GCE A level Further Mathematics (9FM0) – Shadow Paper (Set 1)
9FM0-3B Further Statistics 1

October 2020 Shadow Paper mark scheme

Please note that this mark scheme is not the one used by examiners for making scripts. It is intended more as a guide, indicating where marks are given for correct answers. As such, it may not show follow-through marks (marks that are awarded despite errors being made) or special cases.

It should also be noted that for many questions, there may be alternative methods of finding correct solutions that are not shown here – they will be covered in the formal mark scheme from the original paper.

This document is intended for guidance only and may differ significantly from the examiners' final mark scheme for the original paper which was published in December 2020.

Guidance on the use of codes within this document
M1 – method mark. This mark is generally given for an appropriate method in the context of the question. This mark is given for showing your working and may be awarded even if working is incorrect. A1 – accuracy mark. This mark is generally given for a correct answer following correct working. B1 – working mark. This mark is usually given when working and the answer cannot easily be separated. Some questions require all working to be shown; in such questions, no marks will be given for an answer with no working (even if it is a correct answer).

Question	Scheme	Marks
1(a)(i)	$X \sim \text{Po}(15)$	B1
	$\text{P}(X = 8) = 0.01944\ldots$ awrt **0.0194**	B1
		(2)
(ii)	$\text{P}(X \geq 10) = 1 - \text{P}(X \leq 9)$ [$= 1 - 0.06985\ldots$]	M1
	$= 0.93015\ldots$ awrt **0.930**	A1
		(2)
(b)	$\text{H}_0: \lambda = 3 \quad [\mu = 9]$ $\text{H}_1: \lambda < 3 \quad [\mu < 9]$	B1
	$\text{P}(Y \leq 5 \mid Y \sim \text{Po}(9)) = 0.11569\ldots$ awrt **0.116**	B1
	Not significant / Do not reject H_0 / 5 is not in the CR	M1
	There is not sufficient evidence to suggest a decrease/change in the rate of faulty light bulbs being produced.	A1
		(4)
(c)	Use of Po(9) to attempt critical region	M1
	Critical region is $Y \leq 3$/ H_0 is not rejected when $Y \geq 4$	A1
	True distribution is $W \sim \text{Po}(5)$	B1
	$\text{P}(W \geq 4 \mid W \sim \text{Po}(5)) = 1 - \text{P}(W \leq 3)$ [$= 1 - 0.26503\ldots$]	M1
	$=0.73497\ldots$ awrt **0.735**	A1
		(5)
		(13 marks)

Question	Scheme	Marks
2(a)	requires large *n*/small *p* so not a good approximation	B1
		(1)
(b)	*X* and *Y* must be independent	B1
		(1)
(c)	$P(X + Y < 1.68)$ from Po(9) $[P(X + Y \leq 1)]$	M1
	$= 0.00123\ldots$ awrt **<u>0.0123</u>**	A1
		(2)
		(4 marks)

Question	Scheme	Marks
3(a)	$[X \sim \text{Geo}\left(\frac{1}{6}\right)$ Beth's 3rd roll is the 5th roll overall] $P(X = 5) = \left(\frac{5}{6}\right)^4\left(\frac{1}{6}\right)$	M1
	$= 0.0803755\ldots$ awrt **<u>0.0804</u>**	A1
		(2)
(b)	$P(X \geq 8)\left[= \left(1 - \frac{1}{6}\right)^7\right]$	M1
	$= 0.27908\ldots$ awrt **<u>0.279</u>**	A1
		(2)
(c)	Mean = 6	B1
	Standard deviation $\left[= \sqrt{\frac{1-\frac{1}{6}}{\left(\frac{1}{6}\right)^2}}\right] = \sqrt{30}$ or awrt **<u>5.48</u>**	B1
		(2)
(d)	$P(\text{Beth wins}) = \frac{1}{6} + \left(\frac{5}{6}\right)^2\left(\frac{1}{6}\right) + \left(\frac{5}{6}\right)^4\left(\frac{1}{6}\right) + \ldots$	M1
	Infinite geometric series $= \frac{\frac{1}{6}}{1-\left(\frac{5}{6}\right)^2}$ (oe)	M1
	$= \frac{6}{11}$	A1
		(3)
		(9 marks)

Question	Scheme	Marks
4(a)	$\left[\mathrm{E}(X)=\right](-3)\times\frac{2}{5}+(-1)\times\frac{1}{3}+(3)\times\frac{1}{15}+(9)\times\frac{1}{5}\left[=\frac{7}{15}\right]$ (oe)	M1
	$\left[\mathrm{E}(X^2)=\right](-3)^2\times\frac{2}{5}+(-1)^2\times\frac{1}{3}+(3)^2\times\frac{1}{15}+(9)^2\times\frac{1}{5}\left[=\frac{311}{15}\right]$ (oe)	M1
	$\mathrm{Var}(X)=\mathrm{E}(X^2)-\left[\mathrm{E}(X)^2\right]=\frac{311}{15}-\left(\frac{7}{15}\right)^2=\frac{4616}{225}$ =**<u>20.5..</u>**	A1
		(3)
(b)	<table><tr><td>x</td><td>(–3)</td><td>–1</td><td>3</td><td>9</td></tr><tr><td>y</td><td>18</td><td>2</td><td>7</td><td>13</td></tr><tr><td>p</td><td>$\frac{2}{5}$</td><td>$\frac{1}{3}$</td><td>$\frac{1}{15}$</td><td>$\frac{1}{5}$</td></tr></table>	M1
	$P(Y\geq 8)=P(X=-3)+P(X=4)\left[=\frac{2}{5}+\frac{1}{5}\right]$	M1
	$=\frac{3}{5}$	A1
		(3)
(c)	$E(XY)=(-3)(18)\frac{2}{5}+(-1)(2)\times\frac{1}{3}+(3)(7)\times\frac{1}{15}+(9)(13)\times\frac{1}{5}$	M1
	$=\frac{38}{15}$ oe	A1
		(2)
		(8 marks)

