Question	Scheme	Marks	AOs
1 (a)	H_0; recycling plastics and the distance to the nearest recycling point are independent H_1; recycling plastics and the distance to the nearest recycling point are not independent	B1	1.2
	Degrees of freedom, $\nu = (3-1)\times(2-1) = 2$ Therefore critical value = $\chi^2(0.05) = 5.991$	B1	3.1b
	Test statistic $= \sum \frac{(O_i - E_i)^2}{E_i} = \frac{(64-58.3)^2}{58.3} + \frac{(42-47.7)^2}{47.7} + \frac{(32-29.7)^2}{29.7} + \cdots =$	M1	1.1b
	$8.0988\ldots\ldots =$ awrt 8.10	A1	1.1b
	In critical region, therefore sufficient evidence to reject H_0. Data does **not** support Barbara's belief at the 5% significance level that people recycling plastics is independent of the distance to the nearest recycling point.	A1	3.2a
			(5)
(b)	• Test statistic is halved and • Critical value stays the same	B1	2.4
	This is not in the critical region and therefore Barbara's original conclusions are not valid, data suggests independence.	dB1	2.3
			(2)
			(7)

Notes
a) 1st B1 accept 'no association between …' & 'association between…' in H_0 and H_1 oe but must mention context 2nd B1 must have (ν = 2) awrt 5.99 for critical value (implies correct degrees of freedom) M1 need to see at least 2 terms if A1 not awarded 1st A1 awrt 8.10 (accept p = awrt 0.0174) 2nd A1 cso must have correct comment in context.
b) 1st B1 valid reason why test statistic is halved 2nd B1 dependent on 1st B1 cao

Question	Scheme	Marks	AOs
2 (a)	$\mathbf{P}(X=4)=\frac{7}{10}\times\frac{6}{9}\times\frac{5}{8}$	M1	2.1
	$=\frac{210}{720}\quad=\frac{7}{24}\quad *$	A1*	1.1b
			(2)
(b)	$\mathbf{E}(X)=1\times\frac{3}{10}+2\times\frac{7}{30}+3\times\frac{7}{40}+4\times\frac{7}{24}=$ $\frac{59}{24}=2.458\dot{3}$ $\mathbf{E}(X^2)=1^2\times\frac{3}{10}+2^2\times\frac{7}{30}+3^2\times\frac{7}{40}+4^2\times\frac{7}{24}=$ $\frac{299}{40}=7.475$	M1 A1	3.1b 1.1b
	$\mathrm{Var}(X)=\frac{299}{40}-\left(\frac{59}{24}\right)^2=1.43159\ldots$ awrt 1.43	M1 A1	1.1b 1.1b
			(4)
(c)	Let rv $T=$ number of tokens drawn $T\sim\mathrm{Geo}(0.3)$	B1	3.3
	$Y=T-1$	B1	3.1b
	$\mathrm{Var}(Y)=\mathrm{Var}(T)$	M1	3.4
	$\mathrm{Var}(Y)=\frac{1-p}{p^2}=\frac{0.7}{0.3^2}=\frac{70}{9}=7.\dot{7}$	A1	1.1b
			(4)
			(10)

Notes
a) M1 need $\frac{7}{10}\times\frac{6}{9}\times\ldots$ either using P(GGG), or P(GGGR or GGGG) A1 * full correct solution
b) 1st M1 correct method for $\mathbf{E}(X)$ or $\mathbf{E}(X^2)$ 1st A1 both $\mathbf{E}(X)$ and $\mathbf{E}(X^2)$ correct 2nd M1 full method for $\mathrm{Var}(X)$ involving attempt at $\mathbf{E}(X)$ and $\mathbf{E}(X^2)$ or $\mathbf{E}((X-\mu)^2)$ 2nd A1 awrt 1.43 Correct answer with no working scores 1/4
c) 1st B1 use of Geometric distribution for number of tokens drawn 2nd B1 Establishing link between *Y* and *T* 1st M1 same variance for *Y* and *T* A1 awrt 7.78 Correct answer with no working scores 2/4

