Write your name here

Surname	Other names

Pearson Edexcel GCE

Centre Number

Candidate Number

A level Further Mathematics
Further Statistics 1
Practice Paper 3

You must have:
Mathematical Formulae and Statistical Tables (Pink)

Total Marks

Instructions

- Use black ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- Fill in the boxes at the top of this page with your name, centre number and candidate number.
- Answer all the questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – there may be more space than you need.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are **7** questions in this question paper. The total mark for this paper is **75**.
- The marks for each question are shown in brackets – use this as a guide as to how much time to spend on each question.
- Calculators must not be used for questions marked with a * sign.

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

1. A discrete random variable X has the probability function

$$P(X = x) = \begin{cases} k(1-x)^2 & x = -1, 0, 1 \text{ and } 2 \\ 0 & \text{otherwise.} \end{cases}$$

(a) Show that $k = \frac{1}{6}$.

(3)

(b) Find E(X).

(2)

(c) Show that $E(X^2) = \frac{4}{3}$.

(2)

(d) Find Var(1 – 3X).

(3)

(Total 10 marks)

Mark scheme for Question 1

Examiner comment

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2. A doctor takes a random sample of 100 patients and measures their intake of saturated fats in their food and the level of cholesterol in their blood. The results are summarised in the table below.

Intake of saturated fats \ Cholesterol level	High	Low
High	12	8
Low	26	54

Using a 5% level of significance, test whether or not there is an association between cholesterol level and intake of saturated fats. State your hypotheses and show your working clearly.

(Total 10 marks)

Mark scheme for Question 2

Examiner comment

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3. Each cell of a certain animal contains 11 000 genes. It is known that each gene has a probability 0.0 005 of being damaged.

A cell is chosen at random.

(a) Suggest a suitable model for the distribution of the number of damaged genes in the cell.

(2)

(b) Find the mean and variance of the number of damaged genes in the cell.

(2)

(c) Using a suitable approximation, find the probability that there are at most 2 damaged genes in the cell.

(3)*

(Total 7 marks)

Mark scheme for Question 3

Examiner comment

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*Part (c) would have been 4 marks in the old specification and 3 marks in the new specification.

4. A traffic officer monitors the rate at which vehicles pass a fixed point on a motorway. When the rate exceeds 36 vehicles per minute he must switch on some speed restrictions to improve traffic flow.

(a) Suggest a suitable model to describe the number of vehicles passing the fixed point in a 15 s interval.

(1)

The traffic officer records 12 vehicles passing the fixed point in a 15 s interval.

(b) Stating your hypotheses clearly, and using a 5% level of significance, test whether or not the traffic officer has sufficient evidence to switch on the speed restrictions.

(6)

(c) Using a 5% level of significance, determine the smallest number of vehicles the traffic officer must observe in a 10 s interval in order to have sufficient evidence to switch on the speed restrictions.

(3)

(Total 10 marks)

Mark scheme for Question 4

Examiner comment

5. The probability that John wins a coconut in a game at the fair is 0.15.
John plays a number of games.

(a) Find

(i) the probability of John winning his second coconut on his 7th game.

(2)

(ii) the expected number of games John would need to play in order to win 3 coconuts.

(1)

(b) State two assumptions that you made in part (a).

(2)

Sue plays the same game, but has a different probability of winning a coconut. She plays until she has won r coconuts. The random variable G represents the total number of games Sue plays.

(c) Given that the mean and the standard deviation of G are 18 and 6 respectively, determine whether John or Sue has the greater probability of winning a coconut in a game.

(5)

(Total 10 marks)

Mark scheme for Question 5

Examiner comment

6. A proportion p of letters sent by a company are incorrectly addressed and if p is thought to be greater than 0.05 then action is taken.

Using H_0: $p = 0.05$ and H_1: $p > 0.05$, a manager from the company takes a random sample of 40 letters and rejects H_0 if the number of incorrectly addressed letters is more than 3.

(a) Find the size of this test.

