Pearson Edexcel

Mark Scheme

Mock Paper Set 2

Pearson Edexcel GCE Further Mathematics

Further Statistics 1 Paper 9FM0_3B

Mock paper
Publications Code 9FM0_3B_Mock paper_MS

General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\sqrt{}$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- ∗ The answer is printed on the paper
- ⎡ The second mark is dependent on gaining the first mark

4. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

5. Where a candidate has made multiple responses and indicates which response they wish to submit, examiners should mark this response.
   If there are several attempts at a question which have not been crossed out, examiners should mark the final answer which is the answer that is the most complete.

6. Ignore wrong working or incorrect statements following a correct answer.

7. Mark schemes will firstly show the solution judged to be the most common response expected from candidates. Where appropriate, alternatives answers are provided in the notes. If examiners are not sure if an answer is acceptable, they will check the mark scheme to see if an alternative answer is given for the method used.

General Principles for Core Mathematics Marking

(*But note that specific mark schemes may sometimes override these general principles*)

<u>Method mark for solving 3 term quadratic:</u>

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q), \text{ where } |pq| = |c|$, leading to $x = \ldots$

$(ax^2 + bx + c) = (mx + p)(nx + q), \text{ where } |pq| = |c| \text{ and } |mn| = |a|$, leading to $x = \ldots$

2. Formula

Attempt to use the correct formula (with values for *a*, *b* and *c*)

3. Completing the square

Solving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0, \quad q \neq 0$, leading to $x = \ldots$

<u>Method marks for differentiation and integration:</u>

1. Differentiation

Power of at least one term decreased by 1. $(x^n \to x^{n-1})$

2. Integration

Power of at least one term increased by 1. $(x^n \to x^{n+1})$

<u>Use of a formula</u>

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

<u>Method mark</u> for quoting a correct formula and attempting to use it, even if there are small errors in the substitution of values.

Where the formula is <u>not</u> quoted, the method mark can be gained by implication from <u>correct</u> working with values but may be lost if there is any mistake in the working.

<u>Exact answers</u>

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

1)			
	Using $X \sim \mathrm{B}(120, 0.1)$ $\mathrm{P}(X < 10) = \mathrm{P}(X \leq 9) = 0.2285916 \ldots$ Approximation $X \approx \sim \mathrm{Po}(12)$ Using Poisson approximation $\mathrm{P}(X < 10) = \mathrm{P}(X \leq 9) = 0.2423921 \ldots$ Error $= 0.2423921 \ldots - 0.2285916 \ldots = 0.0138005 \ldots$ = 0.0138*	 B1 B1 M1 A1*	 1.1b 3.1a 1.1b 1.1b
			(4)
			(4)

Notes

1st B1 from calculator awrt 0.23

2nd B1 Po(12) seen or implied

M1 Use of their Poisson approximation

A1* cso

2)			
(a)	$(\mathrm{G}_X(1) =) \frac{1^3}{k}((1+1)^2 + 2\times 1^4) = \frac{6}{k} = 1$ $k = 6$	M1 A1cao	1.1b 1.1b
			(2)
(b)	$\mathrm{G}_X(t) = \frac{t^3}{"6"}(1 + 2t + t^2 + 2t^4) =$ $\frac{t^3}{6} + \frac{t^4}{3} + \frac{t^5}{6} + \frac{t^7}{3}$ Probability distribution x: (3), 4, 5, (7) P(X = x): $(\frac{1}{6})$, $\frac{1}{3}$, $\frac{1}{6}$, $(\frac{1}{3})$ $\mathrm{P}(4 \le X \le 6.5) = \frac{1}{3} + \frac{1}{6} = \frac{1}{2}$	M1 M1 A1 M1 A1cao	3.1a 1.1b 1.1b 3.4 1.1b
			(5)
(c)	$\mathrm{G}_{X_1+X_2}(t) = \mathrm{G}_{X_1}(t) \times \mathrm{G}_{X_2}(t) = \frac{t^6}{36}((1+t)^2 + 2t^4)^2$ oe	B1ft	1.2
			(1)
			(8)

Probability distribution table in (b):

x	(3)	4	5	(7)
P(X = x)	$(\frac{1}{6})$	$\frac{1}{3}$	$\frac{1}{6}$	$(\frac{1}{3})$

Notes

a) M1 Use of $\mathrm{G}_X(1) = 1$

A1 cao

b) 1st M1 expanding /simplifying terms

2nd M1 finding coefficients of each power of t

1st A1 sample space identified and correct coefficients as probabilities of at least $X = 4$ and $X = 5$

3rd M1 use of probability distribution to find required probability

2nd A1 cao

c) B1 squaring PGF ft their value of k.

