

Mock Paper Set 1

**Answer ALL questions. Write your answers in the spaces provided.**

**1.** In a survey, a large number of people were asked whether or not they recycle plastics. In

each case the distance, in metres, from their home to the nearest plastic recycling point

was measured and recorded. The results are shown in Table 1

|  |  |
| --- | --- |
| **Observed frequencies** | **Recycle plastics** |
| **Yes** | **No** |
| **Distance to nearest****plastic recycling point** | **Less than 500 m** | 64 | 42 |
| **500–1000 m** | 32 | 22 |
| **More than 1000 m** | 14 | 26 |

**Table 1**

The Council’s Environment Officer, Barbara, believes that whether or not people recycle

plastics is independent of the distance to the nearest recycling point.

Barbara decides to test if the data from the survey supports her belief. Her expected

frequencies are shown in Table 2

|  |  |
| --- | --- |
| **Expected frequencies** | **Recycle plastics** |
| **Yes** | **No** |
| **Distance to nearest****plastic recycling point** | **Less than 500 m** | 58.3 | 47.7 |
| **500–1000 m** | 29.7 | 24.3 |
| **More than 1000 m** | 22 | 18 |

**Table 2**

(*a*) Carry out a hypothesis test, at the 5% significance level, to see if the data from the

survey support Barbara’s belief. State your hypotheses and the critical value used in

this test.

**(5)**

Barbara explains her results to her assistant, Bill. The data were collected by Bill, who

now confesses to Barbara that he actually only collected data from 100 people and then

doubled all his results.

(*b*) With reference to the test statistic and critical value, explain whether or not Barbara’s

conclusions are still valid.

**(2)**

**(Total for Question 1 is 7 marks)**

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**2.** A bag contains 10 tokens, 3 are red and 7 are green. Tokens are removed, one at a time,

without replacement until a red token is removed or until 4 tokens have been removed,

whichever occurs first.

The discrete random variable *X* represents the number of tokens removed.

(*a*) Show that 

**(2)**

The probability distribution of *X* is given in the table below

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| x | 1 | 2 | 3 | 4 |
| P (*X = x*) |  |  |  |  |

(*b*) Showing your working clearly, find Var(*X* )

**(4)**

The same bag of 10 tokens is now used again. This time a token is removed, the colour is

noted and the token is replaced. This procedure is repeated until a red token is removed.

The discrete random variable *Y* represents the number of green tokens removed.

(*c*) Showing your working clearly, find Var(*Y* )

**(4)**

**(Total for Question 2 is 10 marks)**

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**3.** The discrete random variable *X* has distribution *X* ~ Po(10.8)

(*a*) (i) Find the probability that *X* = 9

(ii) Find the probability that *X* is at least 9

**(3)**

Charlie has a mobile phone. His parents claim that on average he looks at his mobile phone

no more than 3 times every 30 minutes. Charlie’s teacher suspects that this may not be true

and that Charlie looks at his mobile phone more frequently than 3 times every 30 minutes.

She decides to carry out a hypothesis test of her belief, at a 5% significance level.

She observes Charlie during a 1-hour revision class and notes the number of times that he

looks at his mobile phone.

If Charlie actually looks at his mobile phone at an average rate of 5 times every 30 minutes,

(*b*) find the power of the teacher’s hypothesis test, stating her hypotheses.

**(6)**

(*c*) State an assumption the teacher needs to make in order for this test to be valid.

**(1)**

**(Total for Question 3 is 10 marks)**

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**4.** Desmond believes that a particular six‑sided die is biased. He decides to count the

number of rolls required until he first rolls a 5 or a 6. He carries out this experiment

100 times. His results are summarised in the table below

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| Number of rolls to 1st 5 or 6 | 1 | 2 | 3 | 4 | 5 | 6 | 7 or more |
| Observed frequency | 39 | 17 | 17 | 10 | 3 | 3 | 11 |

(*a*) Stating your hypotheses clearly, test, at the 5% significance level, whether or not these

results provide evidence that the die is biased. You should show your working clearly,

including the expected frequencies and the critical value used.

**(10)**

Mai suggests recording the results of rolling the die 100 times and using a goodness of fit

test for a discrete uniform distribution.

(*b*) (i) Give 2 reasons why Mai might think that her test is better than Desmond’s test.

(ii) Give a reason why Desmond might think that his test is better than Mai’s test.

**(3)**

**(Total for Question 4 is 13 marks)**

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**5.** A laboratory checks soil samples for evidence of lead contamination. Past records

suggest that 0.5% of samples test positive for lead contamination.

As part of a new contract the laboratory receives batches of 1000 independent soil samples.

The random variable *X* represents the number of soil samples that test positive for lead

contamination out of a batch of 1000 independent soil samples.

Ushma wants to calculate the probability that at least 10 soil samples will test positive for

lead contamination out of a batch of 1000 independent soil samples.

(*a*) (i) State the distribution of *X*.

(ii) Calculate the percentage error if Ushma uses a Poisson distribution to estimate

this probability.

**(6)**

Ushma decides to use *X* ≥ 9 as the critical region to test the hypotheses

H0 : *p* = 0.005 against H1 : *p* > 0.005

where *p* is the probability that a randomly selected soil sample tests positive for lead

contamination.

Ushma’s colleague Javed suggests an alternative test of size 8%. Their manager Chris

wants to minimise the probability of a Type I error. Given no further information about

Javed’s test,

(*b*) explain which of these tests you would advise Chris to use. Give a reason for your

answer.

**(4)**

**(Total for Question 5 is 10 marks)**

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**6.** Malik is a circus clown. As a part of his act he throws darts at a modified dartboard whilst

blindfolded. He knows that there is a probability of 0.8 that each dart misses the bullseye.

In rehearsals, Malik throws darts at the dartboard whilst blindfolded until his assistant

tells him he has hit the bullseye 3 times.

(*a*) (i) Calculate how many darts Malik should expect to throw in order to hit the bullseye

3 times.

**(3)**

(ii) State two assumptions that need to be made about Malik’s throws of the darts in

order for this calculation to be valid.

**(2)**

Malik rehearses once a day for each of 100 days.

Given that the assumptions in part (a) are true,

(*b*) calculate an estimate of the probability that the mean number of throws required is at

least 16

**(5)**

**(Total for Question 6 is 10 marks)**

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**7.** A discrete random variable *X* has probability generating function G*X*(*t*) = e*λ*(*t* – 1) where *λ* is

a positive constant.

(*a*) Use calculus to show that E(*X* ) = *λ*

**(3)**

A discrete random variable *Y* has probability generating function where *a* and *b* are both positive and *n* is a positive integer.

(*b*) Show that



**(2)**

(*c*) Hence find the range of possible values of *a*

**(2)**

(*d*) In the case where *n* = 4, use calculus to show that 

**(8)**

**(Total for Question 7 is 15 marks)**

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**TOTAL FOR PAPER IS 75 MARKs**