(2)

(b) Find the probability of a Type II error in the case where p is in fact 0.10.

(2)

Table 1 below gives some values, to 2 decimal places, of the power function of this test.

p	0.075	0.100	0.125	0.150	0.175	0.200	0.225
Power	0.35	s	0.75	0.87	0.94	0.97	0.99

Table 1

(c) Write down the value of s.

(1)

A visiting consultant uses an alternative system to test the same hypotheses. A sample of 15 letters is taken. If these are all correctly addressed then H_0 is accepted. If 2 or more are found to have been incorrectly addressed then H_0 is rejected. If only one is found to be incorrectly addressed then a further random sample of 15 is taken and H_0 is rejected if 2 or more are found to have been incorrectly addressed in this second sample, otherwise H_0 is accepted.

(d) Find the size of the test used by the consultant.

(3)

Figure 1 shows the graph of the power function of the test used by the consultant.

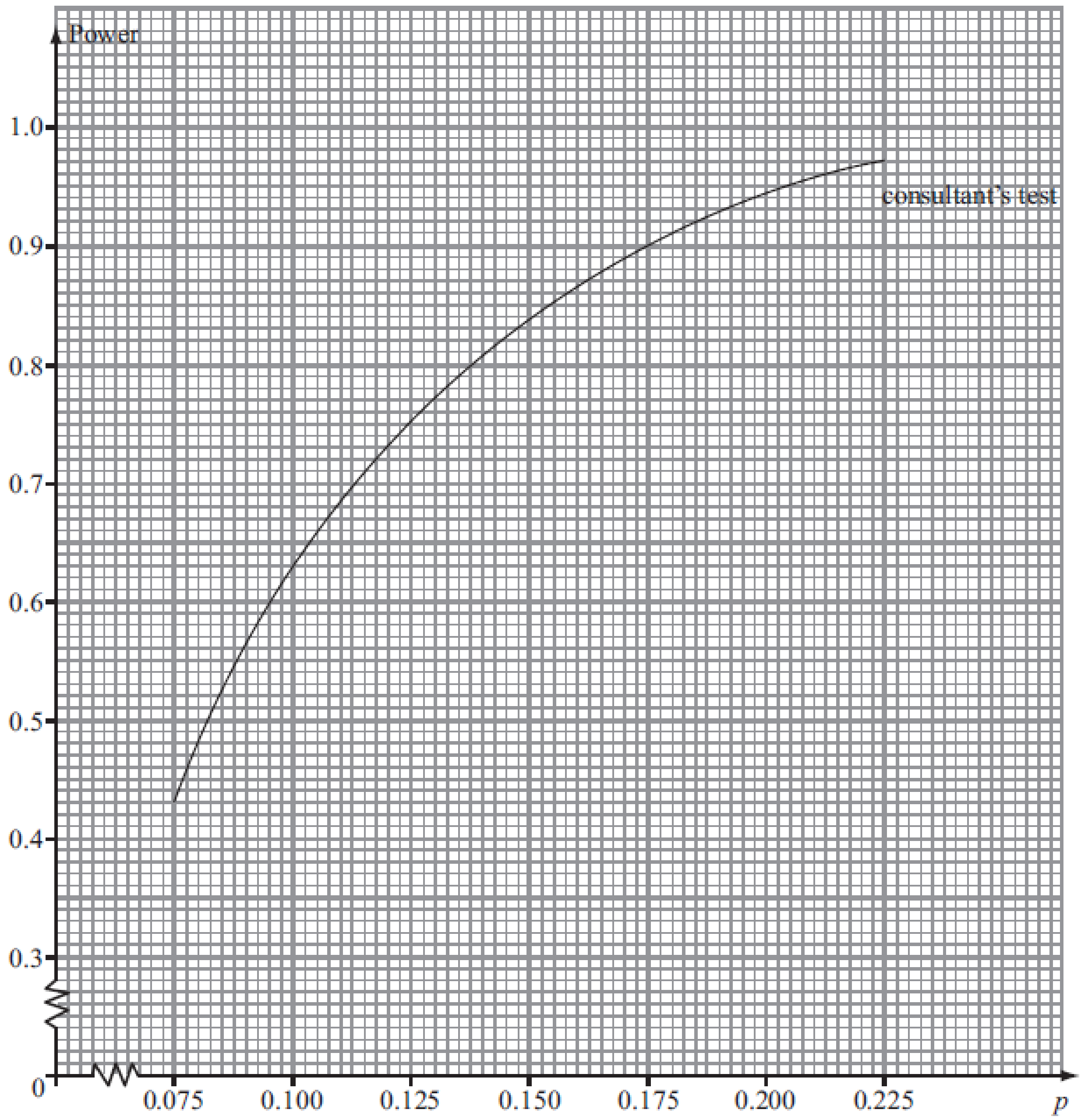

Figure 1

(e) On Figure 1 draw the graph of the power function of the manager's test.

(2)

(f) State, giving your reasons, which test you would recommend.

(2)

(Total 12 marks)

[Mark scheme for Question 6]

[Examiner comment]

7. A shop hires out carpet cleaners by the day. The number of requests X per day to hire a cleaner can be modelled as a Poisson distribution with mean 3.

(a) Find, in terms of e, the probability that on a particular day there will be

(i) exactly 2,

(ii) at least 4

requests to hire a cleaner.

(5)

The random variable Y represents the number of carpet cleaners hired on a particular day. The shop has 4 cleaners.