Question	Scheme	Marks
5(a)	The items are **independent /** There are a **fixed number** of items in a sample/ There are only	
	two outcomes to the item being faulty – either it is either faulty or not / The **probability**	
	of a faulty item is **constant**	B1 B1
		(2)
	$\frac{(0\times2)+(1\times6)+(2\times11)+(3\times19)+(4\times25)+(5\times32)+(6\times16)+(7\times9)}{120\times7}=\frac{504}{840}$	M1
(b)	= 0.6 **	A1cso
		(2)
(c)	$p = 0.6 \quad q = 0.4$	
	$s = 120\times\ 21q^5p^2\ = 120\times21$ x 0.4^5 x $0.6^2 = 9.29$	B1
	$t =\ 120\times35q^3p^4\ = 120\times35$ x 0.4^3 x $0.6^4 = 34.84$	B1
		(2)
	H_0 : A binomial distribution is a suitable model. H_1: A binomial distribution is not a suitable model.	B1

Observed number of samples	19	19	25	32	25
Expected number of samples	11.55	23.22	34.84	31.35	19.04
$\frac{(O-E)^2}{E}$	4.81	0.77	2.78	0.013	1.87
$\frac{O^2}{E}$	31.26	15.55	17.94	32.66	32.83

M2

	$\nu = 5 - 2 = 3$	B1ft
	Critical value for $\chi^2 =\ 11.345$	B1ft
	$\sum\frac{(O-E)^2}{E} = 10.23$ or $\sum\frac{O^2}{E} - N = 130.23 - 120 = 10.23$ Answers which round 10.2-10.3 acceptable	M1A1
	10.23 < 11.345 therefore do not reject H_0 A binomial is a suitable model.	A1

Question	Scheme	Marks
6(a)	P(X = 3) = **0**	B1
		(1)
(b)(i)	$G_X(1) = 1$ $2a + b = 1$	B1
	$\mu = G'(1)$	M1
	$a + 6b = 4$	A1
	$2a + b = 1$ $2a + 12b = 8$	M1
	$a = \frac{2}{11}, \quad b = \frac{7}{11}$	A2
		(6)
(ii)	$G' = a + 6bt^5$	M1
	$G'' = 30bt^4$	M1
	$G''(1) = 30b$	A1
	$30b + 4 - 16$ $= \frac{210}{11} + 4 - 16 =$	M1
	$= \frac{78}{11}$ oe	A1
		(5)
		(12 marks)

Qu.	Scheme	Marks
7(a)	Realising S has a discrete uniform distribution over $\{1,\ldots 5\}$	M1
	$\text{E}(S)=1\times\frac{1}{5}+2\times\frac{1}{5}+3\times\frac{1}{5}+4\times\frac{1}{5}+5\times\frac{1}{5}$	M1
	$\text{Var(S)}=1^2\times\frac{1}{5}+2^2\times\frac{1}{5}+3^2\times\frac{1}{5}+4^2\times\frac{1}{5}+5^2\times\frac{1}{5}-3^2$	M1
	$\text{E}(S)=3$ and $\text{Var}(S)=2$	A1
	$\bar{S}\sim\text{N}(3,....)$	M1
	$\text{Var}(\bar{S})=\dfrac{2}{50}=0.04$, $\bar{S}\sim\text{N}(3,0.04)$	A1
	$\text{P}(\bar{S}<k)=0.05\rightarrow\dfrac{k-3}{\sqrt{0.04}}=-1.6449$	M1
	$k=2.67\ldots$ awrt <u>**2.67**</u>	A1
		(8)
(b)	CLT applies since the sample size is large	B1
	CLT states that the sample mean/ $\bar{S}$ is (approximately) normally distributed	B1
		(2)
(c)	True $\bar{S}\sim\text{N}(3.2,\frac{2.5}{50})$	M1
	$\text{P}(\bar{S}<3.5)+\text{P}(\bar{S}>3.1)$ or $1-\text{P}(2.5<\bar{S}<3.1)$	M1
	Power = awrt <u>**0.3265**</u>	A1
		(3)
(d)	E.g. The increase in sample size would decrease the variance of $\bar{S}$ [leading to an increase in $\text{P}(\bar{S}>3.1)$ and the decrease in $\text{P}(\bar{S}<2.5)$ would be negligible]	B1
	So the power would increase.	dB1
		(2)
		(15 marks)