	$= \binom{7}{2} \times \left(\frac{1}{5}\right)^3 \times \left(\frac{4}{5}\right)^5 = 0.05505$ awrt 0.055	**A1**
		(2)
(c)	$\mathrm{P}(Y \le 10 \mid \ldots) \quad = 1 - \mathrm{P}(Y \ge 11 \mid \text{1st head on 2nd toss})$	**M1**
	$= 1 - \mathrm{P}(\text{0 heads in 8 tosses}) - \mathrm{P}(\text{1 head in 8 tosses})$	**M1**
	$= 1 - 0.8^8 - 8 \times 0.2 \times (0.8)^7$	**A1A1**
	$= 0.49688$	**A1**
		(5)
		(12 marks)

A level Further Mathematics – Further Statistics 1 – Practice Paper 02 – Examiner report –

Examiner comment for Question 1 **(Mark scheme)** **(Return to Question 1)**

1. Parts (a) and (b) were well answered although in part (a) a few candidates wrote down Poisson but forgot to give the value of λ .

Since part (c) is a 'show that' question candidates were required to write down all the steps needed to reach 0.777. Many candidates went straight from 1 – P(Y = 0) to 0.7769 to gain full marks they were required to write down the figures between these two stages (1 – 0.2231).

Most candidates identified the Binomial with n = 6 and p = 0.777 in part (d) but few could progress any further. Many who did try either omitted $^6C_4$ or used $(0.777)^2$ $(1 - 0.777)^4$.

Examiner comment for Question 2 **(Mark scheme)** **(Return to Question 2)**

2. The definitions of a type I and type II error were usually clearly written with many candidates giving the exact definition given in the mark scheme.

In part (b) the main error was to select the critical region, $X \leq 1$ rather than $X \leq 2$ since P($X \leq 2$) is closer to 5% than P($X \leq 1$).

In part (c) most candidates were able to identify the correct probability required for a type II error following their CR in part (b) so gaining the method mark.

Examiner comment for Question 3 **(Mark scheme)** **(Return to Question 3)**

3. This question was accessible to the majority of candidates. Whilst many candidates knew "the two conditions needed to approximate the binomial distribution by the Poisson distribution this knowledge was by no means universal. Some candidates appear to have tried to memorise the conditions for:

- modelling a situation using a Binomial distribution
- modelling a situation using a Poisson distribution
- approximating the binomial distribution with the Poisson distribution
- approximating the binomial distribution with the Normal distribution

but failed to remember which set of conditions applies in which situation.

The response to part (b) was very good, with many candidates gaining full marks. The most common error made was to state the hypotheses as $H_0 : \lambda = 6$ and $H_1 : \lambda > 6$. The question clearly states that we are testing for a 'proportion', so that the null hypotheses should be $H_0 : p = 0.03$ and $H_1 : p > 0.03$. The majority of candidates used the correct Poisson distribution and successfully calculated the probability 0.0201.

It was common to see candidates using the incorrect statement P($X \geq 12$) = 1 – P($X \leq 12$) or calculating 1 – P($X \leq 10$) and writing CR: $X \geq 10$. The candidates who tried to find the critical region were more likely to make an error and it is recommended that the probability route is used.
It is pleasing to see that most candidates finished their answers well with a clear conclusion using the context written in the question.

Examiner comment for Question 4 (Mark scheme) (Return to Question 4)

4. Almost all the candidates used the geometric distribution in this question and the first two parts were answered well. There was some misinterpretation of "at least 3" in part (c) but most obtained the correct answer although not always from simply writing down $(1-p)^2$. The examiner required the answers in part (d) to be given in context. Thus a comment that simply stated that the trials (not the shots) were independent, or that the probability of success (rather than the probability of hitting the target) was constant did not score any marks.

Examiner comment for Question 5 (Mark scheme) (Return to Question 5)

5. This was a good question for most candidates as they recognised it as a standard example of this type of test. In part (a) most candidates now realise that some working has to be shown, and few failed to score here. Part (b) was also well done. The test in part (c) was also done very well, but there are still many candidates who are not aware of the fact that they should not state inappropriate values of the parameters in their hypotheses. Only a minority of candidates failed to group classes and a similarly small number failed to give the correct degrees of freedom.

Examiner comment for Question 6 (Mark scheme) (Return to Question 6)

6. A reasonable start for many meant full marks for the derivation of the given answer. The calculus in part (c) was often confused and incomplete and many variations on the answer to part (d) usually involved t/25 and so gained the first mark but not the second.

Examiner comment for Question 7 (Mark scheme) (Return to Question 7)

7. Most candidates used the negative binomial distribution here (only a small minority tried the geometric) and the formula was usually quoted correctly although the 3 was sometimes missed. There were few problems in manipulating the resulting equation into a quadratic in p and then solving to find $p = \frac{1}{5}$. Part (b) was answered well, but part (c) caused difficulties. The common approach was to use a negative binomial distribution with $r = 2$ and x taking values from 2 to 8 but unfortunately the correct 7 cases were not always used. Some candidates used a binomial distribution with $n = 8$ and $p = \frac{1}{5}$ and then simply found $P(X \geq 2)$ which yielded the answer quite simply.