(b) Show that the probability generating function of Y, $G_Y(t)$ is given by

$$G_Y(t) = e^{-3}(1 + 3t + 4.5t^2 + 4.5t^3 - 13t^4) + t^4.$$

(3)

(c) Use the probability generating function to find the mean and the standard deviation of Y.

(8)

(Total 16 marks)

Mark scheme for Question 7

Examiner comment

TOTAL FOR PAPER: 75 MARKS

A level Further Mathematics – Further Statistics 1 – Practice Paper 03 – Mark scheme –

Mark scheme for Question 1 (Examiner comment) (Return to Question 1)

Question	Scheme	Marks
1(a)	$\begin{array}{\|c\|c\|c\|c\|c\|} \hline x & -1 & 0 & 1 & 2 \\ \hline \mathrm{P}(X=x) & 4k & k & 0 & k \\ \hline \end{array}$	M1
	$4k + k + (0) + k = 1$ (Allow verify approach)	A1
	$6k = 1 \quad \Rightarrow \quad k = \frac{1}{6}$ **(*)**	A1cso
		(3)
(b)	$[\mathrm{E}(X)] = -4k\ (+0+0) + 2k$ <u>or</u> $-2k$ <u>or</u> $-1\times\frac{4}{6}+2\times\frac{1}{6}$	M1
	$= -\frac{1}{3}$ (or $-0.\dot{3}$)	A1
		(2)
(c)	$\left[\mathrm{E}\left(X^2\right)\right] = (-1)^2 \times 4k + (0+0) + 2^2 k$ <u>or</u> $4k+4k$ <u>or</u> $(-1)^2\times\frac{4}{6}+2^2\times\frac{1}{6}$	M1
	$= \frac{4}{3}$ **(*)**	A1cso
		(2)
(d)	[Var(X)]= $\frac{4}{3}-\left(-\frac{1}{3}\right)^2$ <u>or</u> $8k-4k^2 = \left[\frac{11}{9}\right]$ $Y = 1-3X: 4 \quad 1 \quad -2 \quad -5$ Prob: $4k \quad k \quad 0 \quad k$ And $\mathrm{E}(Y) = 12k$	M1
	$\mathrm{Var}(1-3X) = (-3)^2\,\mathrm{Var}(X)$ <u>or</u> $9\mathrm{Var}(X)$ $\mathrm{E}(Y^2) = 90k$ and $\mathrm{Var}(Y) = 90k - 144k^2$	M1
	= 11	A1cao
		(3)
		(10 marks)