3)			
(a)	(Gill, $Y \sim \mathrm{Po}(2.1)$) Jack, $X \sim \mathrm{Bin}(3, 0.6)$ $\mathrm{P}(X = 3) = 0.216$ and $\mathrm{P}(Y = 3) = 0.189011 \ldots$ $\mathrm{P}(X = 3 \cap Y = 3) = 0.216 \times 0.189 \ldots = 0.04082 \ldots$ awrt 0.0408	B1 M1 A1	3.3 3.4 1.1b
			(3)
(b)	$\mathrm{P}(X > Y) =$ $\mathrm{P}(X = 1 \cap Y = 0) + \mathrm{P}(X = 2 \cap Y \leq 1) + \mathrm{P}(X = 3 \cap Y \leq 2) =$ $(0.288 \times 0.12245 \ldots) + (0.432 \times 0.3796 \ldots) + (0.216 \times 0.6496 \ldots)$ $= 0.33958 \ldots$ awrt 0.339/0.340	M1 A1 A1 A1	2.1 1.1b 1.1b 1.1b
			(4)
(c)	E.g. • Constant rate of successful journeys unreasonable as Gill may get tired • Gill cannot complete 2 journeys very close together • Spilling water may make slope slippery and so subsequent journeys more difficult.	B1	3.5b
			(1)
			(8)

Notes

(a) B1 distribution of *X,* Bin(3, 0.6) must include both parameters

M1 use of correct distributions to calculate both probabilities, may be implied by correct answer.

A1 awrt 0.0408

(b) M1 complete method all cases

1st A1 at least one case showing correct calculations

2nd A1 all calculations correct

3rd A1 awrt 0.339 or 0.340

SC correct answer with no working shown scores M1A1A1A0

(c) B1 for any reasonable point made **in context.**

4)			
(a)	E.g. large supply of objects or interesting objects evenly distributed across the field	B1	3.5b
			(1)
(b)	Use of $X \sim \text{Geo}(0.1)$ = number of finds in 1 day	M1	3.3
	$\text{P}(X = 7) = 0.1 \times 0.9^6$	M1	3.4
	=0.0531441 awrt 0.0531	A1	1.1b
			(3)
(c)	$\text{P}(X \geq 8) =$	M1	3.4
	$\frac{\text{P}(X = 8)}{1 - 0.9} = \frac{0.1 \times 0.9^7}{0.1} =$	M1	1.1b
	0.4782969 awrt 0.478	A1	1.1b
			(3)
(d)	$\text{P}(X < 7) = 1 -$ '0.0531…' – '0.478…' = (0.468559)	M1	3.1b
	$Y \sim \text{Bin}(5, '0.468559')$	dM1	3.4
	$\text{P}(Y = 3) = 0.290538 \dots$ awrt 0.291	A1	1.1b
			(3)
(e)	$\text{E}(X) = \frac{1}{0.1} = 10, \text{Var}(X) = \frac{1-0.1}{0.1^2} = 90$	M1 A1	3.3 1.1b
	$\bar{X} \approx \sim \text{N}\left('10', \frac{'90'}{80}\right)$	M1	2.1
	$\text{P}(\bar{X} < 9) = 0.172889 \dots$ awrt 0.173	A1	1.1b
			(4)
			(14)

Notes

(a) B1 any appropriate comment in context

(b) 1st M1 Use of $\text{Geo}(0.1)$, may be implied by next line or correct answer

2nd M1 attempt to use correct probability distribution

A1 awrt 0.0531

(c) 1st M1 correct method, may use $1 - \text{P}(X \leq 7)$

2nd M1 use of sum of infinite GP or $1 -$ sum of first 7 terms

A1 awrt 0.478

Alternate method

1st M1 $\text{P}(X \geq 8) = \text{P}$(no interesting objects in first 7)

