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Shadow Set 1

**1** The number of faulty lightbulbs produced by a machine follows a Poisson

distribution.

Past information shows that there will be a faulty bulb at an average rate of 3 every 20 bulbs.

Using this information,

(*a*) (i) find the probability that exactly 8 light bulbs will be faulty in a random selection of 100 bulbs.

**(2)**

(ii) find the probability that at least 10 bulbs will be faulty in a random selection of 100 bulbs.

**(2)**

Alterations are carried out on the machines that produce the bulbs. Following these alterations, the number of lightbulbs which are faulty over a randomly selected 60 bulbs is found to be 5

(*b*) Test, at the 5% significance level, whether or not there is evidence of a decrease in

the rate of faulty bulbs. State your hypotheses clearly.

**(4)**

A further randomly selected 60 bulbs are tested and the hypothesis test is repeated.

Given that the true rate of faulty bulbs is now 1 every 12 bulbs,

(*c*) calculate the probability of a Type II error.

**(5)**

**(Total for Question 1 is 13 marks)**

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**2** The discrete random variables *W*, *X* and *Y* are distributed as follows

*W* ~ B(10, 0.3) *X* ~ Po(3) *Y* ~ Po(6)

*(a*) Explain whether or not Po(3) would be a good approximation to B(8, 0.3)

**(1)**

(*b*) State the assumption required for *X* + *Y* to be distributed as Po(9)

**(1)**

Given the assumption in part (b) holds,

(*c*) find P(*X* + *Y* < Var(*W*))

**(2)**

**(Total for Question 2 is 4 marks)**

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**3** Beth and Amal are playing a game.

They roll a fair dice.

If Beth rolls a 2 she wins, otherwise Amal rolls the dice. If he rolls a 2 he wins the game. Otherwise they repeat this process until one of them rolls a 2.

(*a*) Find the probability that Beth rolls a 2 on her third go.

**(2)**

(*b*) Find the probability that a 2 is rolled on or after Amal’s fourth go.

**(2)**

(*c*) Find the mean and standard deviation of the number of rolls before a 2 is seen.

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**(2)**

(*d*) Find the probability that Beth wins the game.

**(3)**

**(Total for Question 3 is 9 marks)**

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**4** The discrete random variable *X* has the following probability distribution.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  |  |  |  |  |
|  |  |  |  |  |

(*a*) Find Var(*X*)

**(3)**

The discrete random variable *Y* is defined in terms of the discrete random variable *X*

When *X* is negative, 

When *X* is positive, 

(*b*) Find

**(3)**

(*c*) Find

**(2)**

**(Total for Question 4 is 8 marks)**

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**5**

A total of 120 random samples of 7 items are selected from a production line in a factory and the number of defective items in each sample is recorded. The results are summarised in the table below.

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Number of defective items | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| Observed number of samples | 2 | 6 | 11 | 19 | 25 | 32 | 16 | 9 |

(*a*) Write down two reasons why a binomial distribution may be a suitable model.

**(2)**

(*b*) Show that the probability of a randomly selected item from this sample being faulty
is 0.6.

**(2)**

The research station used a binomial distribution with probability 0.6 of a faulty item. The expected frequencies were calculated to 2 decimal places. The results are as follows:

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Number of defective items | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| Expected number of samples | 0.20 | 2.06 | *s* | 23.22 | *t* | 31.35 | 15.68 | 3.36 |

(*c*) Find the value of *s* and the value of *t*.

**(2)**

(*d*) Stating your hypotheses clearly, test, at the 1% level of significance, whether or not the data can be modelled by a binomial distribution. State your hypothesis clearly.

**(8)**

**(Total for question 5 is 14 marks)**

**6** A discrete random variable *X* has probability generating function given by

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where *a* and *b* are positive constants.

(*a*) Write down the value of 

**(1)**

Given that E(X)= 4

(*b*) (i) find $a$ and $b$

**(6)**

(ii) find Var(*X*)

**(5)**

**(Total for Question 6 is 12 marks)**

**7** A five-sided spinner has sides labelled 1, 2, 3, 4 and 5

The random variable *S* represents the score when the spinner is spun.

Helen rolls the spinner 50 times and the mean score,  , is calculated.

Assuming the spinner is fair and using a suitable approximation,

(*a*) find, to 3 significant figures, the value of *k* such that P(< *k*) = 0.05

**(8)**

(*b*) Explain the relevance of the Central Limit Theorem in part (a).

**(2)**

Helen considers the following hypotheses:

H0: The spinner is fair

H1: The spinner is not fair

If < 2.5 or > 3.1, then H0 will be rejected.

Given that the true distribution of *S* has mean 3.2 and variance 2.5

(*c*) find the power of this test.

**(3)**

(*d*) Describe what would happen to the power of this test if Helen were to increase the

number of spins.

Give a reason for your answer.

**(2)**

**(Total for Question 7 is 15 marks)**

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