Mark scheme for Question 2 (Examiner comment) (Return to Question 2)

<table>
<tr><th>Question</th><th>Scheme</th><th>Marks</th></tr>
<tr><td>2</td><td>

Cholesterol Level	High	Low	
High	7.6	12.4	20
Low	30.4	49.6	80
	38	62	100

</td><td>M1A1</td></tr>
<tr><td></td><td>H_0 : Cholesterol level is independent of intake of saturated fats(no association)
H_1 : Cholesterol level is not independent of intake of saturated fats (association)</td><td>B1</td></tr>
<tr><td></td><td>

O	E	$\frac{(O-E)^2}{E}$	$\frac{O^2}{E}$
12	7.6	2.547… or $\frac{242}{95}$	18.947… or $\frac{360}{19}$
8	12.4	1.56129… or $\frac{242}{155}$	5.161… or $\frac{160}{31}$
26	30.4	0.6368… or $\frac{121}{190}$	22.236… or $\frac{845}{38}$
54	49.6	0.3903… or $\frac{121}{310}$	58.790… or $\frac{3645}{62}$

</td><td>dM1
A1</td></tr>
<tr><td></td><td>$\sum\frac{(O-E)^2}{E}=5.1358234..$ <u>or</u> $\frac{1.2^2}{7.6}+\frac{8^2}{12.4}+\frac{26^2}{30.4}+\frac{54^2}{49.6}-100=5.14$
(awrt 5.14)</td><td>A1</td></tr>
<tr><td></td><td>$v=(2-1)(2-1)=1$</td><td>B1</td></tr>
<tr><td></td><td>$\chi_1^2(0.05)=3.841$</td><td>B1</td></tr>
<tr><td></td><td>$5.14>3.841$ so sufficient evidence to reject H_0 [Condone “accept H_1 ”]</td><td>M1</td></tr>
<tr><td></td><td>Association between cholesterol level and saturated fat intake</td><td>A1</td></tr>
<tr><td></td><td></td><td>(10)</td></tr>
<tr><td colspan="3" align="right">(10 marks)</td></tr>
</table>

Mark scheme for Question 3 **(Examiner comment)** **(Return to Question 3)**

Question	Scheme	Marks
3(a)	$X \sim B(11000, 0.0005)$	**M1A1**
		(2)
(b)	$E(X) = 11000 \times 0.0005 = 5.5$	**B1**
	$\text{Var}(X) = 11000 \times 0.0005 \times (1 - 0.0005)$ $= 5.49725$	**B1**
		(2)
(c)	$X \sim \text{Po}(5.5)$	**M1A1**
	$P(X \leq 2) = 0.0884$	**dM1** **A1**
		(4)*
		(8 marks)
*Part (c) would have been 4 marks in the old specification and 3 marks in the new specification.		

Mark scheme for Question 4 **(Examiner comment)** **(Return to Question 4)**

Question	Scheme	Marks
4(a)	Poisson	**B1**
		(1)
(b)	$H_0 : \mu = 9$ (or $\lambda = 36$) $H_1 : \mu > 9$ (or $\lambda > 36$)	**B1B1**
	$X \sim \text{Po}(9)$ and $P(X \geq 12) = 1 - P(X \leq 11)$ or $P(X \leq 14) = 0.9585$ $P(X \geq 15) = 0.0415$	**M1**
	$= 1\text{-}0.8030 =$ <u>0.197</u> <u>CR $X \geq 15$</u>	**A1**
	$(0.197 > 0.05)$ so not significant/ accept H_0/ Not in CR	**M1d**
	he does not have evidence to switch on the <u>speed restrictions</u> (o.e)	**A1ft**
		(6)
(c)	Let Y = the number of vehicles in 10 s then $Y \sim \text{Po}(6)$	**B1**
	Tables: $P(Y \leq 10) = 0.9574$ so $P(Y \geq 11) = 0.0426$	**M1**
	so needs <u>11</u> vehicles	**A1**
		(3)
		(12 marks)