2nd M1 for 0.9^7

A1 awrt 0.478

(d) 1st M1 attempt to find correct probability, may find as sum of gp

2nd M1 dependent on 1st M1, using their $\mathrm{P}(X < 7)$

A1 awrt 0.291

(e) 1st M1 attempt to find mean and variance

1st A1 both correct

2nd M1 Use of central limit theorem with their mean and variance

2nd A1 (from calculator or otherwise) awrt 0.173

5)			
(a)	$\mathrm{P}(Y=1)=\mathrm{P}(\text{two }+1\text{ moves and one -1 move})=3\times\left(\frac{2}{3}\right)^2\times\frac{1}{3}$ $=\frac{12}{27}=\frac{4}{9}$	M1 A1cso	2.1 1.1b
			(2)
(b)	Possible values of Y = {-3, -1, 1, 3}	B1	3.1b
	Probability distribution of Y <table><tr><td>y</td><td>-3</td><td>-1</td><td>1</td><td>3</td></tr><tr><td>P(Y = y)</td><td>$\frac{1}{27}$</td><td>$\frac{6}{27}$</td><td>$\frac{12}{27}$</td><td>$\frac{8}{27}$</td></tr></table>	B1 B1	1.1b 1.1b
	$\mathrm{E}(Y)=-3\times\frac{1}{27}+-1\times\frac{6}{27}+1\times\frac{12}{27}+3\times\frac{8}{27}=1$ $\mathrm{E}(Y^2)=(-3)^2\times\frac{1}{27}+(-1)^2\times\frac{6}{27}+1^2\times\frac{12}{27}+3^2\times\frac{8}{27}=\frac{99}{27}=\frac{11}{3}$ $\mathrm{Var}(Y)=\frac{11}{3}-1^2=\frac{8}{3}$	M1 dM1 A1cao	3.4 1.1b 1.1b
			(6)
(c)	$D=\lvert Y\rvert$, so all values of D are positive but probability structure the same $\therefore\mathrm{E}(D)>\mathrm{E}(Y)$ and $\mathrm{E}(D^2)=\mathrm{E}(Y^2)$ $\therefore\mathrm{Var}(D)<\mathrm{Var}(Y)$	B1 M1 A1	2.1 2.4 2.2a
			(3)
			(11)

Notes

(a) M1 clear method, could be just calculation shown

A1 cso

(b) 1st B1 complete sample space as list or in probability distribution

2nd B1 correct probabilities for y = -3 and y = 3

3rd B1 all correct

1st M1 correct working using their probabilities, so long as $\sum \mathrm{P}(Y=y)=1$

2nd M1 dependent on 1st M1, for correct method for $\mathrm{E}(Y^2)$

A1 $\frac{8}{3}$ oe cao

Alternate Method

1st B1 Let rv X = number of +1 moves, $X \sim \mathrm{Bin}\left(3, \frac{2}{3}\right)$

2nd B1 $Y = X - (3 - X)$

3rd B1 $Y = 2X - 3$

1st M1 $\mathrm{Var}(Y) = 2^2\mathrm{Var}(X)$

2nd dM1 $\mathrm{Var}(Y) = 2^2 \times 3 \times \frac{2}{3} \times \frac{1}{3}$

A1 $\mathrm{Var}(Y) = \frac{8}{3}$ oe cao

(c) B1 use of $D = |Y|$

M1 correct comparison of both sets of expectations, or valid attempt to calculate $\mathrm{Var}(D)$

A1 full explanation or $\mathrm{Var}(D) = \frac{8}{9}$ and correct conclusion

<table>
<tr><td>6)</td><td></td><td></td><td></td></tr>
<tr><td>(a)</td><td>H_0: Po(2.4) is a suitable model for the number of whale sightings per trip
H_1: Po(2.4) is not a suitable model for the number of whale sightings per trip</td><td>B1</td><td>2.5</td></tr>
<tr><td></td><td>Assuming $\mathrm{H_0}$ is true , expected values are $\mathrm{E}_i = 60 \times \mathrm{e}^{-2.4} \times \frac{2.4^i}{i!}$</td><td>M1</td><td>3.4</td></tr>
<tr><td></td><td><table>
<tr><td>No of sightings</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>≥ 6</td></tr>
<tr><td>Expected frequency</td><td>5.44…</td><td>13.06…</td><td>15.67…</td><td>12.54…</td><td>7.52…</td><td>3.61…</td><td>2.14…</td></tr>
</table></td><td>A1</td><td>1.1b</td></tr>
<tr><td></td><td><table>
<tr><td>No of sightings</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>≥ 5</td></tr>
<tr><td>Observed frequency</td><td>4</td><td>18</td><td>20</td><td>8</td><td>6</td><td>4</td></tr>
<tr><td>Expected frequency</td><td>5.44…</td><td>13.06…</td><td>15.67…</td><td>12.54…</td><td>7.52…</td><td>5.75…</td></tr>
</table>
Combining groups</td><td>M1</td><td>2.1</td></tr>
<tr><td></td><td>$\nu = 6 - 1 = 5$</td><td>B1ft</td><td>1.1b</td></tr>
<tr><td></td><td>Critical value, $\chi^2(0.10) = 9.236$</td><td>B1ft</td><td>1.1a</td></tr>
<tr><td></td><td>Test statistic = $\frac{(4-5.44)^2}{5.44} + \frac{(18-13.06)^2}{13.06} + \frac{(20-15.67)^2}{15.67} + \frac{(8-12.54)^2}{12.54} + \frac{(6-7.52)^2}{7.52} + \frac{(4-5.75)^2}{5.75} =$
$0.3825.. + 1.8655.. + 1.1926.. + 1.6441.. + 0.3088.. + 0.5337.. = 5.927…$
awrt 5.9</td><td>M1