Question	Scheme	Marks	AOs
3 (a)(i)	$\mathbf{P}(X = \mathbf{9}) = 0.11237\ldots$ awrt 0.112	B1	1.1b
(ii)	$\mathbf{P}(X \geq \mathbf{9}) = 1 - \mathbf{P}(X \leq \mathbf{8})$	M1	1.1b
	$= 1 - 0.250228\ldots = 0.749771\ldots$ awrt 0.750	A1	1.1b
			(3)
(b)	Select $X \sim \mathrm{Po}(\mathbf{6})$ and H_0: $\lambda = \mathbf{6}$ and H_1: $\lambda > \mathbf{6}$	M1	3.3
	$[\mathbf{P}(X \leq \mathbf{10}) = 0.957379\ldots]$ $\therefore$ $\mathbf{P}(X \geq \mathbf{11}) = 0.04262\ldots$	M1	3.4
	$X \geq \mathbf{11}$ is critical region	A1	1.1b
	Power of test = $\mathbf{P}(Y \geq 11 \mid \lambda = \mathbf{10}) = 1 - \mathbf{P}(Y \leq 10 \mid \lambda = \mathbf{10}) =$	dM1 A1ft	3.1b 1.1b
	$1 - 0.583039\ldots = 0.41696\ldots$ awrt 0.417	A1	1.1b
			(6)
(c)	Test is only valid if the teacher can assume that Charlie looks at his mobile phone • independently each time, or • at a constant rate, or • randomly (any one of these)	B1	3.5b
			(1)
			(10)

Notes
a) (i) B1 calculator value awrt 0.112 (ii) M1 $1 - \mathbf{P}(X \leq \mathbf{8})$ oe seen or implied A1 awrt 0.750 (condone 0.75)
b) 1st M1 $X \sim \mathrm{Po}(\mathbf{6})$ seen and correct hypotheses (allow μ for λ or $\lambda = \mathbf{3}$ and $\lambda > \mathbf{3}$) 2nd M1 attempt to find critical region using Po(6), upper tail only A1 $X \geq \mathbf{11}$ as critical region seen or implied 3rd M1 complete strategy to find power of test, dependent on 2nd M mark 2nd A1ft on their critical region (must be using Po(10)) 3rd A1 awrt 0.417
c) B1 cao must mention context

<table>
<tr><th>Question</th><th>Scheme</th><th>Marks</th><th>AOs</th></tr>
<tr><td>4 (a)</td><td>$\mathbf{H_0}$: Geo(1/3) is a suitable model for the number of rolls to 1st 5 or 6
$\mathbf{H_1}$: Geo(1/3) is not a suitable model for the number of rolls to 1st 5 or 6</td><td>B1</td><td>2.5</td></tr>
<tr><td></td><td>Assuming $\mathbf{H_0}$ is true , expected values are $\mathbf{E}_i = 100 \times \frac{1}{3} \times \left(\frac{2}{3}\right)^{i-1}$</td><td>M1</td><td>3.4</td></tr>
<tr><td></td><td><table><tr><td>Number of rolls to 1st 5 or 6</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7 or more</td></tr><tr><td>Expected frequency</td><td>33.3…</td><td>22.2…</td><td>14.8…</td><td>9.87…</td><td>6.58…</td><td>4.38…</td><td>8.77….</td></tr></table></td><td>A1
A1</td><td>1.1b
1.1b</td></tr>
<tr><td></td><td><table><tr><td>Number of rolls to 1st 5 or 6</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6 or more</td></tr><tr><td>Observed frequency</td><td>39</td><td>17</td><td>17</td><td>10</td><td>3</td><td>14</td></tr><tr><td>Expected frequency</td><td>33.3…</td><td>22.2…</td><td>14.8…</td><td>9.87…</td><td>6.58…</td><td>13.1…</td></tr></table>Combining last two groups</td><td>M1</td><td>2.1</td></tr>
<tr><td></td><td>$\nu = 6 - 1 = \mathbf{5}$</td><td>B1ft</td><td>1.1b</td></tr>
<tr><td></td><td>Critical value, $\chi^2(0.05) = 11.070$</td><td>B1ft</td><td>1.1a</td></tr>
<tr><td></td><td>Test statistic = $\frac{(39-33.3)^2}{33.3} + \frac{(17-22.2)^2}{22.2} + \frac{(17-14.8)^2}{14.8} + \cdots$</td><td>M1</td><td>1.1b</td></tr>
<tr><td></td><td>= 4.5181… awrt 4.5</td><td>A1</td><td>1.1b</td></tr>
<tr><td></td><td>Not in critical region, insufficient evidence to reject $\mathbf{H_0}$, no significant evidence at the 5% level that the die is biased.</td><td>A1cso</td><td>3.5a</td></tr>
<tr><td></td><td></td><td></td><td>(10)</td></tr>
<tr><td>(b)(i)</td><td>e.g. Mai’s test measures results against the frequencies of all six possible scores, whereas Desmond’s doesn’t.</td><td>B1</td><td>3.5b</td></tr>
<tr><td></td><td>Mai’s test needs a fixed number of rolls, whereas number of rolls for Desmond’s test is unknown.</td><td>B1</td><td>3.5b</td></tr>
<tr><td>(ii)</td><td>Desmond’s test is likely to collect more data.</td><td>B1</td><td>3.5b</td></tr>
<tr><td></td><td></td><td></td><td>(3)</td></tr>
<tr><td></td><td></td><td></td><td>(13)</td></tr>
</table>