Mark scheme for Question 5 (Examiner comment) (Return to Question 5)

Question	Scheme	Marks
5(a)(i)	$\binom{6}{1}(0.15)(0.85)^5(0.15) = 0.059900\ldots$	M1A1
		(2)
(ii)	$\dfrac{3}{0.15} = 20$	B1
		(1)
(b)	Probability that John win a coconut in a game is constant.	B1
	Games are independent.	B1
		(2)
(c)	$\dfrac{r}{p} = 18;\ \dfrac{r(1-p)}{p^2} = 36$	B1B1
	$\therefore\ 18(1-p) = 36p$	M1
	$P = \dfrac{1}{3} > 0.15 \Rightarrow$ Sue	A1A1
		(5)
		(10 marks)

Mark scheme for Question 6 (Examiner comment) (Return to Question 6)

Question	Scheme	Marks
6(a)	[X = no. of incorrectly addressed letters. $X \sim B(40, 0.05)$]	**M1A1**
	$P(X > 3) = 1 - P(X \le 3), = 1 - 0.8619 = 0.1381$ **awrt <u>0.138</u>**	**(2)**
		M1
(b)	$P(\text{Type II Error}) = P(X \le 3 \mid p = 0.10)$	**A1**
	$= 0.4231$ **awrt <u>0.423</u>**	**(2)**
(c)	Power = 1 - P(Type II error) so $s =$ **<u>0.58</u>** (0.5769)	**B1**
		(1)
(d)	Y = no. of incorrectly addressed letters in a sample of 15. $Y \sim B(15, 0.05)$	
	Size = $P(Y \ge 2) + P(Y = 1) \times P(Y \ge 2)$	**M1**
	$= [1 - 0.8290] \times [1 + 0.8290 - 0.4633]$	**A1**
	$= 0.23353\ldots$ **awrt <u>0.23</u>**	**A1**
		(3)
(e)	(use overlay)	**B1B1**
		(2)
(f)	2nd/ consultants test is quicker (since it uses fewer letters)	
	2nd / consult test is more powerful for $p < 0.125$ (and values greater than this should be unlikely)	**B1B1**
		(2)
		(13 marks)

Mark scheme for Question 7 (Examiner comment) (Return to Question 7)

Question	Scheme	Marks
7(a)(i)	$P(X=2) = \frac{e^{-3} \times 3^2}{2!} = 4.5e^{-3}$	M1A1
(ii)	$P(X \geq 4) = 1 - P(X \leq 3), \quad = 1 - e^{-3}\left(1 + 3 + \frac{3^2}{2!} + \frac{3^3}{3!}\right)$	M1A1
	$= 1 - 13e^{-3}$	A1
		(5)
(b)	y: 0 1 2 3 4 x: 0 1 2 3 ≥ 4	
	$P(Y=y)$: e^{-3} $3e^{-3}$ $4.5e^{-3}$ $4.5e^{-3}$ $1 - 13e^{-3}$	B1
	$G_Y(t) = e^{-3}(t^0 + 3t + 4.5t^2 + 4.5t^3) + (1 - 13e^{-3})t^4$	M1
	$= e^{-3}(1 + 3t + 4.5t^2 + 4.5t^3 - 13t^4$ (*)	A1cso
		(3)
(c)	$G'_Y(t) = e^{-3}(3 + 9t + 13.5t^2 - 52t^3 + 4t^4$	M1A1
	$\mu = E(Y) = G'_Y(1) = 4 - 26.5e^{-3}$ or 2.68	A1
	$G''_Y(t) = e^{-3}(9 + 27t - 156t^2) + 12t^2$	M1A1
	$G''_Y(1) = e^{-3}(-120) + 12 = 12 - 120e^{-3}$	A1
	$\sigma^2 = G''_Y(1) + G'_Y(1) - [G'_Y(1)]^2$ (= 1.52...)	M1
	$\sigma = \sqrt{\sigma^2} = 1.23$	A1
		(8)
		(16 marks)

A level Further Mathematics – Further Statistics 1 – Practice Paper 03 – Examiner report –

Examiner comment for Question 1 (Mark scheme) (Return to Question 1)

1. This proved an accessible opening question to the paper. Part (a) was a "show that" and some candidates failed to show sufficient steps. There were two stages required: firstly the probabilities needed evaluating from the given probability function and most managed this successfully. Secondly there should be an *explicit* application of $\sum P(X = x) = 1$ and some candidates failed to show this step clearly.