A1</td><td>1.1b

1.1b</td></tr>
<tr><td></td><td>Test statistic is not in the critical region, so no significant evidence against the owner's claim.</td><td>A1cso</td><td>3.5a</td></tr>
<tr><td></td><td></td><td></td><td>(9)</td></tr>
<tr><td>(b)</td><td>Should be recorded as a single sighting

Since for a Poisson model events occur singly</td><td>B1

B1</td><td>2.4

3.5b</td></tr>
<tr><td></td><td></td><td></td><td>(2)</td></tr>
<tr><td></td><td></td><td></td><td>(11)</td></tr>
</table>

Notes

(a) 1st B1 both hypotheses, must include the 2.4

1st M1 correct use of Poisson distribution, may be implied

1st A1 all correct to 1dp

2nd M1 combining groups

2nd B1 ft their groups

3rd B1 ft their degrees of freedom

3rd M1 at least 2 terms shown (give if test value correct and no working shown)

2nd A1 awrt 5.9

3rd A1 correct conclusion in context, cso

(b) 1st B1 explanation of how to record the sighting

2nd B1 stating necessary assumption for Poisson distribution

7)			
(a)	P(Type I error) = $\mathrm{P}(X \geq c) \approx 0.05$	M1	3.1a
	P(Type I error) = $\sum_{i=c}^{i=\infty} 0.4 \times 0.6^{i-1} \approx 0.05$	M1	2.1
	$\therefore \frac{0.4 \times 0.6^{c-1}}{1-0.6} \approx 0.05$ $0.6^{c-1} = 0.05$	M1	1.1b
	$c - 1 = \frac{\log 0.05}{\log 0.6} \approx 5.86$	M1	1.1b
	$c \approx 6.86$	A1	1.1b
	$c = 6$, P(Type I error) = 0.07776, $c = 7$, P(Type I error) = 0.046656 So critical region is $X \geq 7$ (closest to 0.05)	A1cso	2.2a
			(6)
(b)	$\mathrm{P}(\text{Type II error}) = \mathrm{P}(X < c \mid p = \beta) = \sum_{r=1}^{r=c-1} \beta(1-\beta)^{r-1}$	M1	2.1
	$= \frac{\beta(1-(1-\beta)^6)}{1-(1-\beta)} = 0.5$	A1ft	1.1b
	$1-(1-\beta)^6 = 0.5$ $(1-\beta)^6 = 0.5$ $\beta = 1 - \sqrt[6]{0.5} = 0.1091 \ldots$ awrt 0.109	M1 A1	1.1b 1.1b
			(4)
			(10)

Notes

(a) 1st M1 correct definition of Type I error, may be implied

2nd M1 attempt at correct geometric series

3rd M1 use of infinite GP formula,

Alt method 3rd M1 accept $\mathrm{P}(X \geq c) = 0.6^{c-1} = 0.05$

4th M1 solved using logs

1st A1 awrt 6.9

2nd A1 cso all correct with reason for choosing 7 not 6

(b) 1st M1 correct series for $\mathrm{P}(\text{Type II error})$

1st A1 use of GP summation, ft their c (must be positive integer),

or $\mathrm{P}(X < 7) = (1-(1-\beta)^6) = 0.5$

2nd M1 solving for β

2nd A1 awrt 0.109

8)			
(a)	Geometric distribution	B1	1.2
			(1)
(b)	If $\text{Var}(X) = \left(\text{E}(X)\right)^2$, then $\frac{2(1-p)}{p^2} = \left(\frac{2}{p}\right)^2$, $\therefore 1 - p = 2$	M1	2.1
	$p = -1$	A1	1.1b
	where p is a probability, so impossible	A1	2.4
			(3)
(c)	$$20 = \frac{1-p}{p^2}$$	M1	3.1b
	$$20p^2 + p - 1 = 0$$	A1	1.1b
	$p = 0.2$ (or $p = -0.25$)		
	$$p = 0.2$$	A1	2.2a
			(3)
(d)	$\text{P}(X = 5 \mid Y = 2) = \text{P}(Y_2 = 3)$ $(= 0.6^2 \times 0.4)$	M1	3.1b
	$= 0.144$	A1cao	1.1b
			(2)
			(9)

Notes

(b) M1 setting up equation using correct formulae, can have *r* rather than 2

1st A1 solving to give *p*=-1 (or *p* is negative for general *r* > 1)

2nd A1 correct reason

(c) M1 correct equation

1st A1 rearranging to give correct quadratic

2nd A1 for $p = 0.2$ only, A0 if $p = -0.25$ also given and not rejected

(d) M1 correct method, may be implied by working or correct answer

A1 0.144 only

RJ Freeman (01/04/2019)