Notes
a) 1st B1 H_0 / H_1 must include correct value of parameter. 1st M1 use of geometric distribution (may be implied) 1st A1 at least 3 values correct to 1dp 2nd A1 all correct to 1 dp (allow last 2 groups already combined) 2nd M1 combining last two groups only 2nd B1 ft if not combined groups 3rd B1 ft their v 3rd M1 at least 2 terms shown (give if test value correct and no working shown)3rd A1 awrt 4.5 4th A1 cso
b) (i) B1 valid comment on limitation of Desmond's test compared to Mai's B1 valid comment on number of rolls of die expected compared to Mai's (ii) B1 valid comment on amount of data collected.

Question	Scheme	Marks	AOs
5 (a)(i)	$X \sim \mathbf{B}(1000, 0.005)$	B1cao	3.3
(ii)	$\mathbf{P}(X \geq 10) = 1 - \mathbf{P}(X \leq 9) = 1 - 0.96853476\ldots$	M1	2.1
	= 0.03146523…		
	Approximation $Y \sim \mathbf{Po}(5)$ $\mathbf{P}(Y \geq 10) = 1 - \mathbf{P}(Y \leq 9) = 1 - 0.9681719435\ldots$	M1	3.3
	=0.0318280564…	A1	1.1b
	Percentage error $= \frac{0.031828056 - 0.03146523}{0.03146523} \times 100$ (denominator must be their '0.031465…')	M1	1.1b
	=1.1530… awrt 1.15%	A1	1.1b
			(6)
(b)	Size of Ushma's test = $1 - \mathbf{P}(X \leq 8) = 1 - 0.932397\ldots$	M1	2.1
	$= 0.067602\ldots$	A1	1.1b
	[Size of test = $\mathbf{P}(\text{Type I error})$] Size of Ushma's test < Size of Javed's test (0.08)	B1	2.4
	So implies Ushma's test is better	dB1ft	2.2b
			(4)
			(10)

Notes
a)(i) B1 cao must include correct parameters (ii)1st M1 attempting $\mathbf{P}(X \geq 10)$ using a Binomial distribution 2nd M1 attempting $\mathbf{P}(Y \geq 10)$ using Po(5) 1st A1 both correct to 3 sfs (awrt 0.0315 and 0.0318 respectively) 3rd M1 correct method for percentage error using their probabilities from Bin and Poisson distributions 2nd A1 accept awrt 1.14 – 1.16% with working seen (do not accept 0.0115)
b) M1 attempt at $\mathbf{P}(X \geq 9)$ using $X \sim \mathbf{B}(1000, 0.005)$ or $X \sim \mathbf{Po}(5)$ A1 awrt 0.0676 (or 0.0681 if using $X \sim$ "$\mathbf{Po}$" (5)) 1st B1 valid argument implying smaller size of test is preferable 2nd B1 dependent on 1st B1 ft their size of Ushma's test.