Most knew how to find E(X) in part (b) but some simply added their probabilities and a small minority divided their answer by 4.

Part (c) was another "show that" but most knew what to include here and full marks were often awarded. The use of notation was poor with a large number writing (-1^2) when they actually meant $(-1)^2$. This was not penalised here but more attention to detail will be required as they progress to more advanced mathematical units.

For the final part most candidates now know the effect of coding on the variance and most realised that they needed first to find Var(X) and then multiply by $(-3)^2$ and many correct solutions were seen.

Examiner comment for Question 2 (Mark scheme) (Return to Question 2)

2. This proved to be a friendly opening to the paper with very few failing to show sufficient working and many scoring full marks. Some lost marks for the hypotheses either through laziness (simply stating "no association" for the null hypothesis is not sufficient as we want to see the variables under consideration being mentioned) or for stating them the wrong way around. The calculations were usually correct but some mistakes occurred in the degrees of freedom and the conclusion was not always given in context.

Examiner comment for Question 3 (Mark scheme) (Return to Question 3)

3. This question was generally answered well. A few candidates put the Poisson for (a) and then used Variance = Mean to got 5.5 for the variance. Some candidates rounded incorrectly giving an answer of 5.49 for the variance.

Part (c) was generally answered correctly although a minority of candidates used the normal approximation – most used 2.5 in their standardisation and so got 1 mark out of the 4.

Examiner comment for Question 4 (Mark scheme) (Return to Question 4)

4. There was a very variable response to question 2, with many candidates producing "textbook answers", whilst many others failing to recognise a Poisson distribution in part (a), offered either (or sometimes both) a binomial or a normal model.
The latter candidates either stopped at part (a) or pursued their chosen model to little effect.

In part (b) the vast majority successfully opted for a 1-tailed alternative hypothesis, although some did insist on using the parameter p. The value of $P(X \geq 12)$ or the CR was usually found correctly and most candidates were able to make a successful comparison, thereby leading to a well expressed contextual conclusion. Some candidates whose alternative hypothesis suggested a 2-tailed test, still opted to perform a 1-tailed test.

Examiner comment for Question 5 (Mark scheme) (Return to Question 5)

5. Part (a) was attempted well, but the context was missed in part (b). A common error in part (c) was to use 6 rather than 36 and so accuracy marks were lost as a result.

Examiner comment for Question 6 (Mark scheme) (Return to Question 6)

6. Parts (a), (b), (c) and (e) were answered well with many candidates gaining full marks. In part (d) few candidates realised that they were required to work out $P(Y \geq 2) + P(Y = 1) \times P(Y \geq 2)$. Those that did were generally able to reach the correct answer.

In part (d) several candidates were able to say when each test should be used but were unable to draw a correct conclusion for this situation. Only a few referred to the likelihood of the probability being over 0.125 was small so the consultants test should be used in this case.

Examiner comment for Question 7 (Mark scheme) (Return to Question 7)

7. Part (a) was answered well but a small minority of candidates failed to read the instruction to give their answers in terms of e and simply used the tables. Occasionally the answer to part (ii) was not simplified. Convincing demonstrations were rare in part (b). The difficulty hinged on identifying a correct probability distribution for Y. Those that did give a suitable sample space were usually able to use the results from part (a) and the Poisson distribution to complete this part. The final part was usually answered very well. The differentiation caused few problems and apart from occasional arithmetic errors most candidates were able to find the mean and variance of Y but some failed to take a square root to find the standard deviation as required.