Question	Scheme	Marks	AOs
6 (a)(i)	Use of negative binomial with $r = 3$ and $p = 0.2$	B1	3.3
	Use of $\mu = \frac{r}{p} = \frac{3}{0.2}$	M1	3.4
	= 15	A1	1.1b
			(3)
(ii)	Success of each throw is independent of all previous throws (oe)	B1	2.4
	Probability of hitting the bullseye on each throw remains the same (oe)	B1	2.4
			(2)
(b)	Variance of number of throws required, $\sigma^2 = \frac{r(1-p)}{p^2} = \frac{3 \times 0.8}{0.2^2} = 60$	B1	1.1b
	By CLT need to use $\bar{X} \sim N\left(15, \frac{60}{100}\right)$	M1 A1	3.1b 1.1b
	$P(\bar{X} > 16)$	dM1	3.4
	= 0.09835…. awrt 0.0984	A1	1.1b
			(5)
			(10)

Notes
a)(i) B1 needs correct distribution and correct parameters (ii) 2xB1 must be valid comments in context.
b) 1st M1 use of Normal distribution with their μ and σ^2 1st A1 must be correct values 2nd M1 dependent on use of $\bar{X} \sim N\left(15, \frac{60}{100}\right)$ 2nd A1 awrt 0.0984

Question	Scheme	Marks	AOs
7 (a)	$G_X'(t) = \lambda e^{\lambda(t-1)}$	M1 A1	2.1 1.1b
	$G_X'(1) = \lambda e^{0} = \lambda$ *	A1*cso	1.1b
			(3)
(b)	Need $G_Y(1) = 1$, so $\left(\frac{1}{a} + b\right)^n = 1$	M1	3.1a
	$\therefore \frac{1}{a} + b = 1$ so $b = 1 - \frac{1}{a}$ *	A1*cso	1.1b
			(2)
(c)	$b > 0$, $\therefore 1 - \frac{1}{a} > 0$	M1	3.1a
	$a > 1$	A1	1.1b
			(2)
(d)	$\left[G_Y(t) = \left(\frac{t^2}{a} + b\right)^4\right] \therefore G_Y'(t) = \frac{8t}{a}\left(\frac{t^2}{a} + b\right)^3$	M1	2.1
	$[E(Y) =] G_Y'(1) = \frac{8}{a}\left(\frac{1}{a} + b\right)^3 \quad \left[= \frac{8}{a}\right]$	M1	1.1b
	$G_Y''(t) = \frac{48t^2}{a^2}\left(\frac{t^2}{a} + b\right)^2 + \frac{8}{a}\left(\frac{t^2}{a} + b\right)^3$	M1	2.1
	$E(Y^2) - E(Y) = G_Y''(1) = \frac{48}{a^2}\left(\frac{1}{a} + b\right)^2 + \frac{8}{a}\left(\frac{1}{a} + b\right)^3$ $\left[= \frac{48}{a^2} + \frac{8}{a}\right]$	M1 A1	1.1b 1.1b
	$\text{Var}(Y) = G_Y''(1) + G_Y'(1) - \left(G_Y'(1)\right)^2$ $= \frac{48}{a^2}\left(\frac{1}{a} + b\right)^2 + \frac{8}{a}\left(\frac{1}{a} + b\right)^3 + \frac{8}{a}\left(\frac{1}{a} + b\right)^3 - \left(\frac{8}{a}\left(\frac{1}{a} + b\right)^3\right)^2$	M1	1.2
	Use of $\frac{1}{a} + b = 1$ $\text{Var}(Y) = \frac{48}{a^2} + \frac{8}{a} + \frac{8}{a} - \left(\frac{8}{a}\right)^2$ oe	M1	2.1
	$\text{Var}(Y) = \frac{16}{a} - \frac{16}{a^2} = \frac{16}{a^2}(a-1)$ *	A1*cso	1.1b
			(8)
			(15)

Notes
a) 1st M1 attempt to $G_X'(t)$ 2nd A1 * cso must see convincing use of t=1
b) 1st M1 Use of $G_Y(1) = 1$ A1 * cso need $\frac{1}{a} + b = 1$ oe
c) M1 use of $b > 0$
d) 1st M1 attempt to find $G_Y'(t)$ 2nd M1 use of $E(Y) = G_Y'(1)$ 3rd M1 attempt to find $G_Y''(t)$ 4th M1 use of $E(Y^2)-E(Y) = G_Y''(1)$ 1st A1 $G_Y'(1)$ and $G_Y''(1)$ both correct 5th M1 use of $\text{Var}(Y) = G_Y''(1) + G_Y'(1) - (G_Y'(1))^2$, may be seen earlier 6th M1 Use of $\frac{1}{a} + b = 1$ or substitution for b , may be seen earlier e.g. $E(Y) = \frac{8}{a}$ 2nd A1 * cso all correct.